Fluid Dynamics for Astrophysics
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Lecture - 37
Spherical accretion onto a compact object: Solution for flow properties

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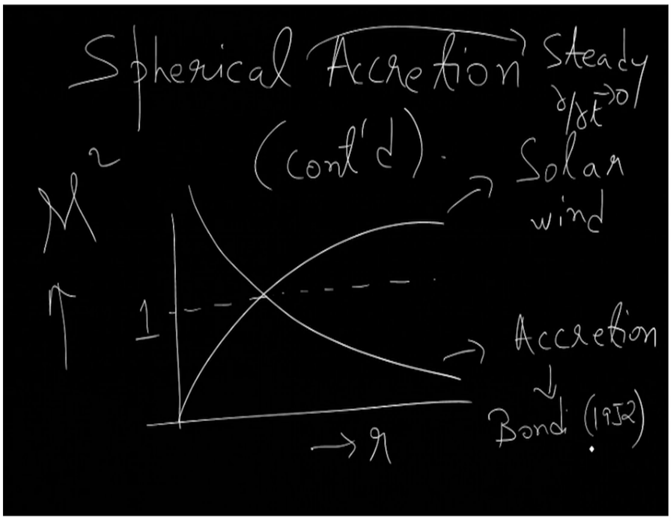

So, hi, we are now continuing with our discussion of Spherical equation right. And remember we have in our discussion which encompassed both the solar wind and accretion you know where we were discussing things like this kind of diagram. You will doubtless, remember this kind of diagram right, where this is the sonic point.

We have already discussed you know what leads to these kinds of transonic solutions. Both of these are transcending solutions because they pass through M square equals 1. And we discussed that this is representative this kind of solution is representative of the solar wind, and this of accretion.

Both spherically symmetric and steady accretion steady in the sense d over dt 0. So, we are now discussing this solution right. So, we would not bother so much about you know going over you know the behavior of the slope at this point. And what the sonic point means and everything because we have already discussed these things.

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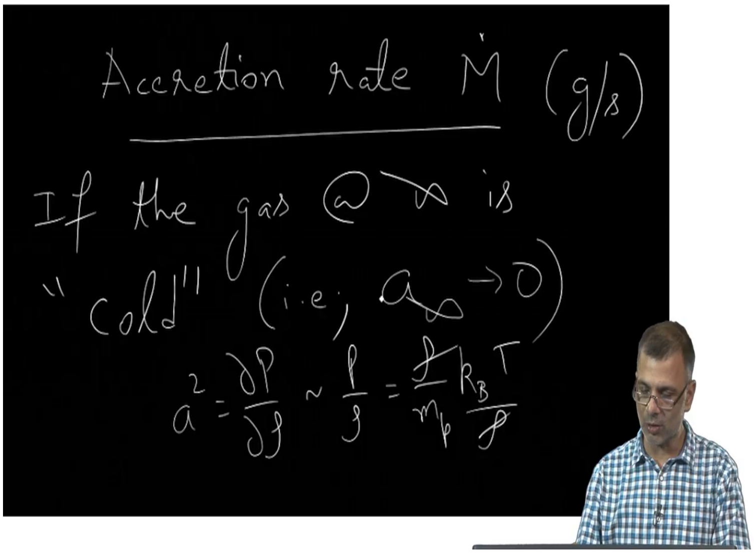

From what we discussed last time you remember, we were placing a lot of emphasis on the accretion rate M dot which is in units of grams per second right or solar masses per second, you know it is like grams per second essentially. And we discussed the fact that you know and why is accretion important in the first place? Well, accretion is important because it is a means of converting gravitational potential energy into observable photons.

In the sense that, it is a means of extracting gravitational potential energy is a very efficient means of extracting gravitational potential energy, and likely converted into via heating converted into observable luminosity. And photons are what we observe in astronomy, which is why it is important to think about accretion problems, right.

And the next thing we talked about is well you know you create one proton, and you release so much you release, so many arcs of energy right so many arcs per second of energy. How about you accrete many many protons? In other words, how about you increase at the accretion rate? Well, attractive as the as it might sound, there is a limiting value to the accretion rate ok.

You cannot, you can only accrete so much matter so that the amount of photons it that are generated via the accretion process is the radiation pressure due to those photons is not high enough to push the accretion accreting matter outwards. And this limiting luminosity or this limiting accretion rate is called the Eddington accretion rate right.

