Fluid Dynamics for Astrophysics
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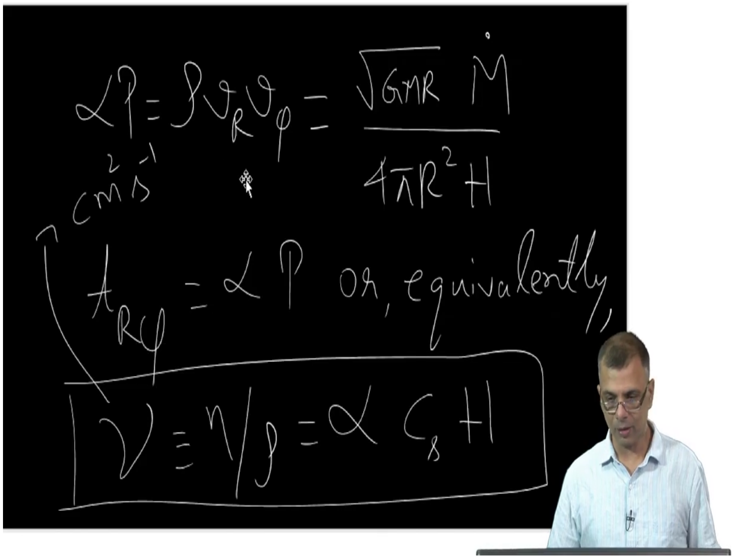
Lecture - 41
Disk accretion: Viscous dissipation and the energy equation, two-temperature criterion



So, this is one of the main things we derived when we were discussing last and you might wonder where this V of R came; alpha P is of course, just t R phi and the main thing we did here was to of course, the V phi we simply substituted by square root of G M over R.

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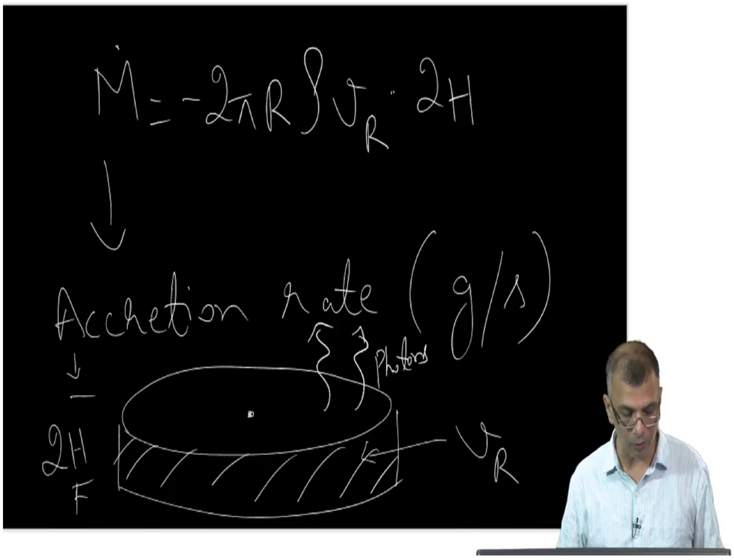

But, this V of R if you realize was simply gotten from the accretion rate which we have written down earlier this is essentially the mass conservation equation, right and this was written as you know minus 2 pi R times rho times V R times 2H, right. So, this is the accretion rate in grams per second, right.

And, so, this simply is saying that you have an accretion disk like. So, right around a central object and stuff is moving in this way yeah with the radial velocity V R slowly sinking in of course, V R is much much smaller than V phi. It is slowly sinking in and it is sinking in through this side surface, yeah. The area of which is 2 pi r times 2H, 2H would be this the height of the disc.

So, 2 pi R times 2H times rho times V R gives you grams per second. So, that is all this equation is telling you. And this was used in eliminating V sub R and that is how you get this equation. So, this is the azimuthal momentum equation and that pretty much you know finishes our discussion of the momentum equation.

And, the main thing the reason it is not so much to as to show you all the equations and so on and so forth one of the main reasons we discussed this was to introduce the famous the Shakura Sunyaev parameterization for the R phi component of the viscous stress tensor and that is this. And, this is where the all important viscosity parameter alpha is introduced and alpha is always between 0 1 and we have discussed what this means right, ok.

