Fluid Dynamics for Astrophysics
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Lecture – 04
Fluid kinematics

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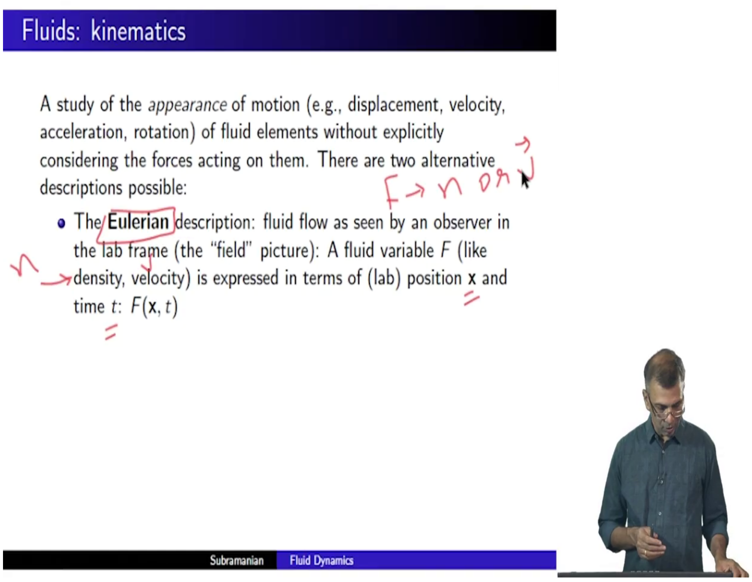

Hi. So, we are back. And we start talking about Fluid kinematics now, right. And so, as I mentioned the kinematics is a study of the appearance of motion of a fluid element by an appearance of motion I mean I know what is the velocity of this fluid element is it bending is it going straight is the fluid element accelerating and so on so forth.

So, we consider this without explicitly worrying about what is causing the motion, without explicitly worrying about the forces acting on them. And before proceeding, it is important to also you know focus a little bit about on this word fluid element this thing. What really does it mean? It is a bit of a fuzzy concept it is you know for instance if you consider this room, which is you know several feet long.

A fluid element would be something that is I do not know about 10 inches you know; a square of about 10 inches on one side that would be acceptable fluid element. It is small in comparison to the room, but still large enough so that the continuum hypothesis holds in other words in a cubic in a cube of about 10 inches on one side there are still many many molecules.

So, that it is meaningful to apply the continuum hypothesis to consider the fluid element as you know as a valid candidate for studying you know to as a continuum. So, it is not as if there are just two or three molecules inside this fluid element there are many many molecules. So, that the number of molecules is much much larger than one. So, it is meaningful to apply the continuum hypothesis to this fluid element, at the same time its small enough such that there can be many many fluid elements inside this room.

So, it is in between it is kind of a mesoscopic you know concept. So, it is important to understand this before going ahead and so, by way of kinematics there are two kinds of descriptions possible. One is called you know the Eulerian description and what this refers to is the fluid flow as seen by an observer in the lab frame. In other words you are standing outside and you are watching a fluid flow in front of you say it is in a pipe or so, so you are standing in the lab and you are watching the fluid flow by in a pipe.

So, you know a fluid variable such as you know density, velocity or something like that is like density or velocity say density n or velocity v. This is expressed in terms of the position x and time t. So, there are two variables that characterize any variable; this F can be density or F can be you know n or v right.

So, depending upon what it is it can acquire the appropriate character. If you are talking about density is a scalar field, if you are talking about velocity it is a vector field right. So, that is Eulerian description.

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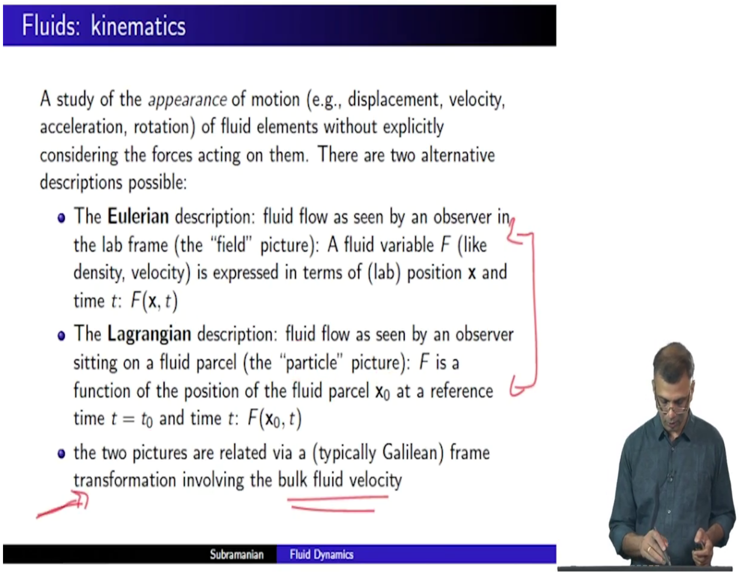

The other description is the, what is called the Lagrangian description and the Lagrangian description is the flow of fluid as seen by an observer sitting on a fluid parcel. So, you are not outside in the lab watching fluid flow by on the other hand you are actually sitting on the parcel of fluid.

Its equivalent to what some people call you know the particle kind of the particle picture. So, in this case F whatever it is F can be density or velocity, it is a function of the position of the fluid parcel x naught, at a reference time t naught and time t. There is really no; you think about this and this will become clear to you.