So, as you can see our discussion about spherical accretion places a lot of emphasis on this thing called accretion rate ok. And we will continue with that kind of accretion, we will continue with that kind of discussion. We will continue with our discussion of accretion rate in the context of steady spherical accretion ok.

So, now, the most naïve, the most natural thing to consider is a situation, where if the gas you have a central body sitting here. And the question is how much matter is it able to attract from infinity right? So, but if the gas at infinity at infinity is cold ok, i.e, the sound speed and from now on I am using a, for sound speed instead of c s squared so simply because the references that I am using say the same thing right.

So, the sound speed at infinity tends to 0. Why is this? You see you remember the formula for sound speed a square is equal to dp d rho in is something like p over rho. And this is equal to rho over m p, this would be n k Boltzmann T over rho. So, essentially the sound speed goes directly as the temperature, well the square of the sound speed goes at the temperature. So, if the gas is cold, the sound speed is also 0 ok.

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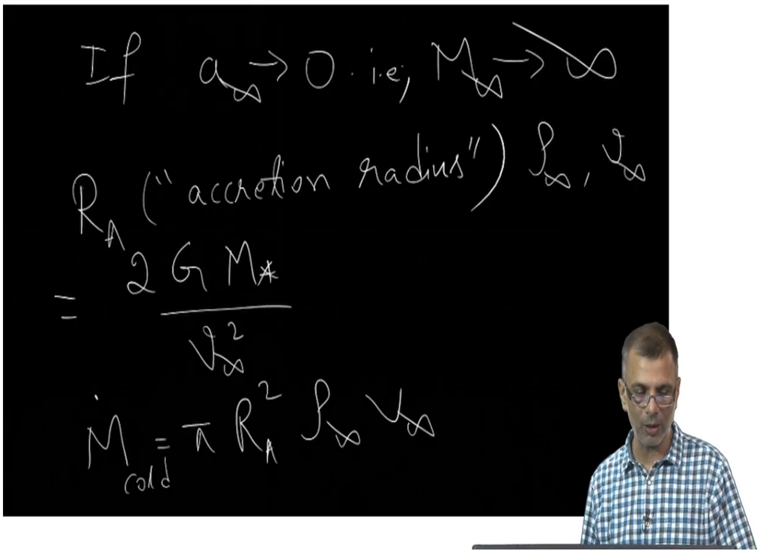

So, let if the gas at infinity is very cold, in other words the sound speed at infinity goes to 0 right. If a infinity goes to 0, i.e., the Mach number at infinity tends to infinity. Here this subscript infinity simply means a very large distance from the central object ok. Then in that case and this was the first kind of you know the first kind of considerations that were done with regard to quasi-spherical accretion.

Then if we define a kind of an accretion radius, this is just a definition, where this R A is an accretion radius ok. If this is defined as 2 G M star, where M star is the, is the mass of the central object over v infinity squared ok. So, the gas has a velocity at infinity you have a density rho infinity, and a velocity v infinity this is the assumption. And a infinity it tends to 0 at infinity.

So, if we define this kind of an accretion radius, this kind this has the dimensions of length you can verify this. If we, define a quantity called R A which we call the accretion radius, then and the reason for calling this the accretion radius will become apparent immediately then the M dot would be equal to pi R A squared rho infinity v infinity ok.

And from here logic of calling this R A is immediately apparent. Except for a factor of 4 is the factor of order of unity. This is like the area of a sphere of radius R A. And then you have you know it is as if matter with density rho infinity and v infinity is falling onto the sphere with the radius R A and that is the accretion rate ok.

But this is the accretion rate for absolutely cold matter, where the Mach number because the sound speed tends to 0, and you have a finite velocity at infinity the Mach number naturally tends to infinity. So, this was one way of thinking. Until, of course, it was realized now another way of saying that the sound speed tends to 0 at infinity is saying that the pressure in infinity is essentially 0.

Why? Well, because the sound the sound speed is essentially pressure over density you have a finite density. So, if the sound speed tends to 0, then the pressure also tends to 0. Well, is this a very realistic thing? Not really, ok, it is hard to think of a situation where you know the pressure of the interstellar medium, and no matter how far it is from the central object, is actually 0.