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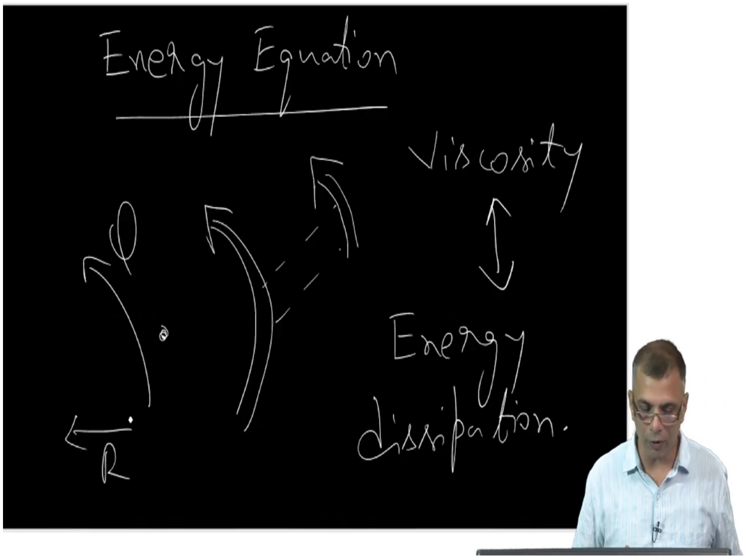

So, now we are now ready to discuss the energy equation. Now, that we are done with mass conservation which is essentially this, right all the three components of the momentum conservation and now, we are ready to discuss the energy equation. And, before doing that though let us sort of step back briefly to try to understand what we are expecting, right.

So, when we were looking at the top view of an accretion disk, you remember this is the picture we had. We had a picture where, the inner parts of the accretion disk that is a central object maybe I should erase that and I should put the central object somewhere here, ok.

So, the inner parts of the accretion disk are moving faster than the outer parts, right and the fact that you have this shear kind of motion points to the fact that there are you know little rubber bands connecting these layers opposing the sliding of this layer over this layer, right.

But, we also know that viscosity is a dissipative thing it appears in the momentum equation yeah, but we also know that viscosity causes energy dissipation. Viscosity and energy dissipation go hand in hand and why are we interested in energy dissipation well that is the whole point, is it not? Accretion disks that look like this, they can be accreting anything can be happening, right.

We are not in a position to go to the active galactic nucleus and see what is going on. What we observe are photons and photons most likely from the accretion disk. Photons why are there photons? Because, accretion disc is heated up. Who is heating it? In other words, who is dissipating the energy? The energy is being dissipated due to viscosity. Hence the importance of the energy equation, ok.

So, because the only way we can observe the accretion disc is via the photons that are being emitted from the accretion disk, very very indirect of course.

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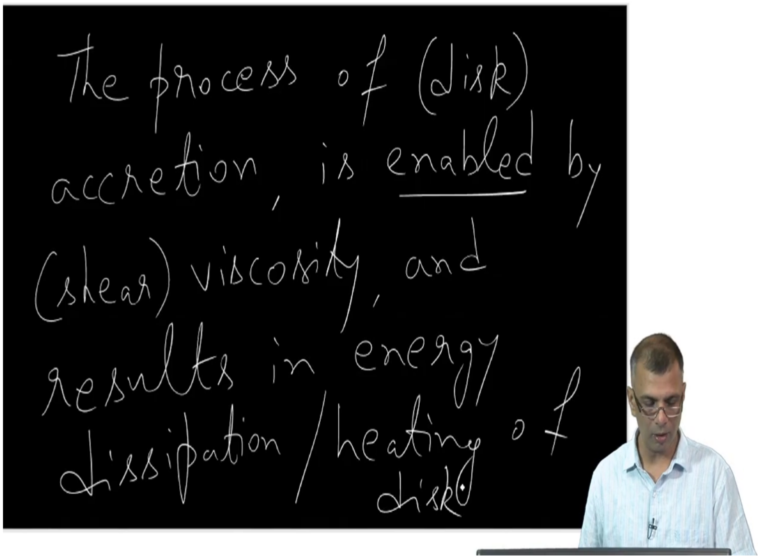

So, to speak the process of accretion process of in this case we are discussing disk accretion is enabled by shear viscosity. Hence and we saw how shear viscosity is parameterized in terms of the alpha parameter, right. So, it is enabled by the shear viscosity ok and results in energy dissipation heating of the disk heating of this disk of this entire disk here.

