**Fluid Dynamics for Astrophysics**
**Prof. Prasad Subramanian**
**Department of Physics**
**Indian Institute of Science Education and Research, Pune**

**Lecture - 35**
**Solar wind: Modifications in Parker's solution**

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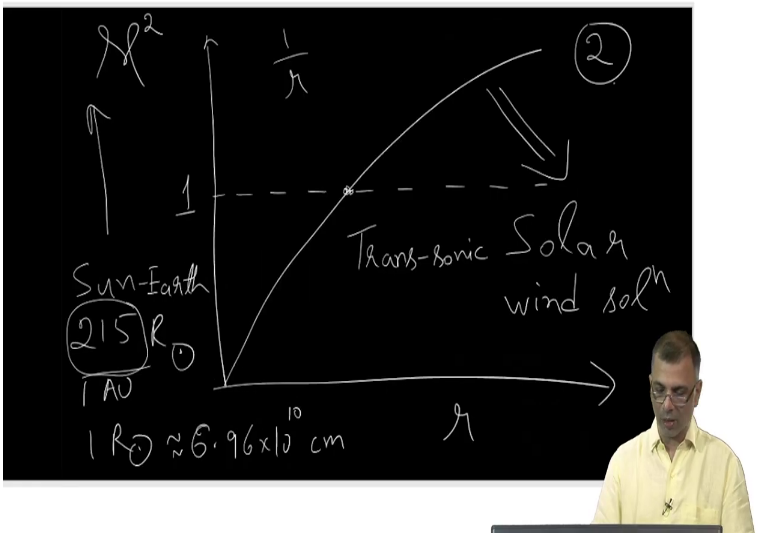


So, hi, let us now briefly review the Parker solar wind solution. And comment on a few facts we already commented on a few complications to the Parker solar wind solution, the fact that you know although it is a nice elegant solution, in reality there are several issues you know associated with it. Several other issues such as magnetic fields, and so on so forth, right.

And but let us now consider a couple of other aspects right. So, just to recap, this is the way you know a typical transonic Parker solution – the ideal Parker solution looks like. So, this would be increasing you know radius from the center. So, the sun would be essentially at the center.

And on the y-axis you have the Mach number square. And these are solutions of the kind 2, we saw all the solutions together on the same graph, now we are concentrating only on the solar wind solution. So, this would represent the solar wind solution, right. And so what does this represent? Well, so the Mach number is plotted on the y-axis, and this is where Mach number equals 1.

So, he this represents a kind of solution, which starts out subsonic at the very center right ok. You know you might wonder at the very center, you see you know because we are taking Newtonian gravity which goes as 1 over r; exactly r equals 0, the gravity blows up. So, what is the problem? So, that there would be a problem. Let us sort of sidestep that problem, let us not go to the very center of the sun.

But, let us simply say that these, represent very small radii ok, say 1 solar radii or something like that, 1 solar radius. Just for context the sun is 1 solar radius that is about 1 solar radius is about 6.96 times 10 raised to 10 centimeter. And the Sun-Earth distance is 215 solar radii ok. So, we are 215 solar radii roughly speaking away from the sun. And this is a typically called 1 astronomical unit, ok. These 215 solar radii, this is typically called 1 astronomical unit.

Just to tell you that you know 1 solar radii is really very small even in the context of the inner solar system, although by you know it is 6.96 times 10 raised to 10 centimeters. So, it is quite large. And the Sun, the Earth would fit very easily, and lots and lots of Earths would fit very easily into the sun.

But by way of distances in the solar system, it is really quite small because you know 215 suns could fit in between the Sun to the Earth ok. So, that is the context that is the kind of picture you should have in your mind. Well, those are the kinds of numbers you should have in your mind by looking at this you know x-axis right.

So, this would be a trans-sonic solar wind solution right ok. Why trans-sonic? Because it passes through it passes smoothly through the sonic point, this is where the sonic point is. Now, before proceeding, you should realize that this is radius and spherical coordinates.

So, you know it is just radius. So, when we say sonic point, we really mean a sonic surface ok, spherically symmetric sonic surface where the Mach number passes smoothly through 1 ok. Now, the one thing now everything that I am I will be talking about right now comes from the following paper Shivamoggi and Rollin.

