Fluid Dynamics for Astrophysics
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Lecture - 36
Spherical accretion onto a compact object: Eddington luminosity and accretion rate

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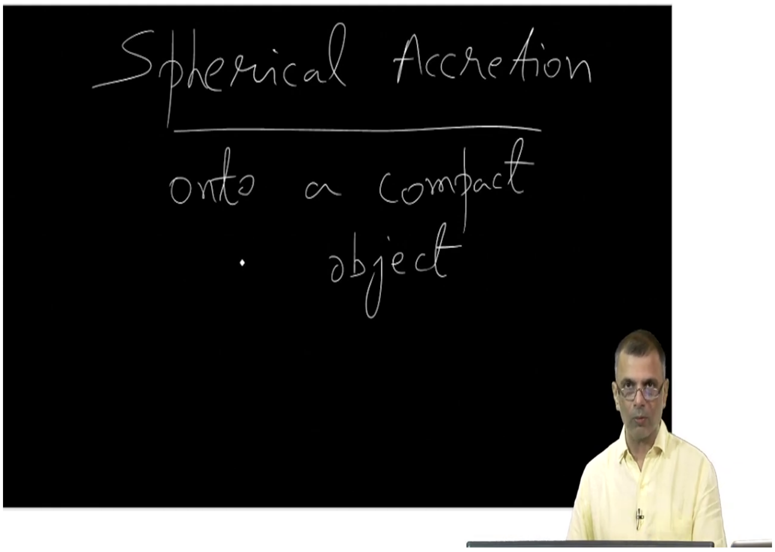

So, now, we are ready to undertake an overview of the opposite of the solar wind which is Spherical accretion. So, in particular spherical accretion onto a compact object; compact object such as you know a black hole or a neutron star or things like this, ok. And, so this is what we will be discussing from now on and we have already looked at the solutions.

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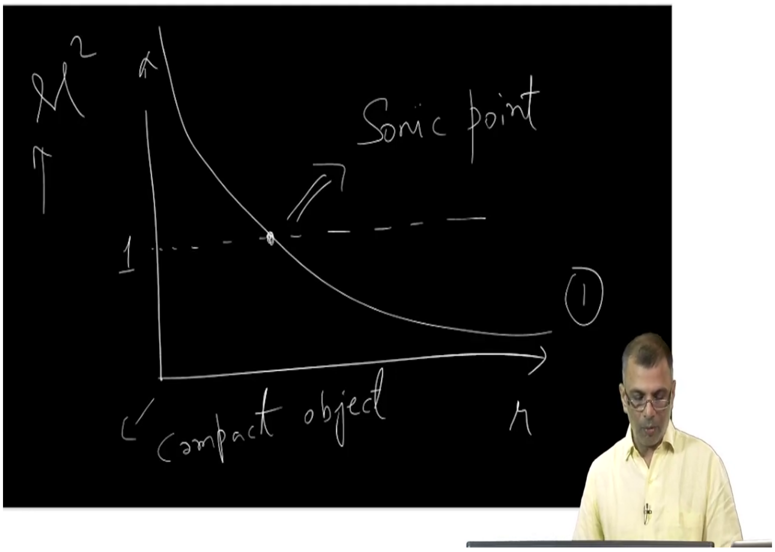

So, since we did the math earlier let me do this let me remind you what kind of solutions we are talking about and we are talking about the kinds of solutions that do not start out subsonic near the you know near the compact object. In fact, it is opposite. They start out subsonic very very far ok very very far in the interstellar medium you know the matter would be subsonic and then as it accretes, it gets faster and faster in particular this might even maybe this is not the best way of drawing this.

I should draw this. It gets it increases without bound in fact. The Mach number in fact, goes to somewhere like infinity, ok and somewhere here would be one. So, this again would be the sonic point or as we have said not quite the sonic point, but the sonic surface in case of spherical symmetry, right.

So, this is the situation we are considering not this kind of curve, but this kind of curve. This would be solutions of the kind 1, solutions that start out subsonic very far away ok. So, the Mach number is significantly less than 1 very far away. It accretes it passes through the sonic point and it becomes supersonic as it approaches the compact object.

The compact object is sitting somewhere here; the compact object which is enabling accretion, which is causing this accretion to happen, right. Now, before all this I think this is astrophysics this is not solar physics this is astrophysics and so, observations are much more indirect ok. So, really this is more reasoning it has to be this way and so on so forth.

So, it is worth pausing a little bit and trying to understand why accretion in particular is of so much astrophysical interest. And, spherical accretion of course, is the simplest of cases which is why we are discussing this. As we go on we will also discuss another kind of accretion called disk accretion ok.