The main thing is to realize that you know one is a description the Lagrangian description is a description where you are the observer is sitting on a fluid parcel and watching changes whereas, the Eulerian description is one in which the observer is in a lab frame and watching changes in the fluid.

The reason we are introducing both of these is in depending upon the situation there are advantages or disadvantages to considering one or the other of the descriptions it is useful to be conversant with both. And so, these two descriptions the Lagrangian and Eulerian description these are related via a simple transformation, typically, a Galilean frame transformation.

There is really no need to you know consider relativity in this case, no need to bother about Lawrence transformation just as simple Galilean transformation typically does and so, it involves and this transformation involves the bulk fluid velocity and we will elaborate on what this is.

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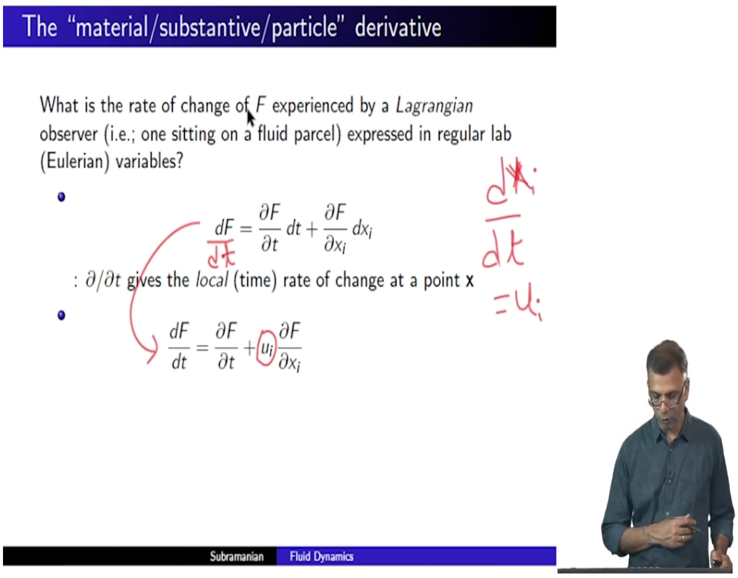

So, this sets the stage for our study of kinematics one of the; one of the main questions one can ask while talking about kinematics is, what is the rate of change of F, this F can be density or velocity or whatever some variable experienced by a Lagrangian observer. In other words remember a Lagrangian observer is one who is sitting on the fluid parcel.

And so, what is the rate of change of F as observed by a Lagrangian observer and how does that relate to the same rate of change as you know observed by someone who is sitting in the lab; in other words someone who is in who is Eulerian right. So, the answer is given by this.

So, the dF the differential you know quantity dF this is what is discerned by the person who is sitting on the fluid parcel the Lagrangian observer. And that is and the change in F whatever it is it might be density, it might be velocity that can be due to a true change in F with respect to time times the dt or it can be a change in F because, the person in the lab frame is watching you know stuff go by it need not be because something is popping up and say the velocity is really changing in time, it can be simply the lab observer can see something changing because, the fluid is flowing ok. And that is what is expressed by this term here.

So, the partial d partial t gives the local time rate of change at a point x. And when you write a straight F, straight dt that is equal to so, what you do is you just divide this. So, you divide a dt here and dt here and that is what this is so, a dF dt. So, you just this dt is simply divided and that d x i d t this is nothing but the velocity u i, where the i refers to the subscript i refers to either x, y or z as the case might be right.

So, that is how you get this u i here, out here, right. And that completes it, that completes the relation between the rate of change of the quantity F as observed by the Lagrangian observer and as observed by the Eulerian observer. The Eulerian observer is essentially you know characterized by the quantities on the right and so, this is.

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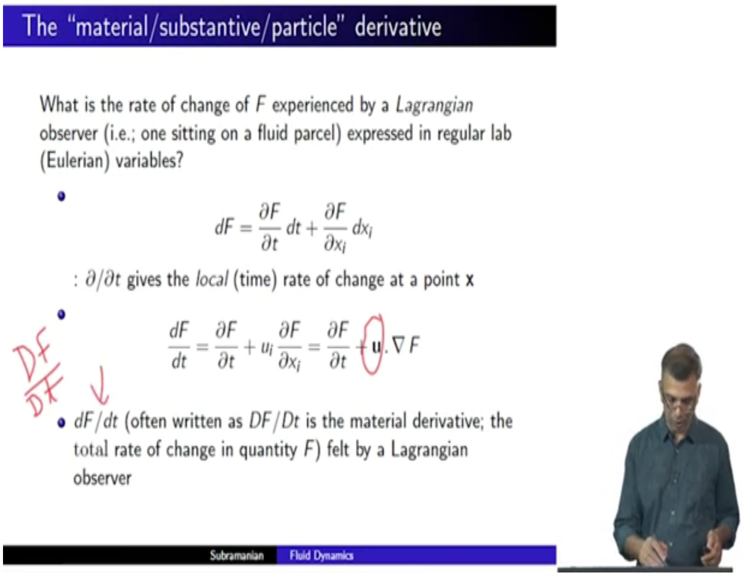

So, another way of writing this quantity in vector form is to say this, this is essentially the same as this where this is a gradient. And so, this is simply a more compact way what is written here is more simply a more compact way of writing what is in the middle of the equation same thing essentially right.