The viscosity which enables the disk accretion to proceed that results in heating of the disk, right and the disc gets heated, its black body temperature rises and it emits electromagnetic radiation which enables us to see the accretion disk. So, therefore, let us now try to see you know what is the rate of dissipation of energy. This is the question that we are trying to address now, ok and where does this viscosity business come in? right.

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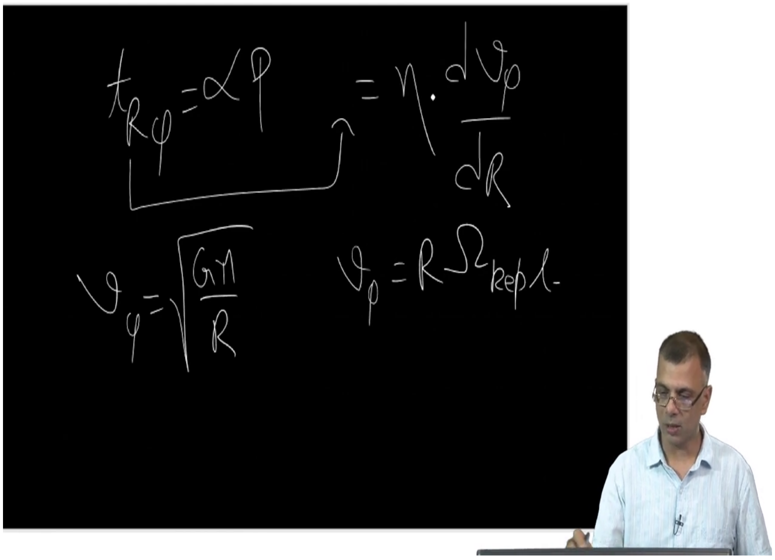

So, let us now go back. You remember that we were writing t R phi is equal to alpha P this is one of the main you know. This is the famous Shakura Sunyaev parameterization and by definition this t R phi is something like eta times d V phi d R, right because this is the definition of the viscous of this R phi component of the stress tensor, right.

So, it is like this there is a d V phi this would be the phi direction. So, this would be the phi direction and this would be the R direction and so, there is a R directed gradient of the phi component of the velocity. So, R directed gradient of the phi component of the velocity and that is exactly how this is you know written. So, this is the basic definition and we parameterized it as alpha P, is it not?

And, since V phi is taken to be the Keplerian value, right and the V phi can also be written as R times omega Keplerian ok. This equality this equals this.

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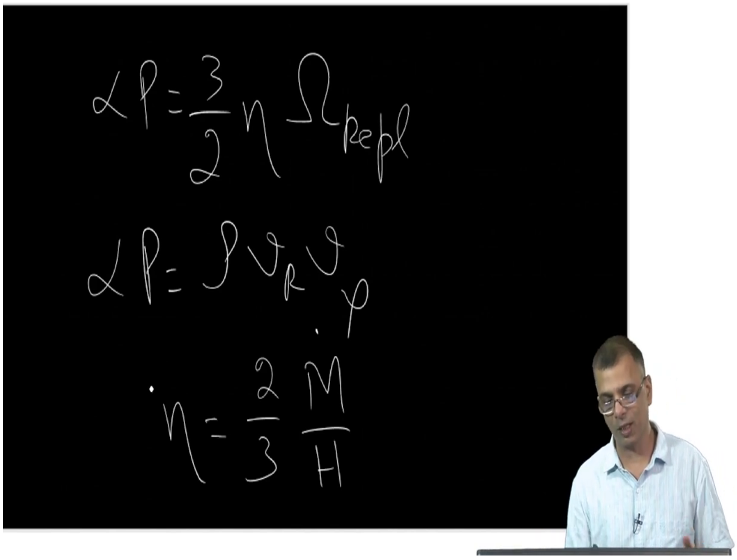

This can be written as alpha P is equal to three halves eta omega Keplerian ok. So, this is one very interesting thing and then we also saw that the alpha P we also saw earlier that the alpha P was equal to rho V R V phi, you remember this. You remember this from here.