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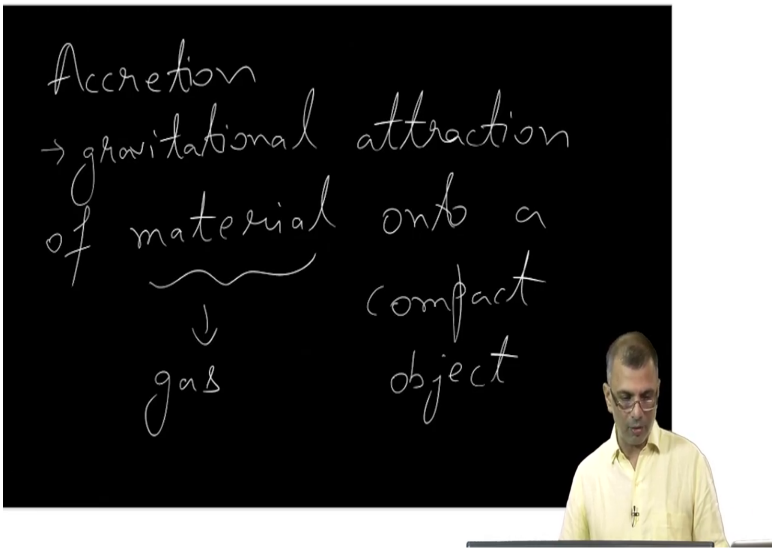

So, accretion is defined as is essentially the gravitational attraction of material I would say in this case we really mean gas ok onto a compact object and the compact object can be a black hole or neutron star that will simply whether it is a black hole or a neutron star will simply influence boundary conditions here, that is all ok.

A black hole has no surface. So, it just follows the material; a neutron star on the other hand does have a solid surface and so, that is that, right.

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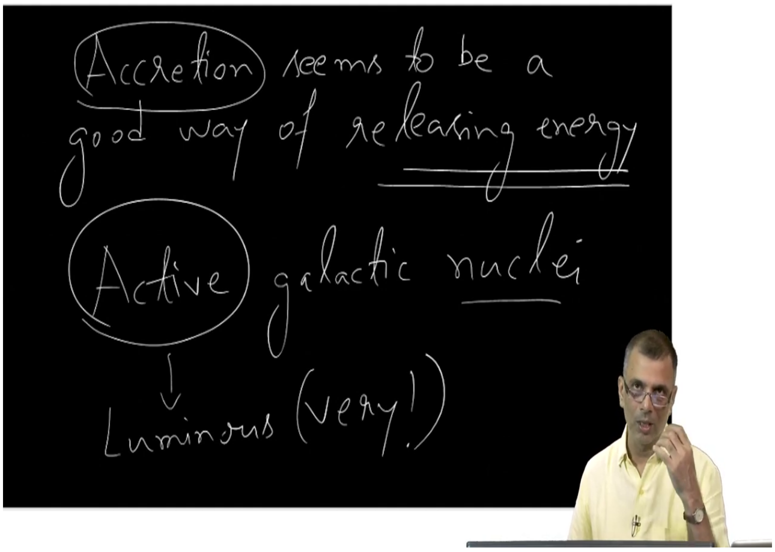

So, the fact is the reason we are interested in accretion is that accretion is a accretion seems to be a good way of releasing energy. What do I mean by this? You see the kinds of objects this paradigm is applied to are called often called active galactic nuclei ok. These are objects that are sitting at the centers of certain kinds of galaxies, so, the hence the name nucleus, ok.

Let me write this down active galactic nuclei ok. So, they are sitting at the centers of galaxies hence the word nucleus and they are not just dormant, they are active. They are active in the sense they are very very luminous very. They are so luminous that they typically outshine the entire galaxy ok.

So, the entire galaxy the light, the photons from the nucleus dominates the entire galaxy and they are very very active. So, naturally the question to ask is what is causing the release of this enormous amount of energy, ok. So, energy release is a very important question in studying these kinds of astrophysical objects, ok.

And, at the center of these nuclei from dynamics of stars and everything and other material, we know that there is typically a very compact object at the centers of these nuclei and many times a black hole, ok. And, a black hole will typically you know will naturally tend to attract matter from around it. It will tend to enable accretion ok.

So, it stands to reason that there is some kind of connection between accretion and energy release, ok. It is these central objects that is naturally will accrete matter ok and they are also very very luminous, right. So, it stands to reason that there has to be some kind of connection between accretion and energy release.

That is the real reason we are interested in accretion and that is the reason we you know we and of course, by way of mathematics you always want to you know look at the simplest situation first and then go on to more complicated situations and spherical accretion transonic spherical accretion is one of the relatively speaking one of the simplest kinds of situations ok, right.