So, this thing, which is also sometimes written as a DF, this is what is called the material derivative. And this is the total rate of change in the quantity F felt by a Lagrangian observer by an observer who is sitting on top of the fluid part parcel. And so, this is the way you relate the rate of change in a Lagrangian frame to the rate of change in an Eulerian frame.

So, that is really so, and as you can see this is simply an, transformation that involves the fluid velocity u. So, this is what I meant when I said, when I said this is what I meant by this statement ok.

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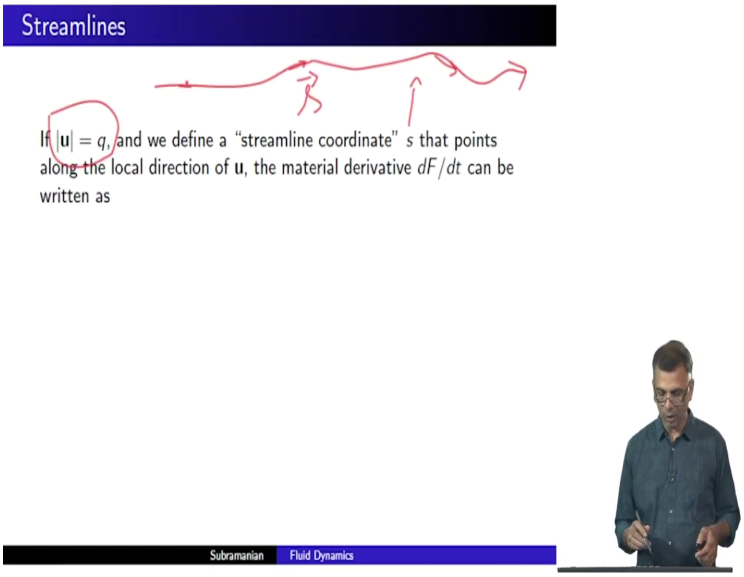

So, having said this now, we move on to what we define as a streamline. Now, this is something that is intuitively obvious to you. Suppose you consider a flowing fluid and you inject a coloured dye in it, a few say the fluid is colourless. And you inject piece of red dye in it and suppose of the one thing I want to emphasize before going forward is that in fluids many of these things are intuitively already obvious to you.

For instance, if I told you the flow is laminar or the flow is turbulent these are things that are intuitively obvious to you from everyday experience and it is very important to keep this intuition in mind. The reason I brought this up now is because many of you will immediately understand what a streamline is intuitively you will understand what a streamline is, it is this and I could define it a little further by saying it that it is suppose you a colourless fluid is flowing and you inject a few part a few particles of red dye in it.

It is essentially the path traced out by this red dye that is what a streamline is and let us say the fluid is flowing in a slow regular fashion so that the flow is laminar without really having define what laminar it is you still understand what this is. So, this is what a streamline is.

So, let us try to define it you know in a mathematical way really not that hard and so, the prelude to that is this statement here you know if the modulus of the velocity u, the velocity is written as u if the modulus of the velocity is some quantity q. And we define a streamline coordinate s ok.

So, you know a streamline would be this and the streamline could be bending like this. So, this s would be simply moving along the stream lines neither x nor y is a combination ok. So, the streamline coordinate is one that points along the local direction of u. So, here it would be here, here it would be here, and like this and so on so forth this so, this would be essentially the direction of s ok.

So, why are we talking about this? This is because the material derivative which we talked about in the previous slide, this quantity here can be written in terms of the streamlined coordinates likes well. This is again the definition of the material derivative from the previous slide.

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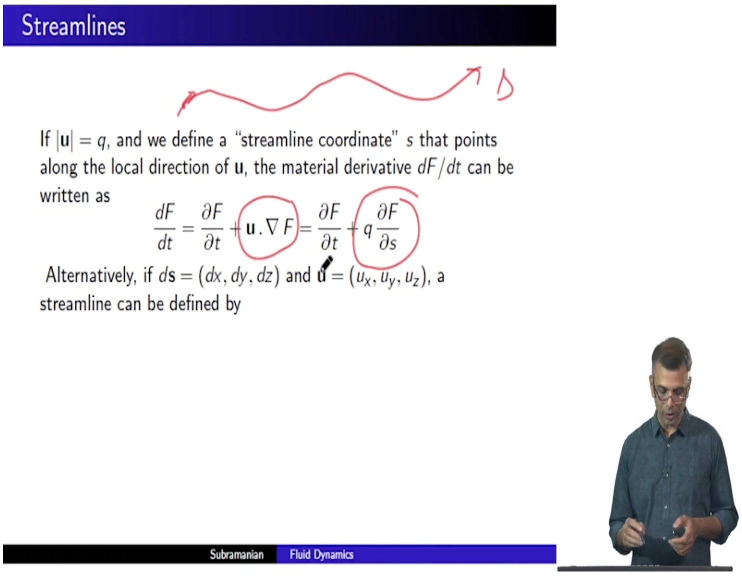

And this can be easily written in other words, this guy can be written as this, similarly easy. So, you do not have to deal with a gradient you can simply deal with a simple derivative like so, d F ds where, the s is of course, along the streamline; neither x nor y, but along the streamline, thinking about a two dimensional flow. So, you see the equation on the; the second equation here d F dt plus q d F ds this is a little easier than the equation in the middle.