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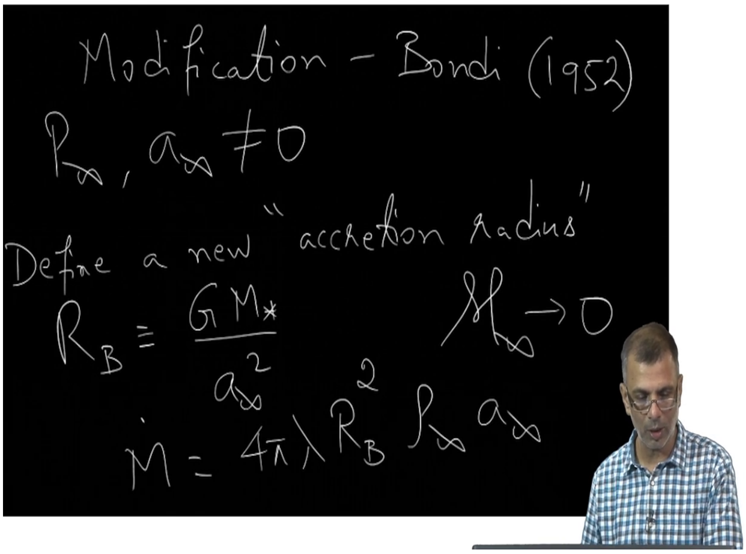

Therefore, modification was introduced by due to Bondi Hermann Bondi famous paper in 1952 ok, where he proposed that the P infinity the pressure at infinity and the sound speed at infinity are not 0; in other words, the gas is not cold at infinity ok.

And in this case, if we now propose again a radius an accretion radius along the same kind of along the same kind of logic ok R we call this R B for R Bondi, and then you will have the same numerator except here instead of v infinity square I am going to have a infinity square ok.

So, we define a new accretion radius R B as over a infinity square. Now, notice the similarity between this formula and this formula, is exactly the same except and the denominator you do not have v infinity square you have a infinity square, and there is a reason for this ok. There is a reason for this. It will become apparent in a minute.

But the up the immediate upshot of this is that the accretion rate is now we can we there is a factor of order this is a factor of dimensionless number of order unity, this is not so terribly important for the purposes of discussion, I have just written it down for completeness. But it is essentially the same thing it is like a 4 pi R squared except now instead of R A you have R B squared rho infinity and a infinity.

So, it is as if gas with a speed that is equal to the sound speed at infinity and density which is some density that exists at infinity is impinging on a sphere with radius R sub B. And the amount of matter that is coming through the sphere per second that is the accretion rate ok. And in this case what we are saying is that as you go to infinity, we are talking about this kind of solution right.

So, as you go to infinity very very large r, the velocity is essentially 0, although the sound speed is finite right. Therefore, in this kind of situation what happens is the Mach number at infinity is not infinity in fact the Mach number at infinity tends to 0 and that is what this kind of solution is representing ok.

And therefore, this kind of solution is due to Bondi ok alright. So, that is that. And now let us discuss a couple of other interesting phenomena that are due to that arise from this kind of con consideration ok, why is this called an accretion radius I mean you know so on so forth.

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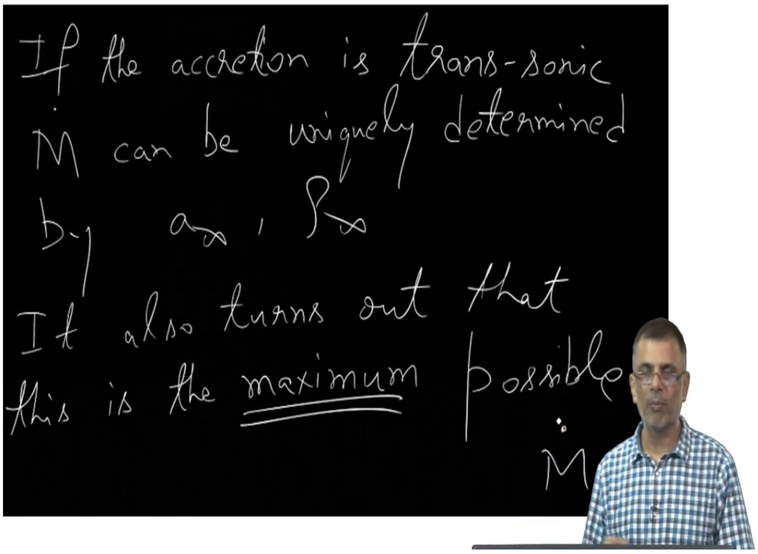

Now, the main thing to note is that if the accretion is transonic is and what do I mean by transonic, I mean like this ok. If the accretion is such that you start with a very low Mach number, pass smoothly through a true sonic point, your subsonic all the way here and you pass smoothly through the sonic point, and then becomes supersonic.