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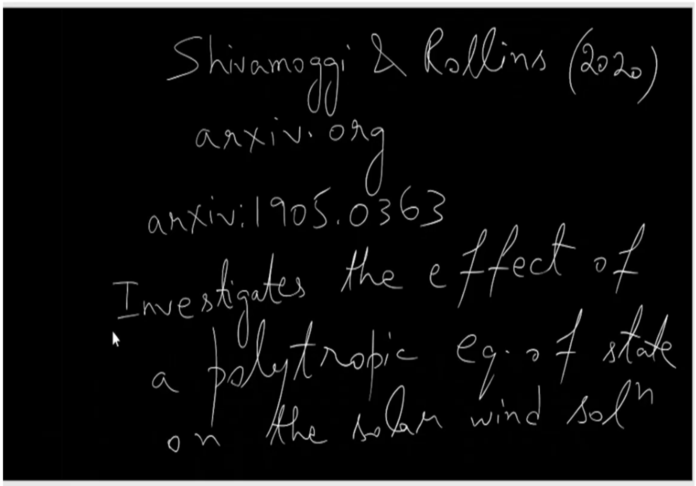


And so I should give Shivamoggi and Rollins. And this can be downloaded from the ok; this can be downloaded from this site. And the reference is 1905 dot 0363, this is the reference ok. So, this is where I am getting the material where I am talking about right now. This is very recent. So, I thought I would this is a research paper. So, I thought I would discuss the gist of this research paper for a little bit, ok.

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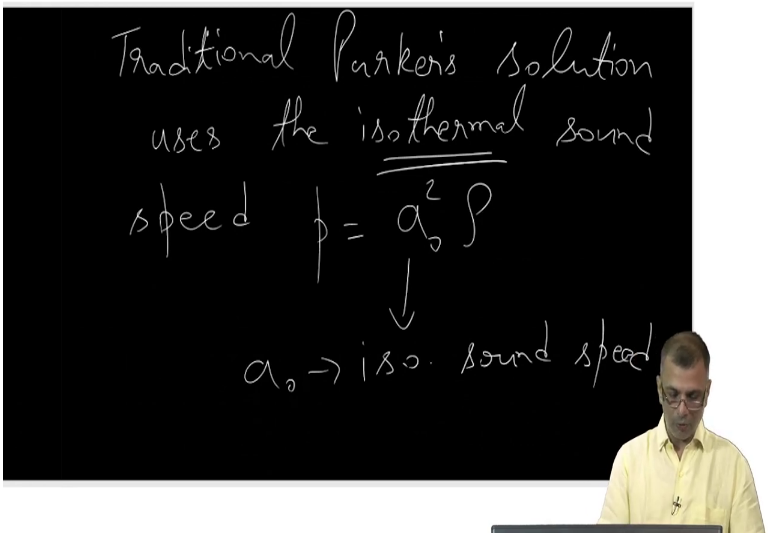


Now, remember the traditional Parker's; the traditional Parker's solution uses the isothermal sound speed ok, where p is related to rho using p equals a naught squared rho, where a naught is the isothermal sound speed. This is how we did our derivation ok.

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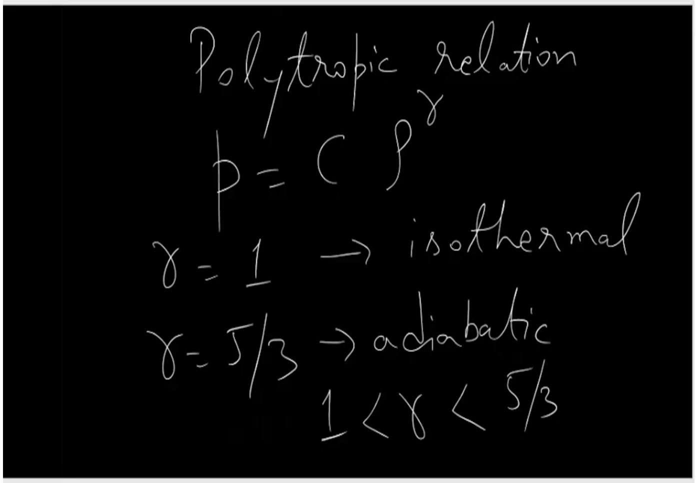


And another way of writing it is, if you use a polytropic relation between p and rho, and we write p equals C rho raised to gamma. Gamma equals 1 is isothermal; gamma equals five-thirds is adiabatic ok. And in using this kind of sound speed, we are assuming gamma equals 1, of course, and that is what the traditional Parker's definition you know uses. In general, of course, gamma is somewhere in between 1 and five-thirds ok.