Another way of defining a streamline is if the ds, if a differential you know element along the streamline is characterized by you know dx, dy, dz and you know also the velocity is characterized by ux, uy, uz, a streamline can be defined by this.

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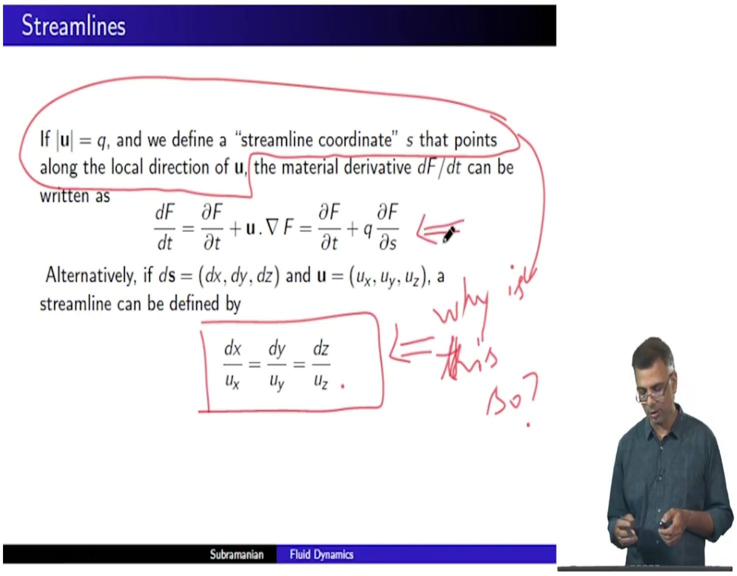

This is how a streamline is defined and I urge you to try to figure this out why this is so. How does this follow from this definition of a streamline? How does this definition follow from this definition of a streamline? So, this is yet another way of thinking about a streamline. From a mathematical point of view the advantage of thinking in streamline coordinates is the convenience that it offers in simplifying the material derivative.

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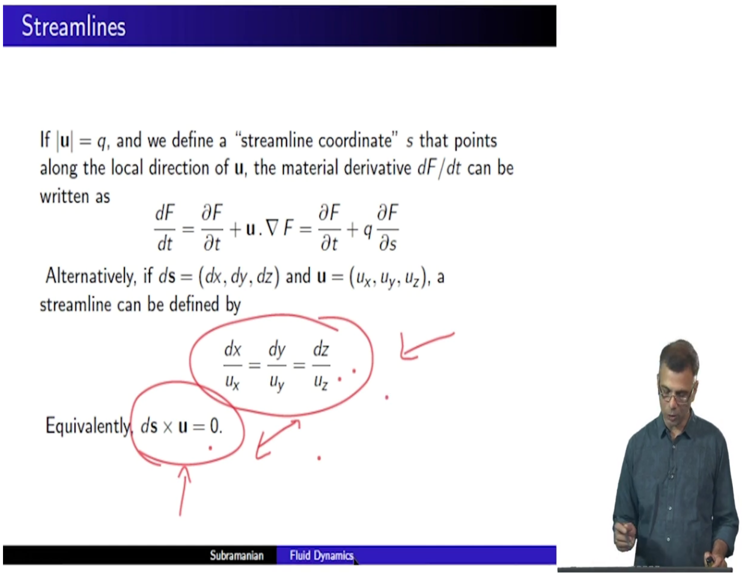

Equivalently another yet another way of thinking about it and this follows directly from this definition ds cross u is equal to 0. This and this follow quite simply from each other. And I urge you to think about how this sorry how this follows from this, ok. And maybe I will assign this is one of the thinking points and it is useful not that hard to figure out.

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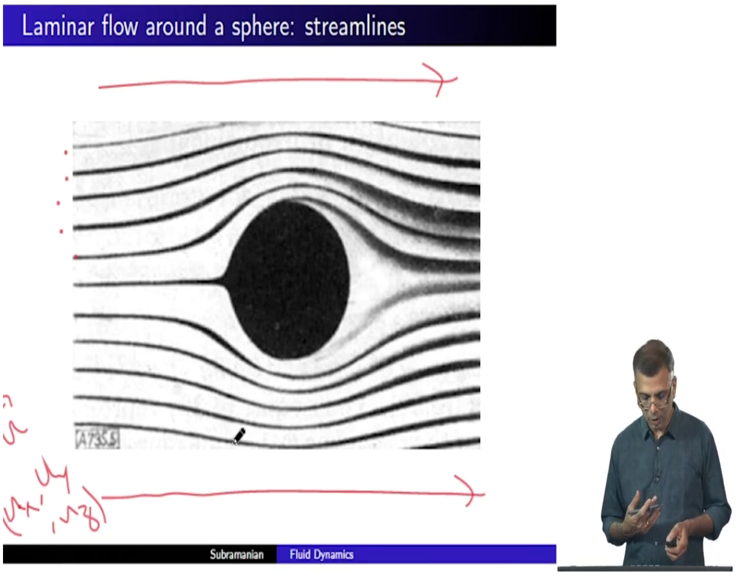

So, here is an example. Again like I said this is an example of how you are already familiar with if I gave you this picture and said that these are the streamlines of laminar flow around a sphere, you would have no difficulty whatsoever and accepting it. It sounds right, yeah. Think of a sphere immersed in a fluid that is you know flowing relatively slowly in a steady manner and think of you know little parcels of dye injected here, here sorry injected here, here and here and so on so forth.