If the accretion is like that, then M dot can be uniquely determined by the sounds speed at infinity and the density at infinity that is evident from here. You see some at the sounds speed at infinity and the density at infinity. Although of course, you have this R B squared, but this R B squared also involves just the sounds speed at infinity you see.

So, therefore, this statement stands you give me you know gas that is pretty much stationary ok at infinity. And you tell me what the sound speed is at that very large distance this one.

And you tell me what the density is at that very large distance? I will tell you what the M dot is I will immediately tell you what the M dot is ok.

If it is a transonic flow, and it also turns out that this is this kind of M dot is the maximum possible M dot. And this is a very important statement this can be shown, but it involves a little bit of you know mathematics that we do not really have time to go into. I urge you to look into, I urge you to look into the following paper.

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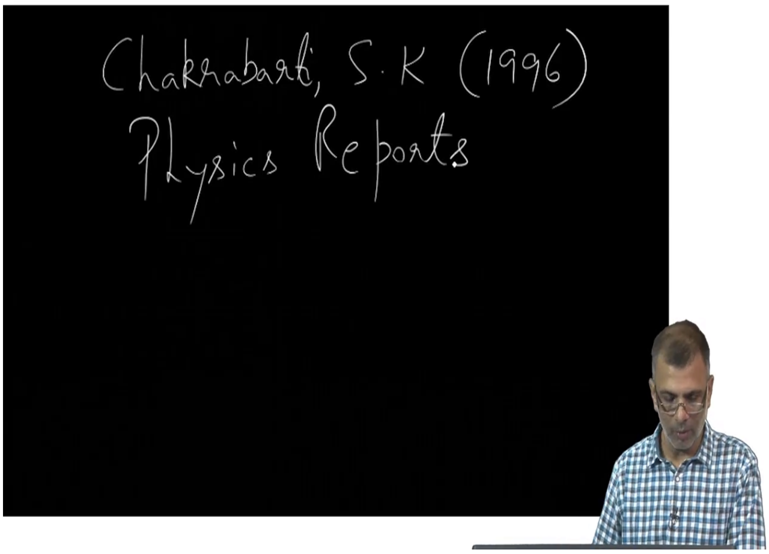

I urge you to look at 1996 Physics Reports ok. The proof of statements like this, the proof of this kind of statement is given in that paper and a lot more ok. So, that is a very comprehensive review article I strongly urge you to you know this article is a very comprehensive review article, I urge you to look at it.

But let us for a moment not bother about how to show this, but let us think about the implication of this. This is why we are so interested in transonic accretion. You see from what we have been discussing since yesterday since the last time we met, it should be apparent that you are really always interested in maximizing M dot ok.

You really want to pile on as much M dot as possible ok because that is going to give you the maximum possible luminosity ok. So, there is a problem you cannot pile on more than the Eddington limit. Yes, there is that. But nonetheless so that that is one that is that is a limitation from one point of view that is a maximum ok.

But another point of you still want to maximize it to the extent possible without exceeding the limit Eddington limit, and this is a way of achieving the maximum possible M dot. What is it? To ensure that the accretion is transonic ok. If you ensure that the accretion that the path the accretion is transonic, then you are guaranteed that you are accreting the maximum possible M dot. You cannot have an M dot that is larger than this.

And then of course, you have to ensure that this maximum possible M dot is still lower than the Eddington limit otherwise. And for spherical accretion turns out that that itself is not a problem ok alright. So, this is one interesting statement right. And I showed you that we would be talking a little bit about what is called the Bernoulli integral. And here we go.