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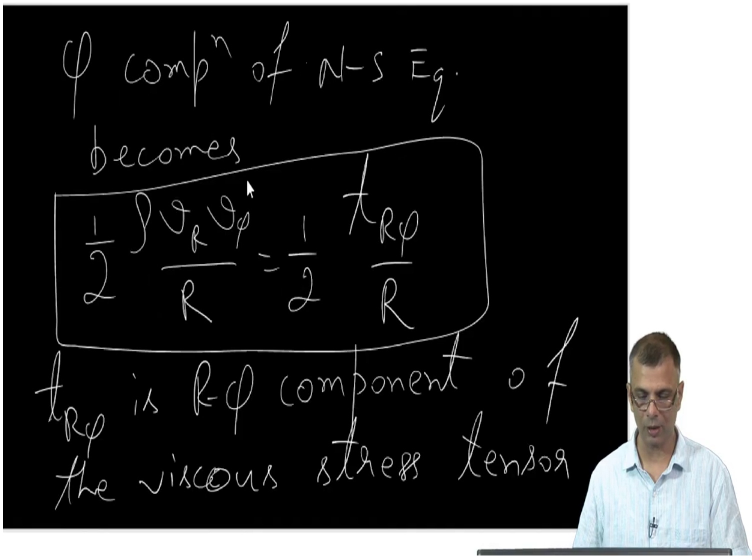

This came from the basic momentum equation like this.

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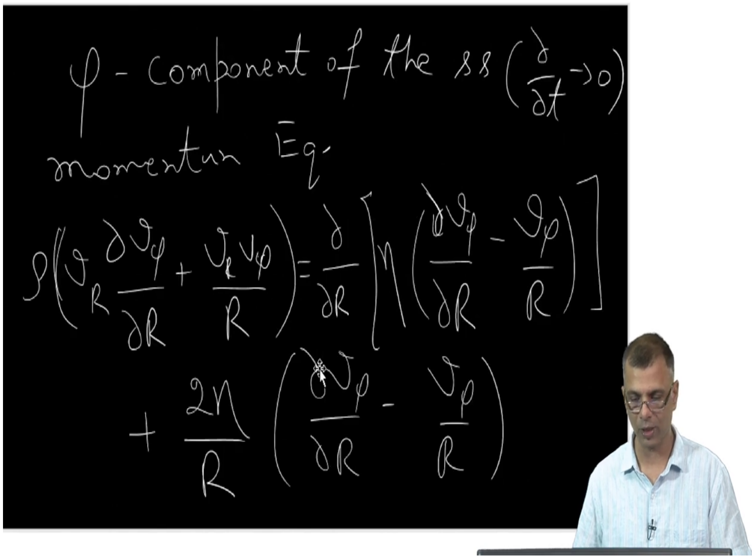

Where did this come from? This simply came from the phi component of the this just simply came from here, right. And, this was simplified using this we simplified it to be this. So, essentially rho V R V phi is nothing, but t R phi which in turn is alpha P. So, we are back to this. So, we equate this to this we equate rho V R V phi to this guy, right.

So, and this helps us get a very interesting result ok and again substituting for the radial velocity in terms of the accretion rate like this using the accretion rate to get rid of this V R radial velocity, we get the following very interesting expression very interesting quality for the viscosity parameter. This is equal to two thirds M dot over H. Why is this interesting?

This is because you see the M dot is a global thing. It is essentially the amount of grams per second that the central object is about able to swallow, right and H is the thickness of the accretion disk. These are all global things and this is being related to the microphysical properties of the accreting matter which is embodied by eta ok.