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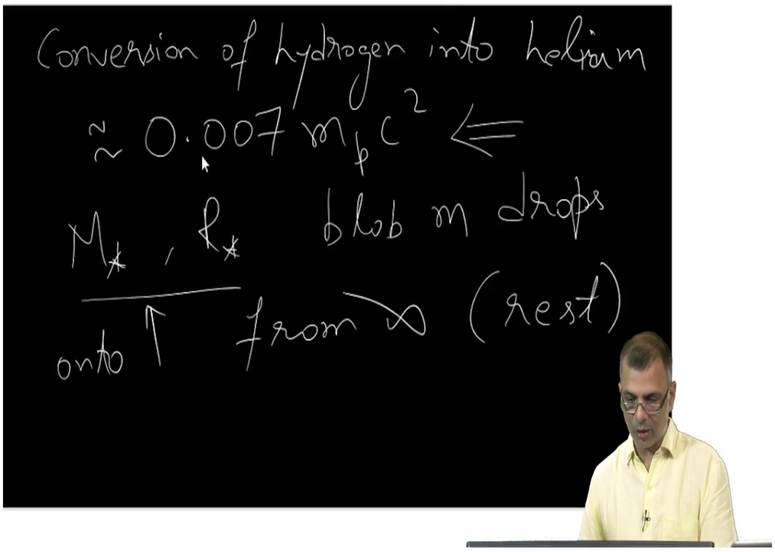

So, let us now look at a couple of scenarios simple scenarios, right. So, conversion nuclear fusion, ok; so, conversion of hydrogen into helium this is one of the best known sources of energy, right. I mean in the center of the sun this is what is happening this releases per hydrogen atom this typically releases something like 0.007 m p c squared, where mp is the mass of a proton, ok.

So, let us keep this as a benchmark and let us try to see if accretion is a more efficient process or a less efficient process, ok. So, now consider a object of mass M star a compact object of mass M star and radius R star, ok. Now, let us consider a blob. So, this would be the compact object the central object a blob comprising of just hydrogen of mass m ok drops onto this object from infinity and it was at rest at infinity.

It was at rest at infinity and it is you know its being attracted and in it drops off. It sort of you know accretes onto this central object which has a mass M star and a radius R star ok. Now, what is the kind of energy that we would expect to be released? Well, the kind of energy you are essentially converting the gravitational potential energy into. So, let us simply estimate the gravitational potential energy, right.

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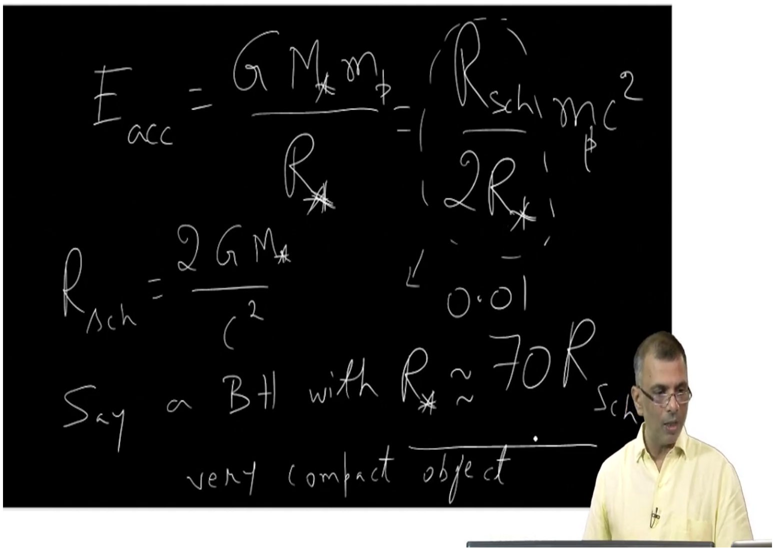

And, that would be we write this as E accreted and this is simply equal to GM star m, where M star is the mass of the central object, m is the mass of the blob which is accreting right over R star, that is it. Now, we will try to show that this for really compact objects this can be really large, ok. This can substantially exceed this 0.007 mc squared, ok. In particular, let us just say this m is equal to mp 1 per proton, alright.

So, let us now define something called a Schwarzschild radius which is let us this is just a definition GM star over C squared. This is just an a normalizing unit ok, now turns out that in units of this radii if you write it here this becomes R Schwarzschild over 2R star right mc squared.

So, you really should be comparing this one with 0.007 this is mc squared is the same mc squared that you saw here, this mpc squared. Let me write this as mp. So, that there is no right. So, now, suppose there was a compact object say consider a black hole with R star some 70 Schwarzschild radii. In other words, this ratio is something like 70 ok. So, this would be 1 over 70. So, this would become essentially 1 over 140, say 1 over 100, right.