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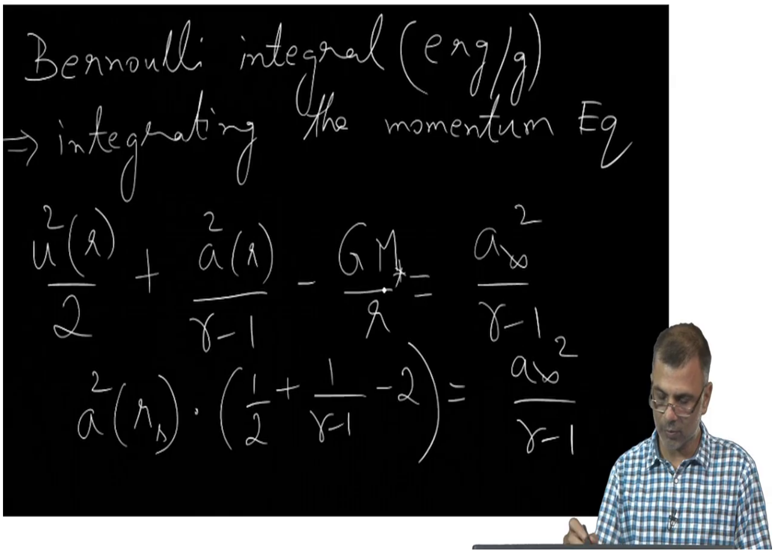

So, a lot of this can be found in this paper. So, remember the Bernoulli integral, this is essentially the specific energy ok. This is obtained by integrating the momentum equation which we have written down many, many times before; especially in this context, in the context of you know quasi-spherical equation ok. And you integrate that and the result would be something like this.

This is the kinetic energy per gram, and this is the internal energy per gram, and this is the gravitational potential energy per gram ok. So, this would be at any given r ok. Now, and this is conserved you agree that is the whole point of the Bernoulli integral this is conserved. This is the same at any r, no matter what the r is the various terms will balance each other, so that the sum is always the same.

Now, how what would this term? So, this would be equal to whatever its value is at infinity. And what would be its value at infinity? Well, at infinity the gravitational potential is essentially 0 because r is very, very large. So, this essentially goes to 0. And we have said that at infinity the gas is essentially at rest right, there is really no v infinity ok.

So, the gas is. So, pardon me, I am switching between u and v, but you get the context right. So, the gas is essentially interested this is not there only this term survives. So, this would be equal to a infinity squared over gamma minus 1 ok. This immediately gives you a relation between a infinity, and the a at the sonic point ok.

The constancy of this ok immediately gives you a relation between the a infinity and a and the sonic point. So, it is like ok sorry this is not into, this is actually this is a multiplication. So, again this, this is again the Bernoulli integral. And the Bernoulli integral at the sonic point is this. Why is that? Because at the sonic point you know there is no need to write a separate u because by definition the u is equal to a ok.

And there is a the there is a relation that that expresses the r in terms of a right at the sonic point there is a whole point right d u d r is it becomes a 0 over 0. So, this is this again is a Bernoulli integral at a infinity at the sonic point, and that can be immediately related to a infinity right.

And so what this is essentially saying is that you immediately have a relation between a infinity and a the sonic, having prescribed a infinity and rho infinity you know everything else that is what I am trying to demonstrate here ok. You can immediately also have a relation like this.

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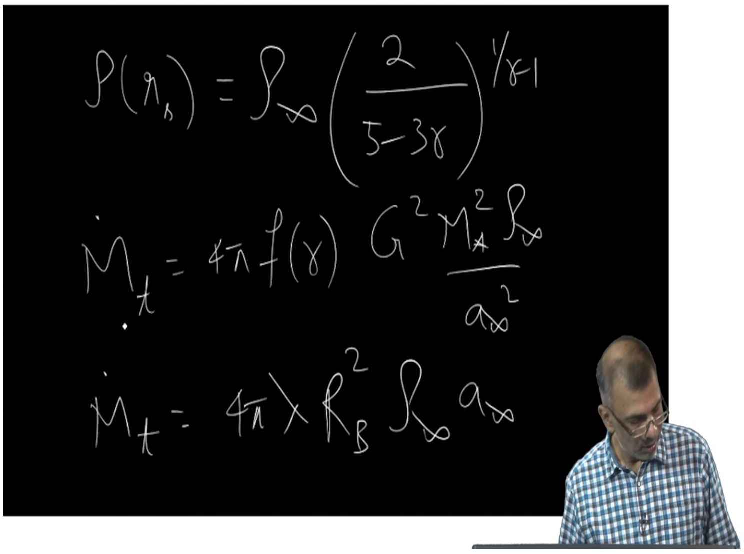

The rho right the rho at the sonic point is equal to the rho infinity ok times 2 over all of these all of these formulas I am simply writing down it is really no sweat to derive any of these using what we have done earlier ok. So, this now gives you transonic accretion rate which would be M sub t equals 4 pi some function of gamma ok, there are lots of gammas floating around and we would not bother too much G square M star square rho infinity over a infinity cubed.