And C is of course, an arbitrary constant. So, this is the general polytrophic relation. And we will see what this paper does is it sees it investigates the effect of a polytrophic equation of state on the solar wind solution. Does the solar wind solution become faster, so you know on Parker's solar wind solution? That is what this paper does.

In other words, it tries to see if this gamma was not exactly equal to 1, yeah, if it was between 1 and five-thirds what happens ok. Now, why are we so interested? Why is this problem even interesting? You know people write papers, people you know come up with interesting mathematical consequences and so on so forth. But it is very important to know to figure out why it is interesting, why is this even worth paying attention to right?

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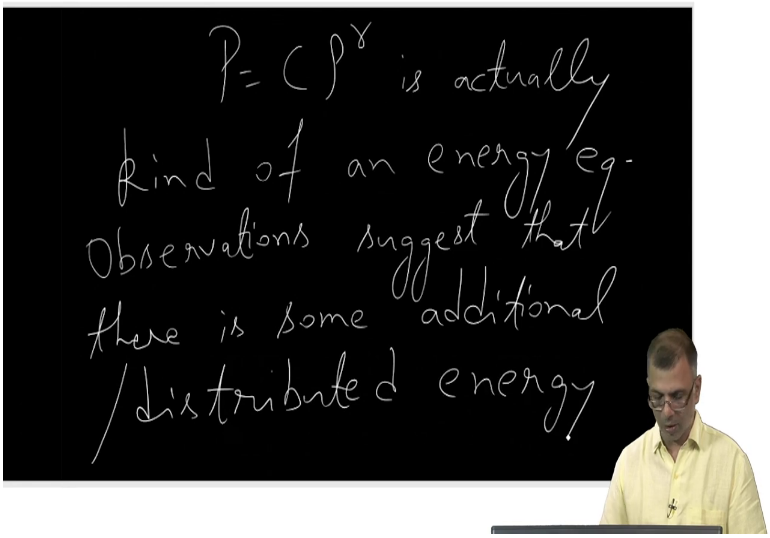


The reason it is worth paying attention to is because you know the polytrophic the equation of state this one is actually kind of an energy equation. If you remember we never used the energy equation anywhere in deriving the transonic solution. We only use the conservation of mass and the conservation of momentum; those are the only two equations we used.

It is only via this kind of a closure relation between P and rho that we introduce the idea of, you know energy conservation. And why is this energy equation important, or why is it that we even need to investigate anything other than this? That is because observations review right, observations reveal that the velocity profile predicted by Parker's isothermal – this kind of solar wind solution are a little too low in comparison to observations ok.

And the temperatures, in particular, the temperatures are a little too low in comparison to what this kind Parker solar wind solution together with this kind of a relation predicts ok. So, the net upshot is that observations suggest that there is some kind of additional or a distributed , energy deposition in the solar wind over and above what a Parker solution predicts.

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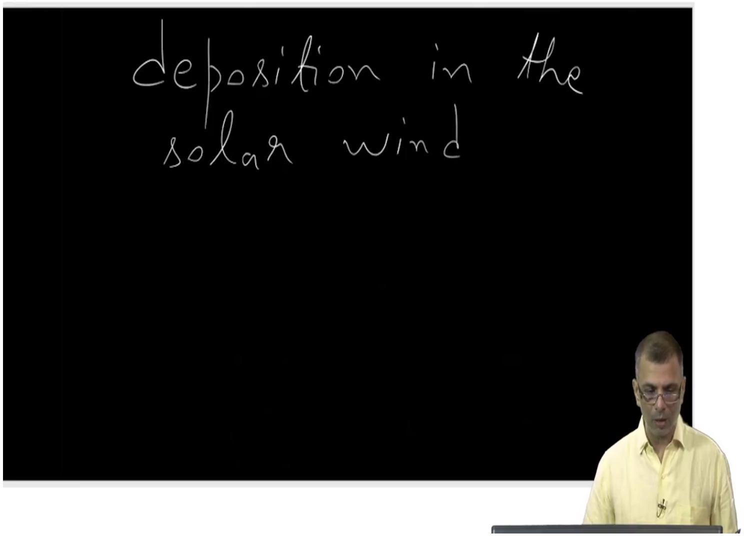