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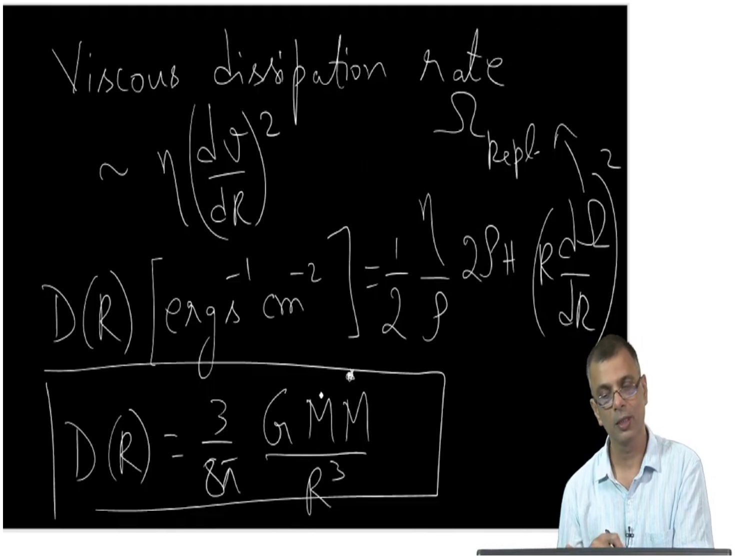

So, and if you remember the dissipation rate the viscous dissipation rate, if you go back to our discussion of the energy equation the viscous dissipation rate is given by something like this. In particular, if we write down an expression for D of R which is given by ergs per second per centimeter squared ok.

This would essentially represent the number of ergs per second per centimeter squared the energy flux coming out from the face of the accretion disk which is essentially you know what we would observe. Not directly the energy flux that is dissipated due to you know dissipation, but the energy flux which enables the heating and therefore, the photons that are indirectly you know result a product of this kind of viscous dissipation, right.

So, this would be given by using an expression that is very analogous to this would be one half eta over rho 2 rho H R d omega d R quantity squared and this omega is essentially omega Keplerian.

And, if you substitute these rows cancel out and if you substitute for this omega Keplerian and if you substitute for this eta using this expression you get the following nice expression, for the number of ergs per second per centimeter squared that is potentially available via viscous dissipation ok available to heat the disc and the heated disc of course, radiates ok.

And, so, that would be essentially D of R is equal to 3 over 8 pi. This is a fairly simple algebra. I urge you to work through this I have given you all the steps. This is very this is a very nice expression; nice because it does not depend upon the microphysics, this expression does not depend upon the microphysics of the viscosity at all, ok.

You give me a certain black hole or a certain compact object with this mass, right which is sucking in matter at a certain rate M dot. And, at a particular radius this is the amount of ergs per second per centimeter squared that is available via viscous dissipation and this expression is independent of all the microphysics. It is very interesting, ok.

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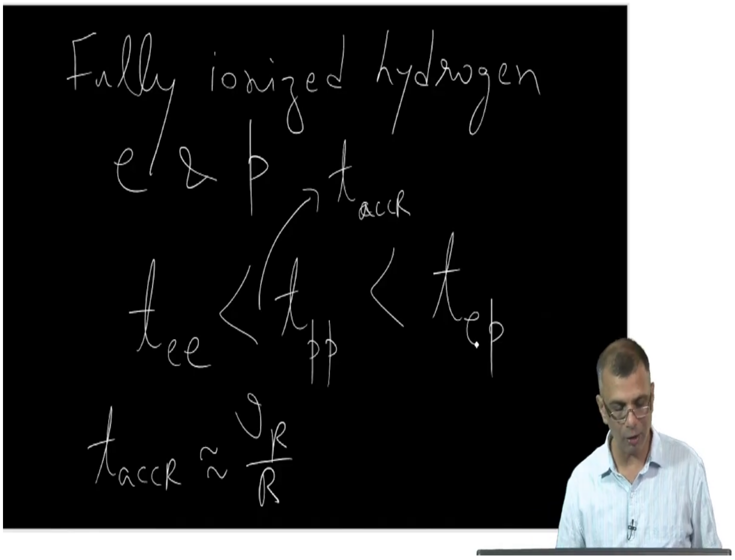

So, this is essentially the upshot of the energy equation ok. So, having now talked about the basics of disk accretion, the basics of quasi Keplerian disk accretion let us now move to slightly more detailed topics and one of the detailed topics is as follows.

M any times what happens is as matter falls in you see, there is not enough time for. So, let us simply talk about an electron proton plasma a fully ionized hydrogen. Let us neglect the presence of other heavy elements. So, only electrons and protons let us talk about this, right.