And think of the path that you know this dye traces and that is what it will look like. Because of the presence of the sphere out here the fluid sort of tends to avoid the sphere and go on like this and far from the sphere the streamlines are almost straight like this; straight like this as though the presence of the sphere did not affect it at all ok.

So, all this talk about streamlines of all these definitions this definition and this definition here all of this is the practical use of this definition. How would you write down the equation of the streamline? Well, this is how you this is how the equation of a streamline.

If you know the velocity of the velocity field is characterized by you know u x like that, and that is how you know the velocity field is characterized and instead if you wanted to write in write it in terms of stream coordinates this is how you would write it. So, this is the practical you know use of a streamline.

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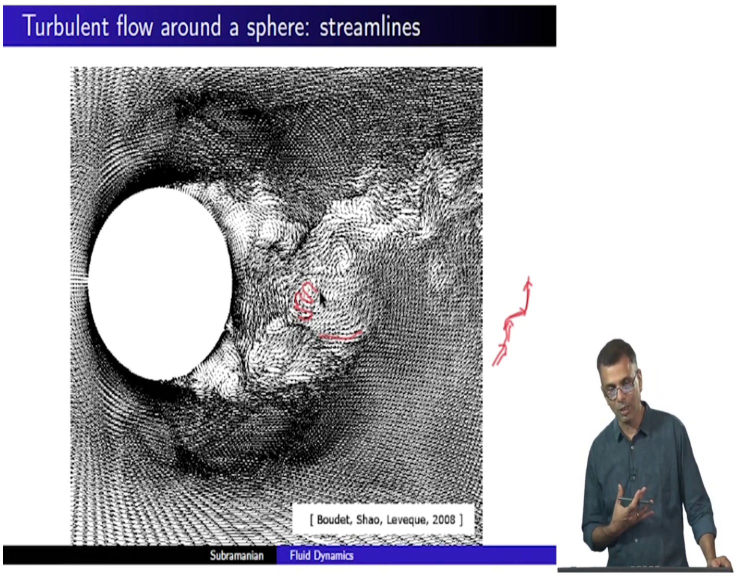

And now, if I showed you this picture instead right, and told you that this these are streamlines of turbulent flow around the sphere; again you would I mean intuitively you would have no difficulty in accepting this statement. You are sure enough you know this sort of looks like; you know all these little things that you see here are essentially these are little vectors like this, and like this, and like this, and like this. That is what these things are here and you join these little vectors around you get a streamline.

And these stream lines are all tangled at the back and this represents a turbulent velocity field, where you know the velocity is changing in a rapid unpredictable manner. And so, if you join the little arrows all throughout like so, for instance like this, you would get a streamline, a streamline that is often quite tangled like this.

And obviously, and the equations of these I mean the appearance of these streamlines are not as regular as what it would be for a laminar flow, but you know nonetheless I just wanted to show you this picture to show to illustrate the point that, these things are intuitively obvious without really going into the details of what constitutes turbulence and so on so forth.

And turbulence is one advanced topic in fluid dynamics, we will probably not have enough time to cover in this course, where is they are say one of the; one of the last frontiers of classical physics. Very very interesting field with very wide variety of applications and as some people have very rightly said turbulence is a norm rather than exception.

Consider a tap a faucet in your home and you turn on the water. What do you think is the character of the water flowing out of the of out of the faucet? Of course, it depends upon how regular the surface of the faucet is, the you know tip of the faucet, but generally you turn the water on slowly and the flow is generally laminar.

But you increase you know you increase the tap essentially you rotate the tap counter clockwise even more and the water flow starts to become irregular and you can imagine the streamlines of flow becoming irregular and tangled and resembling you know the streamlines here.

So, this just wanted to illustrate how these concepts are really quite intuitive and obvious to you and that is one of the beauty of studying fluid mechanics. You are merely systematizing the concepts that, you are already familiar with at an intuitive level, right.

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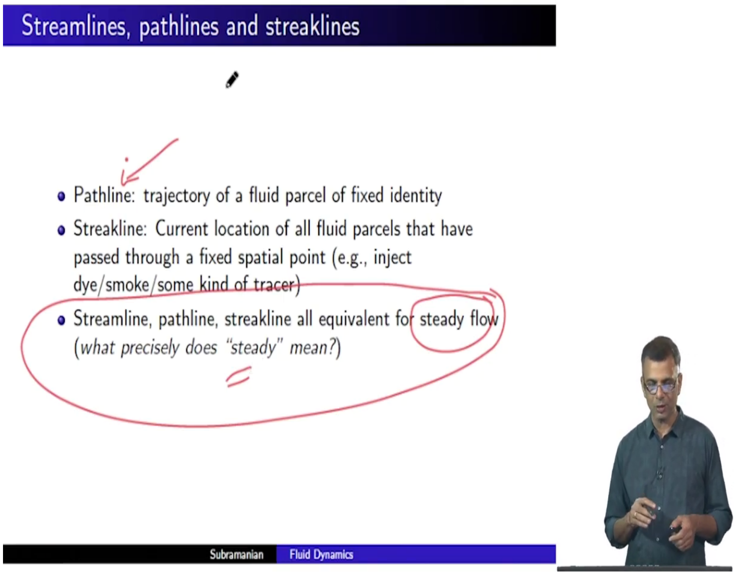

So, again without splitting too much without splitting hairs I will just sort of lay out these definitions in front of you. There is a technically a difference between what is called a pathline, which is the trajectory of a fluid parcel of fixed identity ok. Whereas, a streakline is something that shows the current location of all fluid particles that have passed through a fixed spatial point. In other words for instance inject dye or smoke or some kind of tracer.