And this by the way if I am writing M star there, I really should be writing M star here also ok. So, now, this formula is essentially the same as this formula as this formula ok. It is essentially the same; there is really no difference ok. I might as well write a subscript t here ok, it is the same formula. And it is just written in a slightly different guys.

And the point is this is the maximum possible accretion rate ok. And the maximum possible accretion rate is achieved by ensuring transonic equation. This is the same as writing equals right 4 pi lambda where this lambda and this are pretty much the same thing, R B squared rho infinity a infinity ok this and this are the same thing. Now, let us also look at you know a few interesting facts about this accretion radius.

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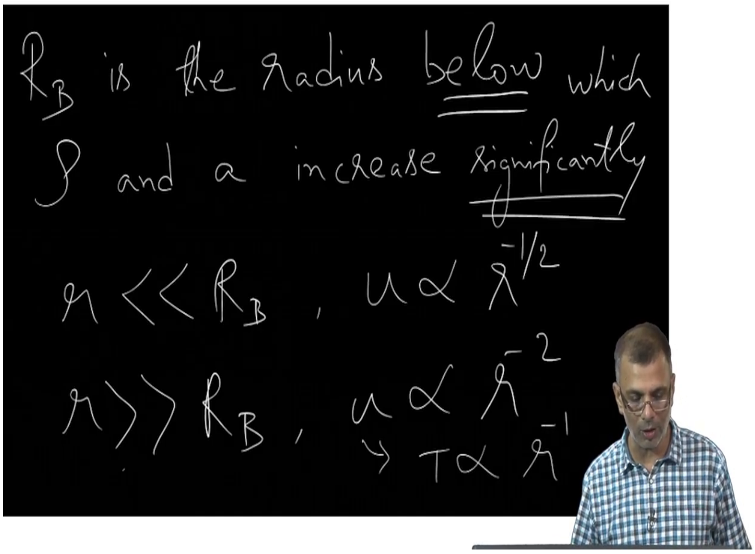

The accretion radius is the radius below which the density and sound speed increase significantly. So, up until, up until the R B you know the density and the sound speed are pretty much the same as what it would they were at infinity. It is only within this R B that the density and sound speed increase in increase significantly ok. And you know and the other thing is for r much smaller than R B, the info velocity goes as r raised to minus one-half. For r much much larger than R B the in fall velocity goes as r raised to minus 2.

Now, what, what is this saying? You see you have in fall ok, and you know it is falling in, but by the time it is by the time you are you it is falling at a certain rate ok. But by the time you cross this accretion radius, the in fall rate sort of you know slows down. This is a sharper this is a sharper you know exponent than this one right. And so and for this here the temperature is proportional to r raised to minus 1.

So, what happens is below the accretion radius the in fall velocity kind of slows down and the density and the sound speed increase significantly. And as a result the temperature, the temperature increases the temperature is no longer the same as it was at infinity the temperature increases as r raised to 1 minus 1.

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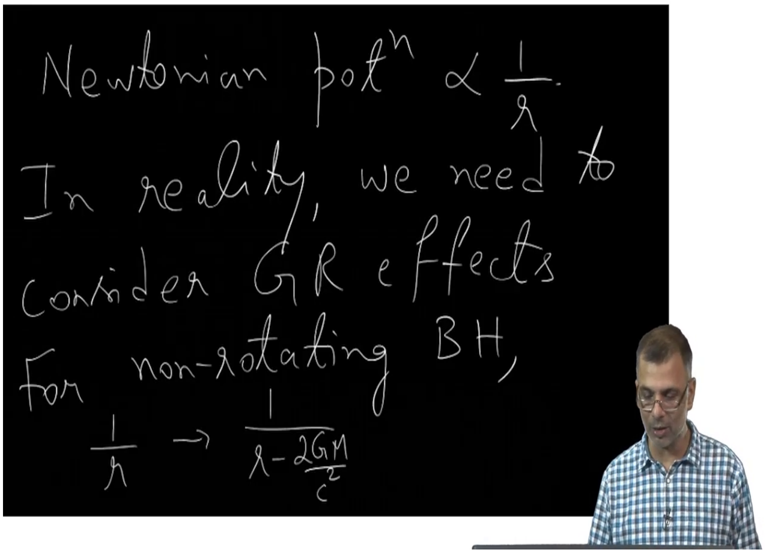