Now, turns out that the time scale it takes for electrons and electrons to equilibrate to a Maxwellian distribution via coulomb collisions ok, is most of the time it is shorter than the time scale it takes for protons and protons to equilibrate to a Maxwellian distribution in other words to have a well defined temperature also via coulomb collisions and this in turn is shorter than the time scale it takes for electrons and protons to equilibrate amongst each other via coulomb collisions ok.

Now, this is something that is universally that is always true ok, this is always true. The reason I am hinting at this is because sometimes what happens is there is enough time during the accretion process remember the accretion process is one where matter is swept in radially during the accretion process there is enough time for electrons and electrons to.

So, there would be t accretion right a t accretion which is given by some kind of V R over R right the radial velocity divided by the local radius at a given. So, this would be the time scale at which over which a stuff accretes ok. And, given this kind of time scale there might be enough time for the electrons in electrons to equilibrate in other words the t accretion is larger than the t ee.

So, by the time given parcel of gas moves in ok there is enough time for electrons and electrons to equilibrate among themselves via coulomb collision. So, the there is a well defined electron temperature. There might be enough time for the protons to protons to equilibrate among each other; maybe may not be that is not always the case ok this is not clear.

But, definitely there is not enough time for the electrons and protons to equilibrate amongst each other. So, the t accretion would be somewhere here ok which is saying that there is enough time for definite electron temperature to be established ok. And, if the t accretion is somewhere here ok, then there is not even enough time for the protons to collide amongst each other and establish a certain temperature equilibrium.

But, it is not clear whether the t accretion is on this side or this side of the equality, but definitely t accretion is smaller than t ep. In other words, there is not enough time.

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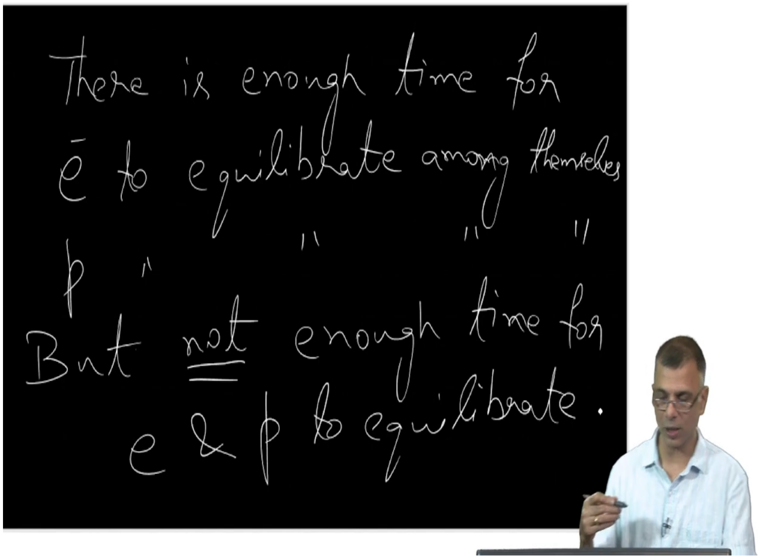

There is most of the time there is enough time for electrons among themselves for protons to equilibrate among themselves maybe, but not enough time, but not enough time for e and p to equilibrate.

In other words, there can be situations where the V R is such that the V R is such that there is not enough time for the electrons and protons to equilibrate among themselves. There is not enough time for interspecies equilibration. There is enough time for intra species equilibration, but not enough time for interspecies equilibration.

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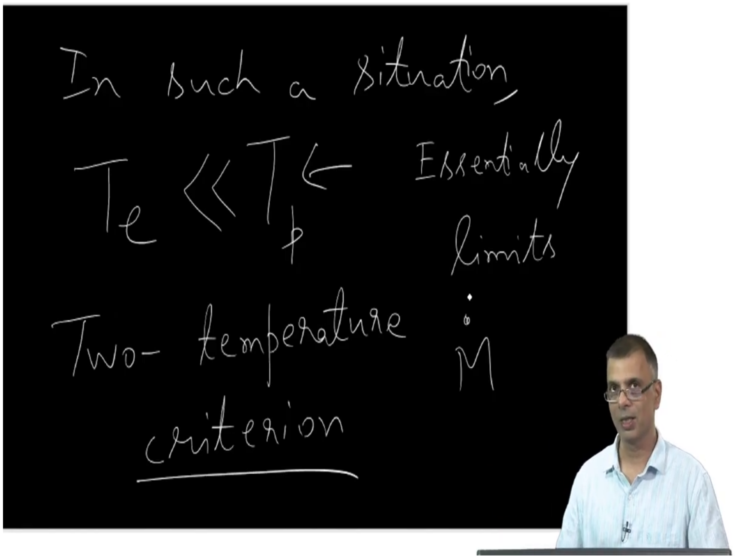