So, if its 1 over 100 then this is something like 0.01. So, compare this 0.01 with 0.007. It is already more efficient than a nuclear conversion of hydrogen into helium. So, this is the whole point. Therefore, and what is this I mean the assumption of course, is that this is essentially just the gravitational potential energy.

So, what we are saying, the point we are trying to make here is that the gravitational potential energy if it can be released and there is a big F of course, if we can somehow tap this gravitational potential energy, the gravitational potential energy arising from accretion of you know gas from at rest from infinity onto a compact object.

An object which is so compact that the you know the radius is something like 70 Schwarzschild radius ok that can be potentially more efficient than you know than nuclear fusion. However, this is actually a very compact object, ok. This is something to be this is a very compact object. You compare it, in other words the density of such an object is very very high.

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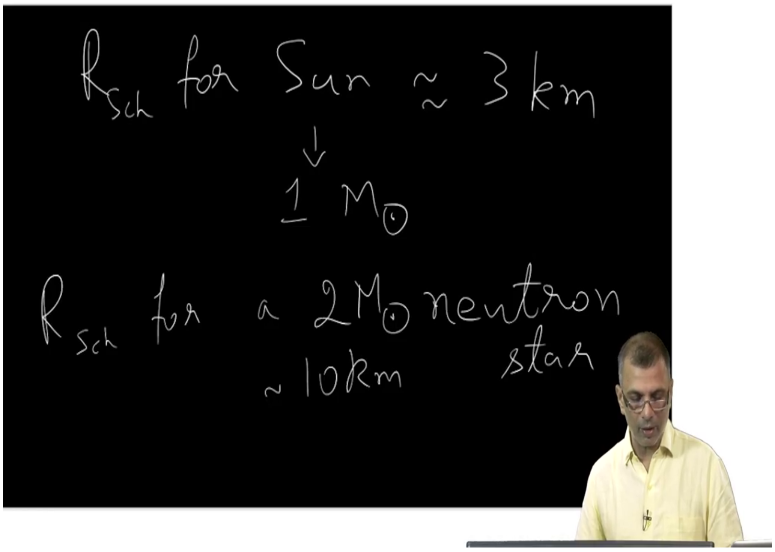

If you compare it with the sun ok I will tell you the Schwarzschild radius for the sun is something like 3 kilometers, ok and the sun of course, has one solar mass ok. However, the R Schwarzschild for a white dwarf 1 white dwarf is something like. So, this is something like 3 kilometers and or neutron star say 1. Let me erase this. Let me erase both of these.

Let me write down ok this would be something like 10 kilometer ok the R star. So, the point is you need really compact objects in order to get accretion in order to enable accretion in order to extract energy via this process.

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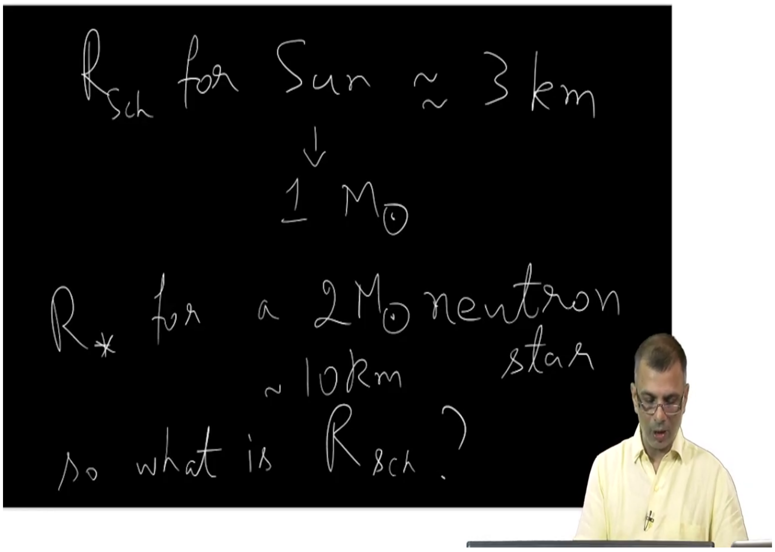

So, the reason we are interested in accretion the reason I should mention before that is I should say R star for a 2 solar mass neutron star is something like 10 kilometer. So, what is R Schwarzschild, ok, you can calculate this just from this formula and you will find that R Schwarzschild is much smaller, ok. So, this ratio will become more and more favorable the more compact the object ok.