And this is really what we have been talking about as a streamline all this while. And importantly it is a streamline, a pathline and a streakline are all equivalent for what is called a steady flow for us for a flow that is like this. Nonetheless while it is kind of intuitively obvious what a steady flow means it is useful to think about what precisely steady means.

Even though it is intuitively obvious to you from a mathematical point of view it is useful to think or even from a conceptual point of view it is useful to think about what precisely the word steady means. I do not want to dwell too much upon the differences between these definitions, I just wanted to make you aware of it and we will move forward.

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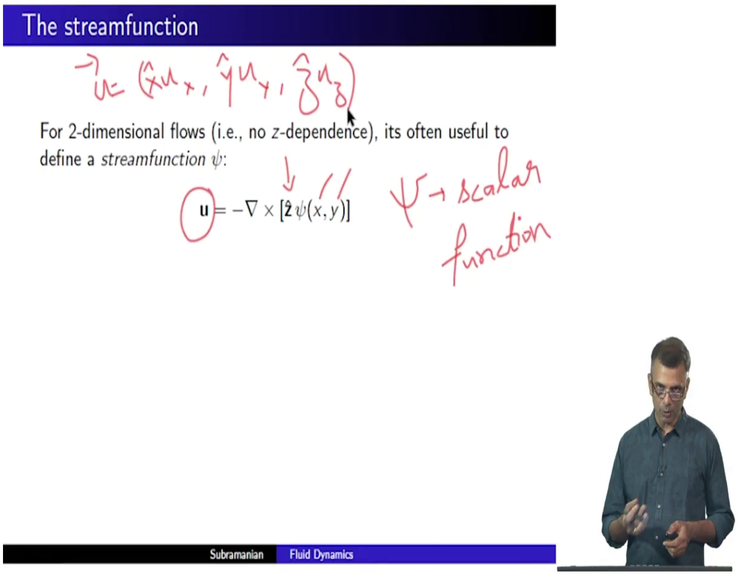

Let us talk about something that is called a stream function. In this case it is defined by its called psi and the way and let us now talk only about 2 dimensional flows. And so, you know there are only two coordinates x and y like this. And the stream function this is how the stream function is defined. You first of all, you assign you take the curl and this is a scalar function, the psi is a scalar function. And the z this imparts a vector character to it.

And you take the curl of this quantity and that gives you the velocity field, which is a vector field of course, u is right that is what this is. So, this is how a stream function is defined. There is a reason for this definition, there is a reason for popping this on you, we will come to that we will see why it is useful.

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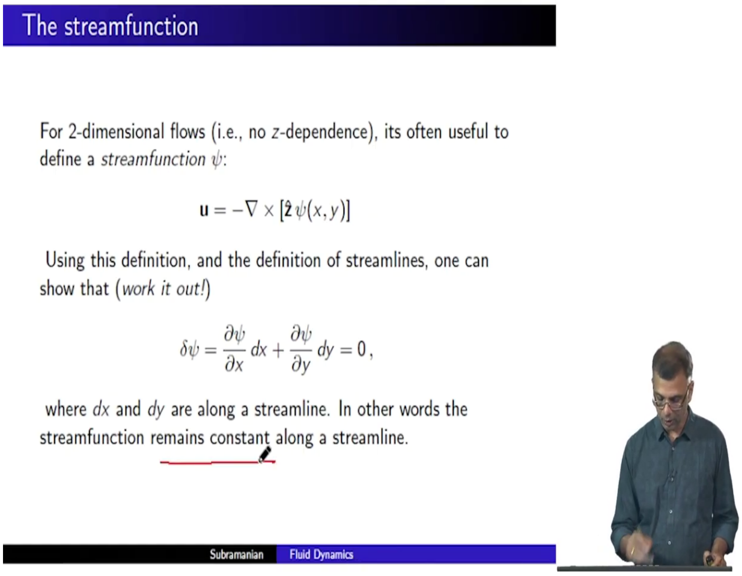

And using this definition of the stream function and the definition of streamlines you can show and I urge you to work it out. You can show that, a differential change in deep psi which is of course, simply you know the sum of the differential change in psi with respect to x times dx plus the same thing for the y coordinate is equal to 0, as long as dx and dy are along a stream line.

So, another way of saying this is that psi is that function which is constant which is conserved along a streamline ok. So, this is one way of defining it this is another way of defining it ok. And you can immediately see conserved quantities are precious in physics.

And so, you can start to now appreciate the logic of introducing this stream function, maybe not yet there, but you can start to appreciate the fact that, there might be some use for this stream function because it looks like it is a conserved quantity. And I urge you not to take my word for it. I urge you to work it out from the definition of the streamline, which is given here, this or this either of these two definitions it is possible to do it either way. So, that is the definition of a stream function, this is what I just said. In other words the stream function remains constant along a stream line.