And things of course, blow up when you reach r equals 0 that is because we are considering a Newtonian potential a Newtonian potential which goes as 1 over r. In reality especially for a black hole, we need to consider we need to consider general relativistic effects which are quite complicated ok.

But at least for a non-rotating black hole is sufficient to for a non-rotating black hole, it is a in order to mimic general relativistic effects, it is sufficient to replace the 1 over r potential that we have been using all throughout this kind of this kind of potential its sufficient to replace the 1 over r potential with a potential that goes as 1 over r minus 2 G M over c squared.

It is if it is sufficient to replace this that kind of mimics the general relativistic effects to a to a satisfactory extent. I do not have time to go into why this is so, this is called this is what is called a pseudonytonium potential ok.

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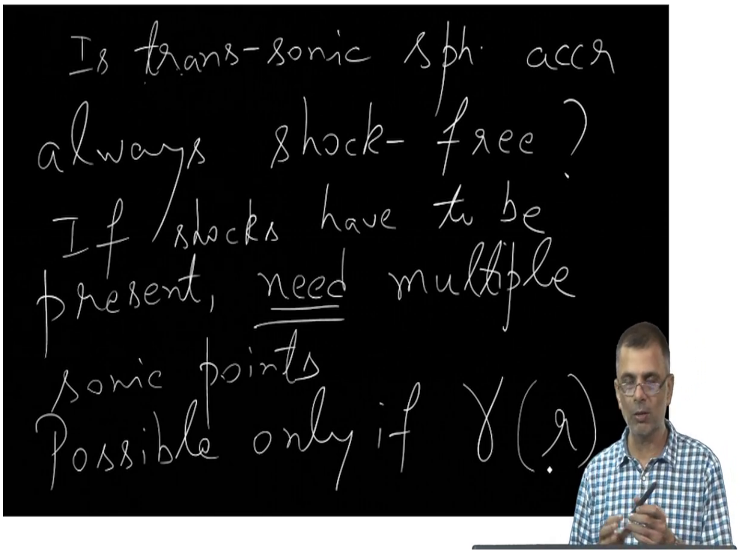

And now one last topic before we wrap up can one is trans-sonic is a trans-sonic spherical equation spherical accretion always smooth? In other words, always shock free ok? In order to understand this, and that is what it certainly looks like right; in order to understand this let us look at this. Now, you see this is trans-sonic accretion right. So, you have one sonic point right.

And now suppose there was a shock right let us say. And by definition by the time you accrete the gas has to be supersonic, there is no other way ok. The final the boundary condition at you know very small radii has to be a Mach number larger than 1; there is no other way ok.

So, suppose there was a shock. And the shock will always happen after that after the gas becomes supersonics, in other words it will happen after this critical point after this sonic point right. So, suppose there was a shock here. What does the shock do? The shock makes the supersonic flow transition to a subsonic flow.

So, you would have this curve you drop down to a value less than 1. But now you see you and then you cannot accrete anymore, you cannot accrete onto the central object anymore in a smooth fashion that is not possible ok, because you know the inner boundary condition demands that the you know at very small radii the flow is supersonic.

Therefore, if you insist that there is a shock here, there has to be another sonic point through which the flow passes. And therefore, the flow becomes supersonic yet again ok. If you need to have shocks, if shocks have to be present you need multiple sonic points not just one ok, one sonic point will not do. And multiple sonic points are generally not possible in spherical accretion. It is possible only if the polytropic index ok is not constant throughout the flow, but the polytropic index is a function of distance.

If you vary the polytropic index, in other words, what you do is if you include additional sources of heating and cooling such that the polytropic index does not remain constant. At whatever value it is, it does not remain constant if you make it vary as a function of r, then it is possible to have multiple sonic points, and then you are possible it is possible to have shocks ok. So, this is all I have to say for now. And we will stop.

Thank you.