Parker solution simply takes a very hot solar wind, and it let us it cool ok. As the solar wind cools, it accelerates. The thermal energy is converted loss lessly so to speak is converted nicely into the bulk kinetic energy of the flow that is what this solution with the isothermal equation of state predicts.

Now, turns out that this does not quite you know agree with the observations, observations suggest that there is some kind of additional and by distributed I mean all throughout the solar wind that is what I mean. All throughout the solar wind as in it is not at 1 radius or 2 radii or so on so forth.

It seems like there is some additional energy deposition for a while as the wind flows out ok. So, and this can be captured via a polytrophic equation a state like this ok, in particular by varying the value of gamma ok. So, that is it really, so the upshot is that one goes through this entire exercise and says instead of writing.

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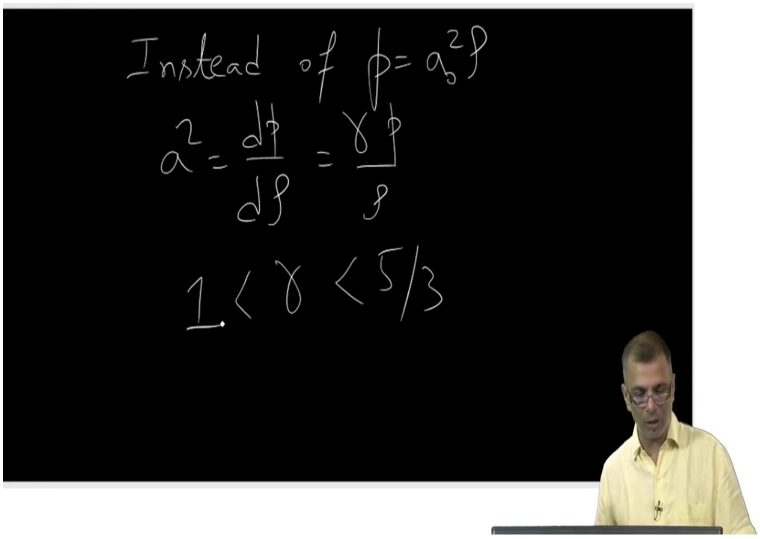

Instead of writing, and I beg your pardon I instead of writing p equals a naught squared rho, we write a square equals dp d rho equals gamma p over rho you write this. So, this would be new definition of sound speed ok where the gamma can be between five-thirds and 1 alright.

And then one goes through the entire derivation just like that, no difference ok, one just goes through the derivations as it is. And the upshot is essentially I am not going to you know reproduce the entire all the calculations, but the upshot is a graph something that looks something like this. The net result is a graph that looks something like this.

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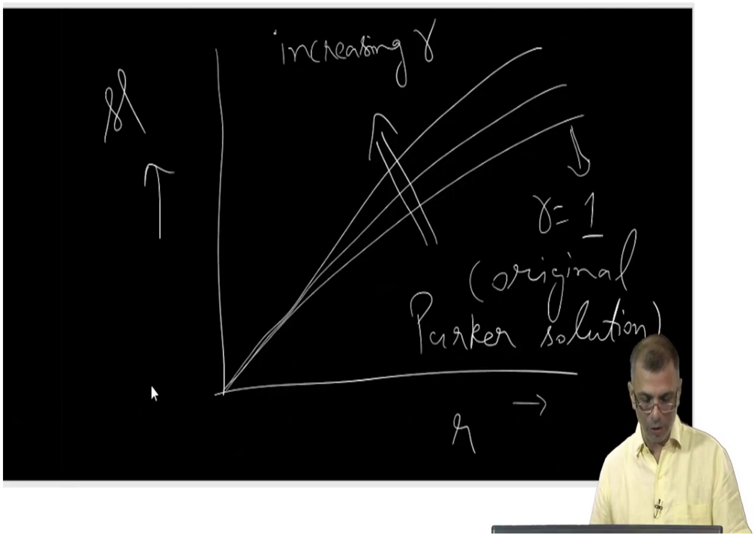