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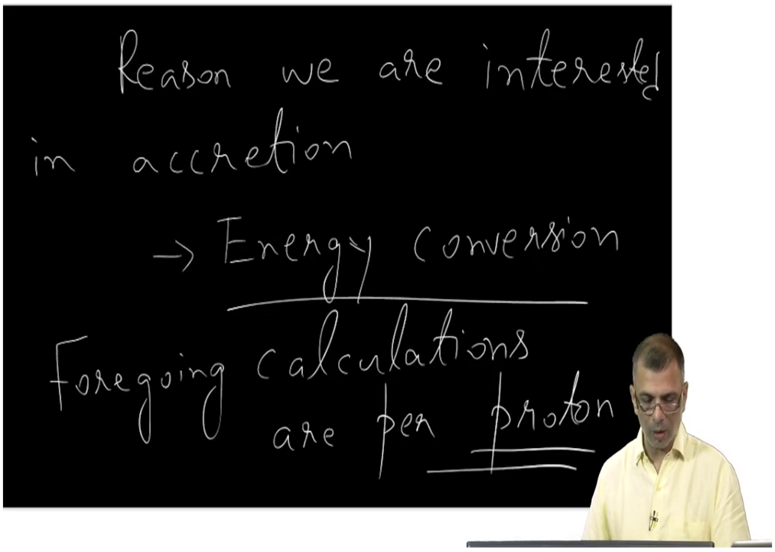

So, really the reason we are interested in accretion because of energy conversion, that is the basic reason we are interested in. This is the real reason we are interested in accretion, ok. Now, naturally you can imagine. So, this entire calculation that we went through was per proton, right.

We went through this entire calculation as a foregoing calculation are per proton, right because you see everywhere I wrote m sub p. So, all of this is for one proton. Now, naturally you can say. So, now, you see I suppose I try to accrete a large amount of matter ok many many protons naturally my E accretion will grow up, naturally, ok.

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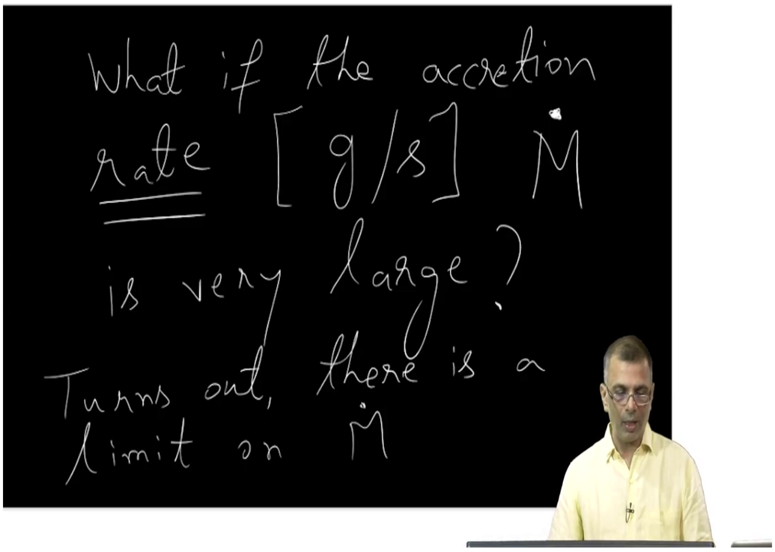

So, in particular what if so, what if the accretion rate in other words grams per second M dot and this dot denotes a time derivative, ok. So, the number of grams per second that I am accreting is what if the accretion rate is very large, ok. What if I am able to accrete many many protons within a second right onto the compact object that will surely you know give me a lot of potential energy release.

This is of course, assuming that I am able to tap that potential energy and convert it into luminosity. That is another matter, but let us not discuss that for the moment, ok. What if the accretion rate is very large? Ok. Turns out there is a limit on M dot there is a limit on the accretion rate M dot, ok and that limit is as follows um. Suppose the in falling matter comprises just of ionized hydrogen. So, there are just protons and electrons, right.

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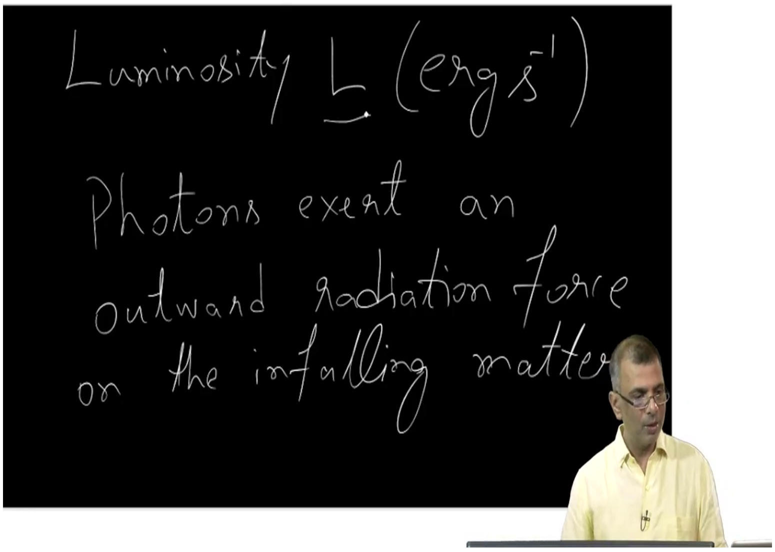