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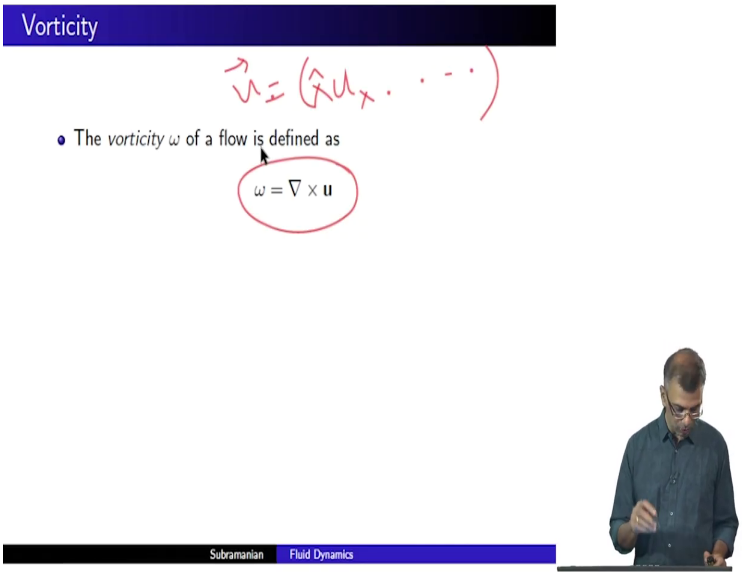

The other quantity that would be useful in talking about fluid kinematics is the vorticity of a flow which is defined as simply the curl of the velocity vector of the velocity field rather. And then the u is again the u is simply and so on so forth. So, this is what is called the vorticity that is the definition.

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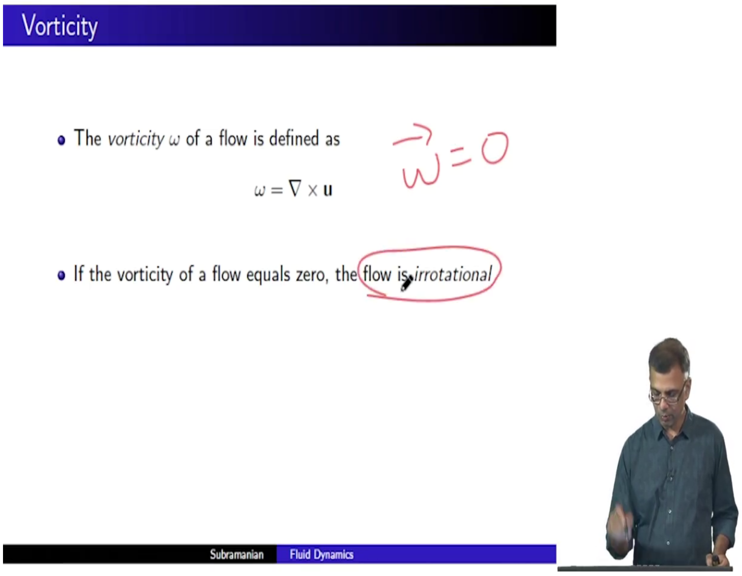

And if the vorticity of a flow is 0, if omega is 0 we say that the flow is irrotational.

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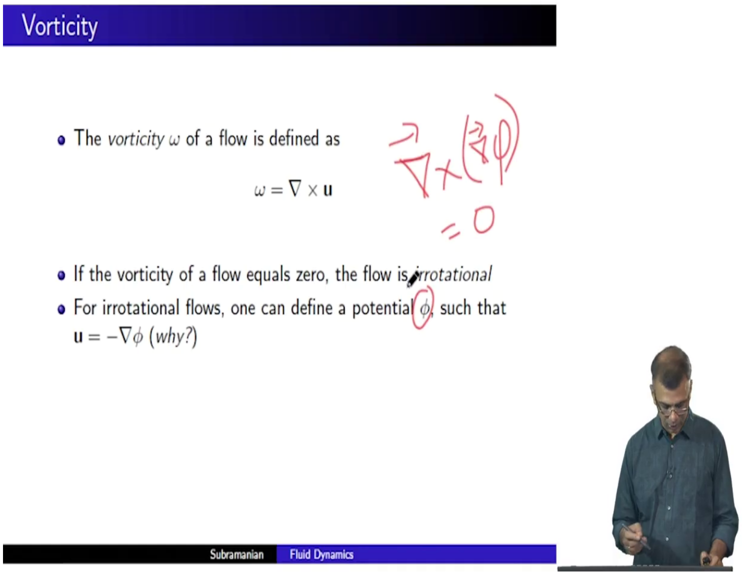

And for irrotational flows, one can it is a very useful thing one can define something called the potential, such that the velocity field is given by the gradient of this potential. Why is this? Simply because the in general curl of any of anything is always equal to 0, ok. This is something that arises from basic vector calculus, the curl of a gradient is always equal to 0.

So, if the curl of the velocity is equal to 0, it is possible to express a velocity as a gradient of something called a scalar potential and you know there is a lot of use for this quantity phi, which we will see in a minute.