You would have r normalized to some you know and this would be the Mach number. And all the solutions like this. And this would be, for instance, this would be gamma equals 1 ok, this curve ok. And in this direction you would have increasing gamma ok. So, this would be the isothermal this is the original Parker wind solution ok, this kind of a curve.

And if you increase the value of gamma to above 1, remember the adiabatic is gamma equals five-thirds, which is something like 2.2 r, and if you increase the gamma even a little bit ok something like 1.1 or one point o 1, and 1.1 or something, what happens is, what you find is that the Mach number increases. You can see that the Mach number for this solution is larger than the Mach number for this solution.

And what distinguishes this from this? The gamma here the gamma for this solution is slightly larger than the gamma for this solution, ok.

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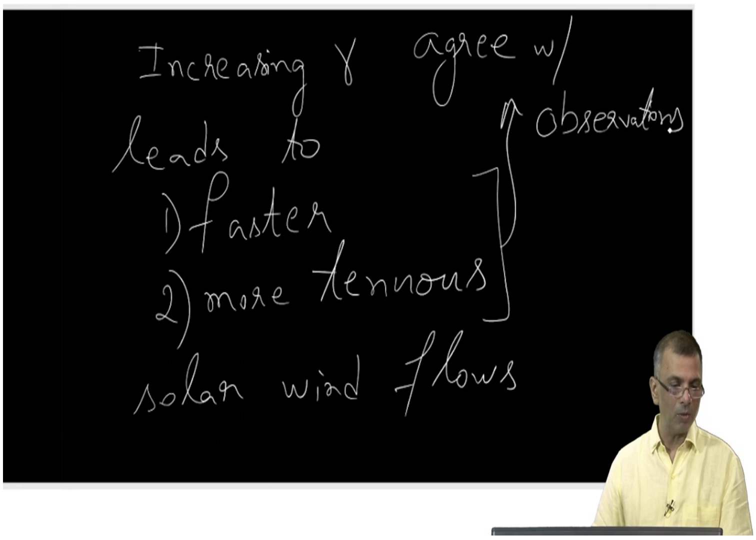

So, what the upshot the main conclusion of this paper is that you know increasing gamma leads to faster and more tenuous solar wind flows. Why faster? Well, here you go, this is what the solutions look like. So, you increase gamma from here to here, and clearly the for a given radius the Mach number here is larger than the Mach number here. And the Mach number here is even larger than the Mach number here. So, that you know justifies this.

And if you go through the solutions again, you will find that the densities when you increase the gamma the densities drop; in other words the solar wind solutions are more tenuous ok. And these kind of agree with observations, ok. So, this is one modification from the standard Parker wind solution that I thought I would point out. There are some other interesting modifications ok. And I will discuss them one by one.

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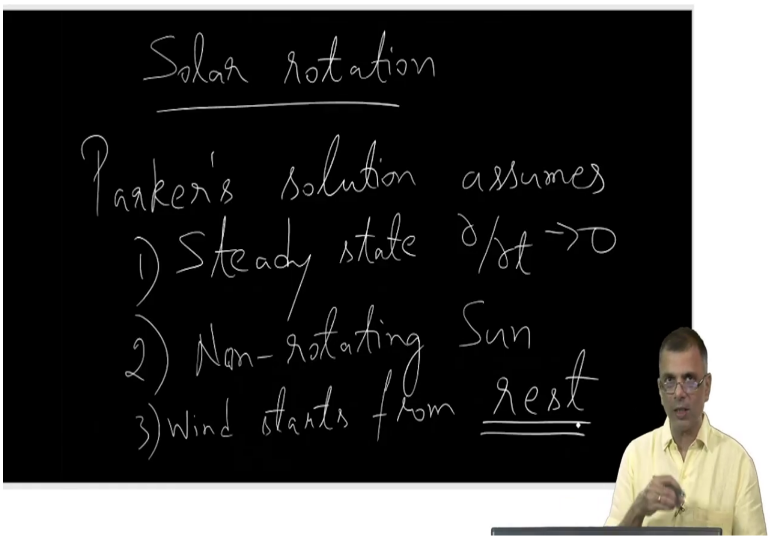