And, the central object luminosity of the central object and the luminosity is essentially ergs per second. This is the observed quantity ok. The luminosity of the central object is some L ok. So, the luminosity the object is luminous because it is emitting photons ok. So, the luminosity is so many ergs per second is due to photons ok and so, the photons exert an outward opposing you know outward radiation force on the in falling matter.

So, the point is you are accreting stuff ok you are accreting stuff because you want to extract the gravitational potential energy. And, suppose, you are able to do that somehow as we will discover the way in which gravitational potential energy is extracted is via viscosity either sheer viscosity in terms of accretion which is relevant to accretion disks or bulk viscosity which is relevant to spherical accretion that is the only way you can extract you can heat up the matter you remember viscosity is a dissipative mechanism.

So, you heat up the matter and when matter gets hot it emits photons and that results in luminosity is not it, but the problem with the rub is that this luminosity you know the object becomes luminous, it emits photons right and the photons carry momentum. We know this photons might not have mass, but they have momentum, right and the photons since their momentum they can press they can exert a force on the in-falling matter, ok.

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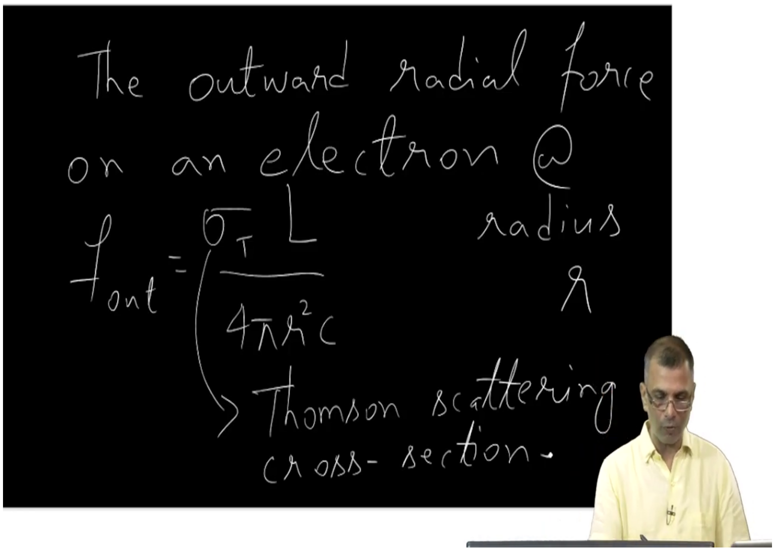

So, the outward radial force on an electron, now the mass is provided by the photon, but really electrons are the ones which radiate, ok and they are the ones which interact most effectively with photons, ok. So, on an electron due to this radiation is equal to f out is equal to this is the Thomson cross-section sigma t and there is the luminosity over 4 pi r squared c at radius r of course.

At a given radius this is the outward you know and this is the Thomson scattering cross section, ok. So, now, what about an electron? You see the electron is feeling the outward force, but the electron also drags a proton with it ok because the electrons and protons are bound via coulomb forces.

So, the electron is feeling this outward force, but it also that outward force is transmitted to a proton via the coulomb coupling and electrons and protons that better remain together in order to maintain charge neutrality, ok. So, essentially the outward radiation force is also felt by the protons, ok. It is also felt by the protons and these two forces you know the inward gravitational force.