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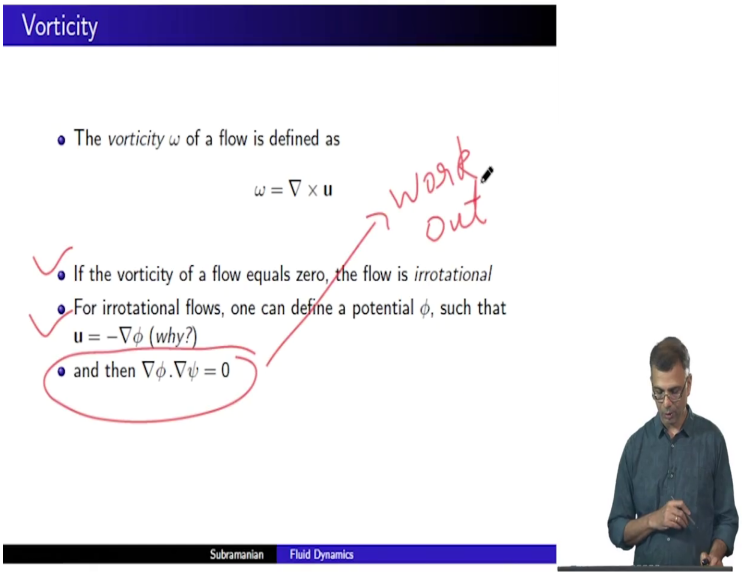

And therefore, from these two definitions; from these two definitions it is possible to show that, this is true. This is yet another property that I would urge you to work it out.

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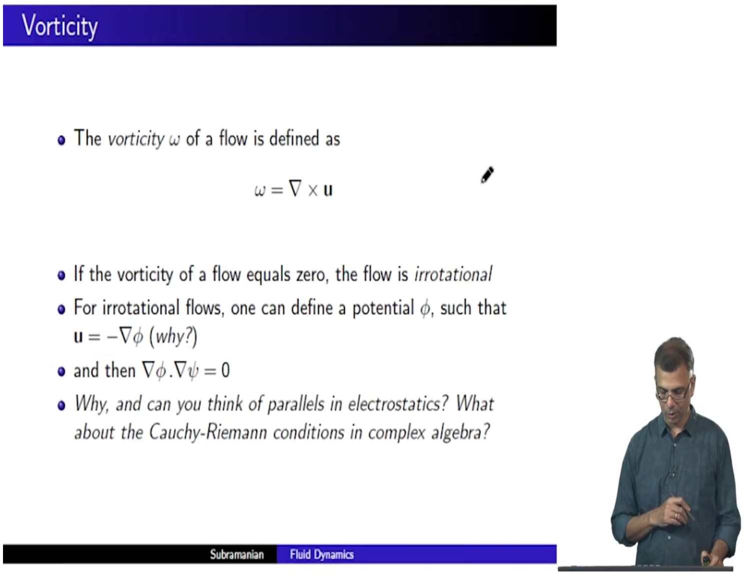

And you can think of parallels in electrostatics, you can think of a phi which would be immediately obvious, you can think of a psi which might not be immediately obvious to you, but really there are parallels in electrostatics if you think about it. And there are parallels to the Cauchy Riemann conditions in complex algebra really in a way you can think of introducing the phi and the psi simply so, that you can apply a complex algebra easily to studying 2 dimensional fluid flows. It simplifies the math greatly and allows you to use certain elegant results which would you know which simplifies life essentially right.

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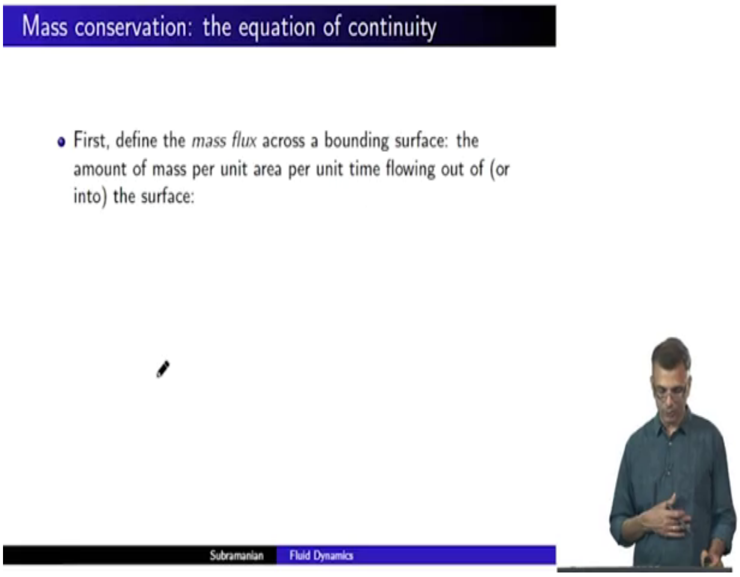

So, having sort of laid down the basics of fluid kinematics we will next go on to start talking about conservation equations. And we will start with the momentum conservation equation which is simply f equals m. Well, we will start with mass conservation which essentially says that mass can neither be created nor destroyed and then we will start talking about momentum conservation, which is essentially f equals ma.

And so, these are the basics of fluid dynamics and we will spend a fair amount of time introducing these conservation equations to you and especially, the momentum conservation equation in different guises which are called the Euler equation. If there is no viscosity, it is called the Navier-Stokes equation if there is a viscosity.

And there is a reason we introduced viscosity fairly early on because it is useful to understand the character of these equations as and when we understand the physical import when we come across them. So, that is what we will take up in the next module.

Thank you.