The other interesting modification has to do with solar rotation ok. Now, what about this? You see Parker solar wind solution assumes one steady state which is to say if you remember nowhere, we essentially all these partial over partial t's were all taken to be 0 in all our equations if you remember ok, so that is what I mean by steady state, and a non-rotating sun, right.

This is what Parker's solution assumes. It assumes that you have you know a steady state and that is true for all the solutions that we discussed tested is this is common, and we are not going to relax this in what we are discussing now. But, we are going to try to you know examine the effects of the fact that the sun actually rotates, it is well observed. It is well observed that the sun is not you know stationary, and it is actually rotating.

This is well-known ok. So, what about it? So, how would it you know possibly modify this whole thing? And also the fact that the other thing is Parker solution also assumes that the wind starts from rest; in other words the initial velocity of the solar wind is strictly equal to 0 that is the reason you know traditional solution this kind of solution actually starts from 0.

But, surely that is not the case, surely because there is some you know finite pressure you know the wind cannot be starting from 0.

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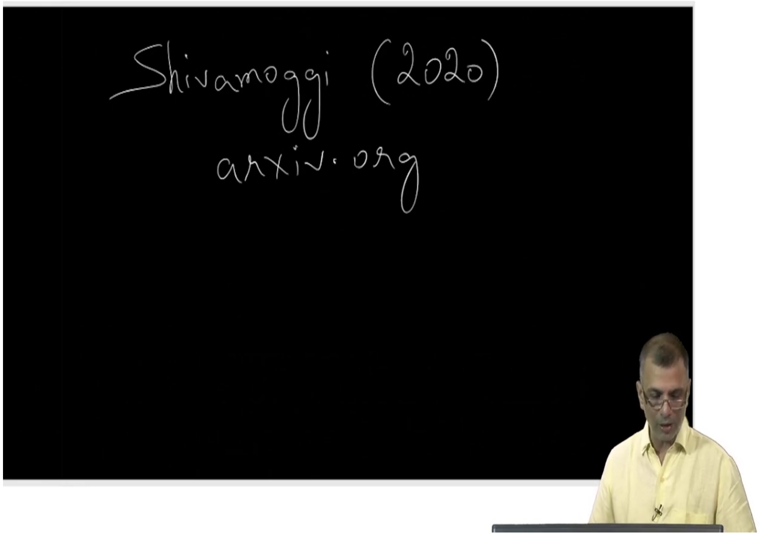


So, what this, the following discussion is taken from a paper also by Shivamoggi, but it is a different paper ok. And it can be again downloaded from and right. So, this can also be downloaded from the archive right. So, suppose let us relax this assumption first. Let us relax this assumption the fact that the wind starts from rest. Let us relax this assumption. And how are we going relax this assumption? Well, in the following way.

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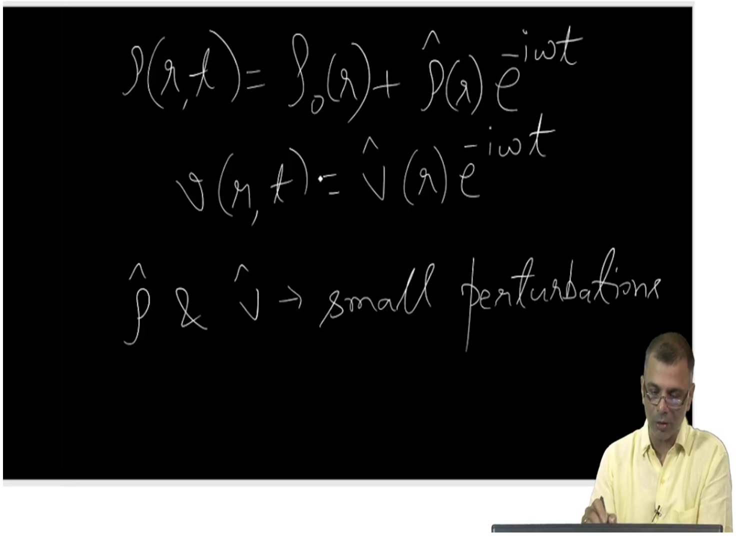