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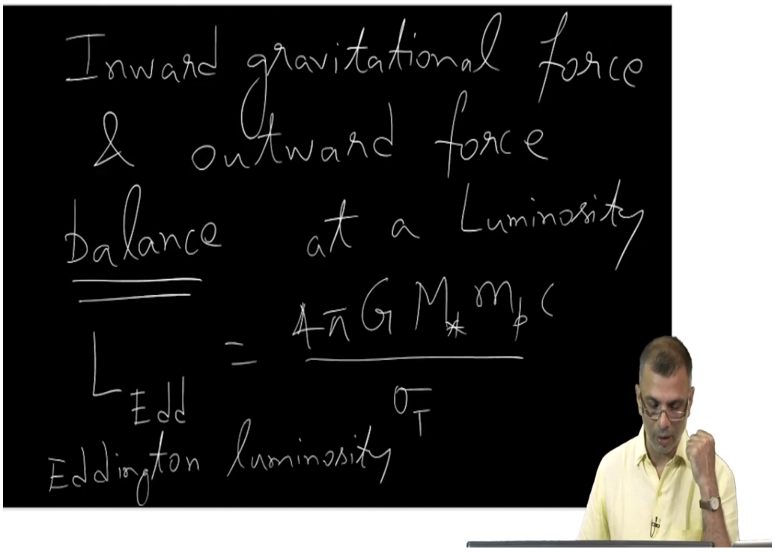

The inward which is of course, the inward gravitational force is mostly felt by the protons ok and outward the outward force due to photons due to photon coupling is felt by the electrons, but indirectly due to coulomb coupling with protons the protons also feel this force they balance at an Eddington luminosity at a luminosity called L Edd.

This is the critical luminosity at which these two balance and Edd stands for Eddington in honor of the famous astrophysicists Arthur Eddington ok and this is given by 4 pi G M star, where M star is the mass of the central object mp c over sigma T ok. And so, when the luminosity of the central object is larger than this value the surrounding hydrogen gas which is trying to accrete will be blown away by radiation pressure.

So, you cannot accrete beyond a certain and so, therefore, so, this is what is called the Eddington this is called the Eddington luminosity this one. So, what does this have to do with the accretion rate?

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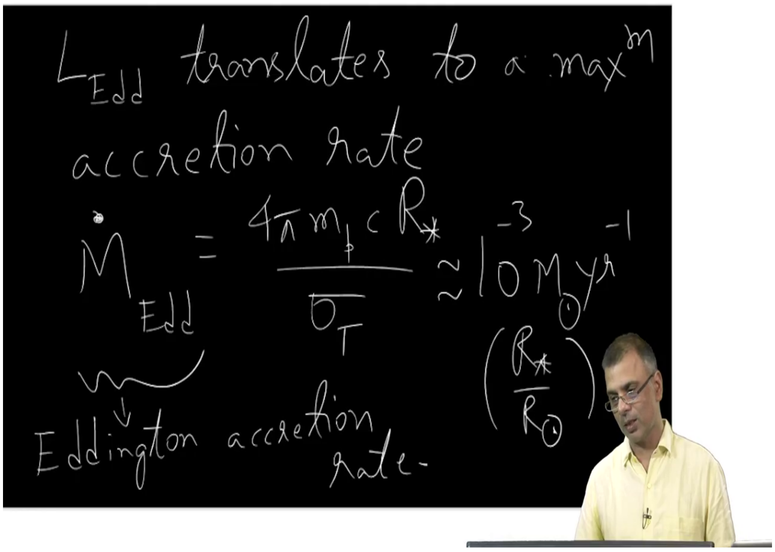

Well, if you work it out it will turn out that translates to well translates to a maximum accretion rate accretion rate M dot grams per second right the dot represents grams per second Eddington is equal to 4 pi mp c. So, this follows directly from the formula for the energy released during the accretion process and this L Eddington ok.

So, this is a maximum accretion rate and as it turns out in practical units this is something like 10 raised to minus 3 solar masses per year times R star over R sun. What this is saying? If you look at this kind of formula what this is saying is that if your object has a radius R star which is something like 1 solar radius ok it can at most accrete 10 raised to minus 3 solar masses per year. That is all it can accrete.

If we decrease anything more than this ok it will be blown up it cannot I mean the matter cannot settle down on the surface the matter will be blown off by radiation pressure ok. So, you cannot be too greedy you cannot simply try to accrete many many more protons if you want to increase the luminosity; there is a natural limiting thing, ok. So, this represents the upper limit on the accretion rate.

If of course, the compact object is larger ok R star is larger, then you can accrete more, but still there is a upper limit. You cannot accrete more than the Eddington accretion rate ok. So, this is really I mean this is one of the interesting things that arises and this is something that will not was not evident to us. We never had to discuss about accretion rates or for that matter and the amount of grams per second that was carried away in our in our discussion of solar wind.