In such a situation, what happens is the electrons equilibrate to form an electron temperature and that is typically lower than the temperature attained by the protons ok and if there was enough time for the protons and electrons to equilibrate among themselves, then the T p would be the same as T e, but there is not enough time you see that is the whole point.

There is not enough time for electrons and protons to equilibrate among themselves. Therefore, many times this equality is very is very severe and this is what is called the two-temperature criterion ok. Many times this happens. So, the plasma that is accreting is not a single temperature plasma, the proton temperatures are typically much higher than the electron temperature.

Now, you might ask why are that proton temperatures typically higher that is because viscous dissipation is what is the primary agent that is responsible for you know heating the plasma, right. Now, viscous dissipation is a momentum based process ok and so, viscous dissipation essentially since it is a momentum based process it appears in the momentum equation we know this, right.

So, since viscous dissipation is a momentum based process what happens is it preferentially gives more energy to the heavier species which are the protons ok and protons typically are heavier and therefore, they radiate much slower and this is something that you will realize after you know after you study radiation processes, ok.

So, protons radiate much more slowly as opposed to electrons. So, first of all protons preferentially get the energy protons are the ones which are preferentially getting the energy; electrons are the ones which are not preferentially getting the energy. And, to make matters worse electrons are radiating faster ok.

So, naturally there is a natural you know the there is always already a tendency by which protons will tend to remain hotter. They are getting energy faster and they are not radiating they are not giving away energy fast either. So, they are keeping it bottled up ok.

More so than electrons therefore, protons do tend to be hotter than the electrons what is more to make matters worse there is not enough time for the protons and electrons to equilibrate amongst each other before a given parcel of plasma is swept into the black hole ok. This time scale is too small, this it is somewhere here.

So, this these three things conspire amongst each other to leave the protons much hotter than the electrons and this is what is called the two-temperature criterion and this essentially limits the M dot ok and this is what it all boils down to the M dot the accretion rate has an upper limit.

Now, we have already encountered an upper limit on the accretion rate and that is the Eddington accretion rate this is yet another upper limit on the accretion rate ok. If the accretion rate is any larger than this then you will always have a two-temperature plasma ok.

So, if you are so greedy that you want to accrete stuff really fast right then you are bound to end up with a two-temperature plasma and this is important because the two-temperature plasma is a collision less plasma ok. When the temperature of the protons is so high what happens is, the mean free path of the protons is much larger many times it becomes larger than the disk height. It becomes larger than this larger than the disk height.

And, so, when the mean free part of the protons is larger than the disk height there are problems, right. We know that the entire fluid picture breaks down and so, you have to take recourse to not collisions between protons and protons themselves, but collisions between protons and other kinds of scattering centers that could be you know maybe turbulent scattering centers or something like that in order to even meaningfully talk about proton temperature ok.

And, so, these are essentially the point I am trying to make is that although we have been talking about accretion disk viscosity and how important it is to the entire accretion process. And many times it becomes a slightly complicated situation especially for you know plasma such as a two-temperature plasma and in situations where the where the proton temperature is very high say in excess of about 10 raised to 11 to 10 raised to 12 Kelvin, ok.

And, in such situations the whole concept of accretion disk viscosity becomes considerably more complicated and our classical descriptions of viscosity no longer hold, but that is a subject that is you know somewhat advanced and we would not bother about it.

So, this pretty much closes our discussion of accretion disks and soon enough we will move on from this subject and we will start discussing other things other astrophysical applications of fluid dynamics such as supernovae and supernova shocks and so on so forth so, that is all for now.

Thank you.