What we are going to say is that instead of let us write the density as a function of radius, and now instead of taking a strictly stationary solution let us allow the density to be a function of t via the following manner. The background density is still steady state, but we introduce a small perturbation right, the density, and the velocity is also a function of the radius and of time and that is we assume that there is a very small velocity, which is a function of r, and it is a function of time through this. This is the assumption ok.

So, what is the deal here? You have a background density, and you have a perturbation. So, rho naught this and this are small perturbations ok. And we know while linearizing and everything we know what the meaning of this word is small perturbations we know this. In other words, this entire term is very small in comparison with this. So, we allow for the fact that the solar wind starts with a small velocity v hat from the surface ok. And we see what happens ok.

So, what we do is instead of in the mass and momentum conservation equation we substitute this rho and this v ok. And we know that the background density you know satisfies the background equations, and it was assumed that the starting velocity was 0 ok. So, now the starting velocity is some small quantity.

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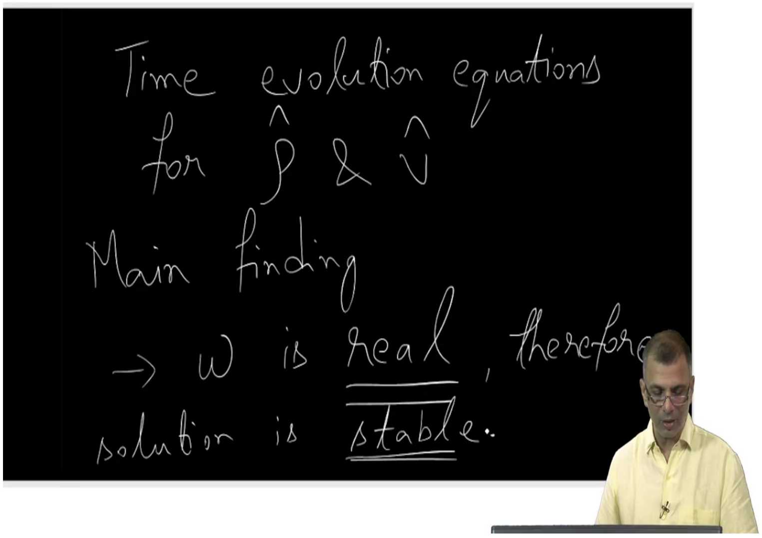


So, what we would do is one would find out evolution the logic of this is one finds out the evolution the time evolution equations for the perturbed density and the perturbed velocity ok. And the main finding, the main finding is that omega is real. What does this mean? Let us look at this equation again omega is real and this is the assumption.

So, if omega is real, this is oscillatory. The e raised to i omega t this is oscillatory. What if omega was not real? If omega was not real, this would not be oscillatory, this would be either a growing or a you know or a damp solution. And in particular if it was a growing solution, you know there would be a problem if it was a growing solution what it would imply is that the presence of a nonzero velocity ok, if it was a growing solution.

In other words if omega was not real, if it had a complex part, it would imply that the presence of a finite starting velocity is enough to render the Parker solution unstable ok. Whereas, that is not the case omega is real, therefore, the solution is stable time stable. In the sense that as a function of time the small perturbation does not grow ok it only oscillates it does not grow with time ok.

So, in that sense, these perturbations are stable. And so therefore, you can allow for a finite starting velocity. This is the main upshot of you know this discussion.