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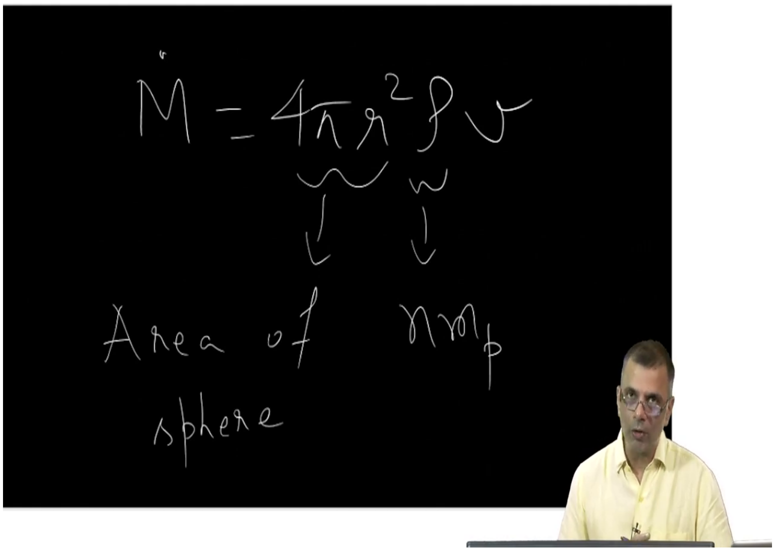

And, if you remember the accretion rate in terms of what we were discussing is simply m dot is simply equal to 4 pi r squared rho v right turns out that this is grams per second, right. So, 4 pi r squared is area of the sphere and rho would be essentially this would be area of a sphere and rho would be the density which turns out to be the number density times the proton mass assuming that it is fully ionized hydrogen and v is of course, the velocity.

So, everything really our whole discussion of accretion will hinge on this accretion rate because it is a very important thing after all. We are interested in accretion because we are interested in how much mass can be created and therefore, how luminous the object can be.

But, of course, we have just discovered that there is an upper limit to the amount of mass that can be accreted per second and that is the Eddington luminosity and of course, the Eddington luminosity this derivation of the Eddington luminosity presupposes spherical accretion ok.

So, that is appropriate to our discussion right now. But, people generally use the concept of Eddington luminosity even when they talk about say you know accretion disks. It is just a useful sort of a useful thing a useful benchmark to compare against ok. So, as matter falls in ok matter gains speed, matter becomes less dense so on so forth and but however, this whole thing remains constant. So, as it gains speed it becomes less dense and this also decreases.

So, therefore, this is the kind of thing we will be discussing and we have already discussed the basic solutions and so, as we go ahead what we will do is we will take recourse to something called again this is something that was inherent in our discussion of the solar wind also, but because it was not so physically important we never really paid too much attention to it.

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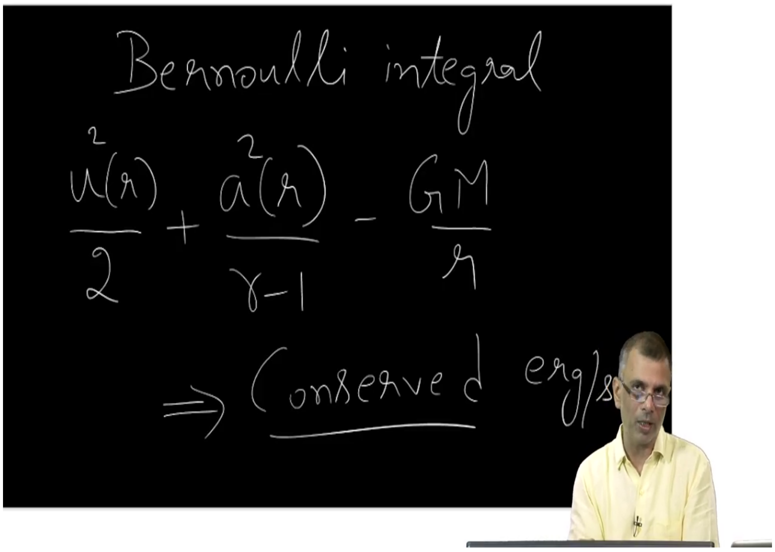

And, we will take recourse to the Bernoulli to discussions of what is called the Bernoulli integral which we have seen earlier right which is essentially the energy per second. So, it would essentially be at any given radius it would be u over 2 right plus p over rho which essentially in terms of the sound speed it would become like this is of course, assuming a polytropic relation of state minus and this is always conserved ok.

So, everything that we have been doing so far by way of the sonic point how du dr behaves and so on so forth is all the same. It is just that the emphases are slightly different ok. So, we will not repeat the discussion, ok. We will assume that you know you are already familiar with the discussion that has taken place, but we will recast the conserved quantities and everything in terms of this Bernoulli integral ok and this.

So, when we take up this discussion once again we will use this conserved energy this is conserved ergs per second. We will use this quantity and derive several useful several interesting properties that have specifically to do with accretion with spherical accretion onto a compact object.

Thank you.