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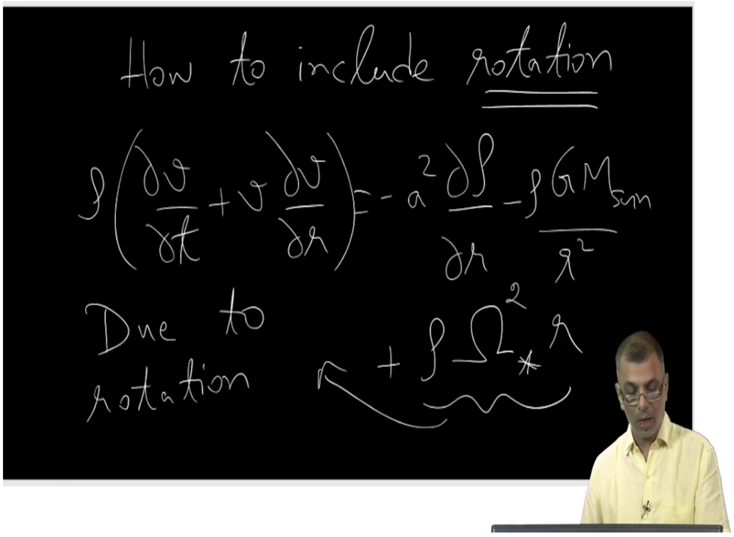


The other upshot of the discussion, the other main thing about the discussion is how to include rotation, how to include rotation. The way you include rotation is you modify the momentum equation which to look like the sound speed is generally called a in this paper. So, I am simply writing a.

So, a is essentially what we used to call c s squared ok. So, this would essentially be the you know this would be the pressure gradient term. So, this entire thing is m a of course this would be the pressure gradient term. And you do not see pressure because you have the sound speed, so it is expressed in terms of the density. We have seen all this earlier and G M sun. This would be the gravity term right.

And now, you have a very important addition, you have a centrifugal term this is due to rotation. This term is due to rotation. And omega star is the rotational velocity of the sun.

This is how your include rotation ok. Now, the recipe is the same. You assume this kind of solution ok.

You assume a small perturbation and density and you assume a small perturbation and velocity. And you put it through the two equations mass conservation and momentum conservation, except now the momentum conservation equation is modified like this.

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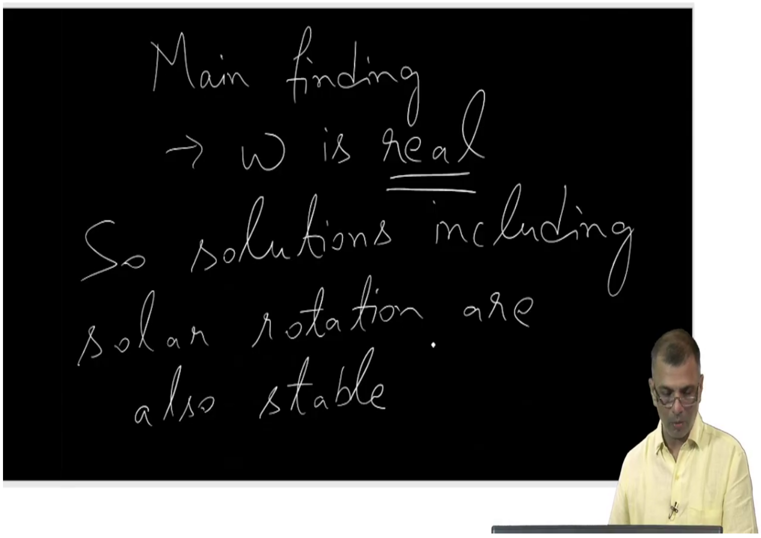


And the finding and the main finding is that omega is again real. So, solutions including solar wind solar rotation are also stable for the same reason because you know you look at this because omega is real the perturbations are only oscillatory, they are not growing or damping without in particular growing is the thing that we are focusing on, the perturbations are not growing without bound, and therefore, the solutions are stable.

So, these are the two modifications to the Parker solar wind solution, spherically symmetric again we are not considering the effects of magnetic field, which disturbs spherical symmetry we talked about this the last time. There is a fast solar wind and a slow solar wind.

But here I thought I would present these couple of interesting results that were published quite recently, and these include the effects of a non-zero starting velocity and includes the effect of solar rotation, and tries to investigate the effects of these physical situations on the stability of Parker solar wind solution.

Thank you.