Fluid Dynamics for Astrophysics
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Lecture - 32
Spherically symmetric transonic flows

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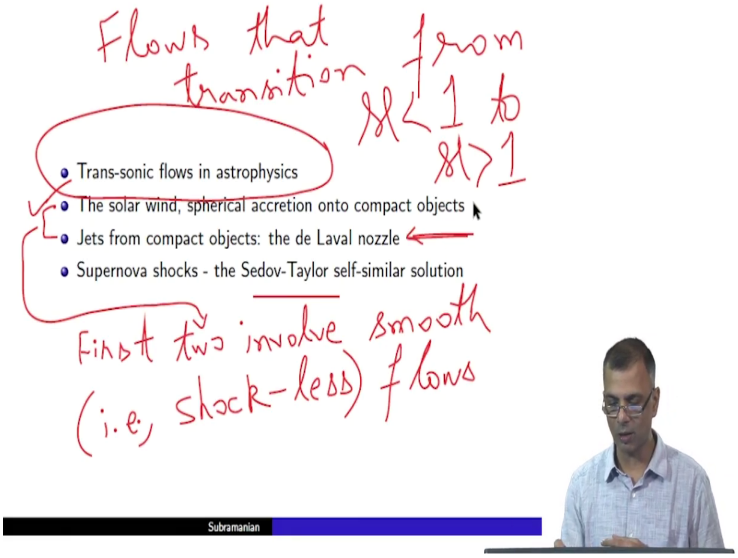

So, hi. We have now, we are now in the middle of discussing astrophysical applications. You will be happy to know that we are comfortably passed the stage where you know we have discussed most of the basics of fluid dynamics that are needed to understand certain applications to astrophysics.

Specifically, we have already, you know we are discussing this thing trans-sonic flows in astrophysics, right. In other words, flows that transition from subsonic to supersonic that is what trans-sonic means, right, ok.

So, the this is what we are discussing. And we have there are several applications and we have already you know just discussed this, jets from compact objects and the application of the concept of a the Laval nozzle to that, right. And the next thing, we will discuss is this. We will discuss two things at the same time, two very disparate, two very different astrophysical situations the solar wind and spherical accretion onto compact objects.

We will discuss this both at the same time. And then we will discuss supernova shocks and in particular we will discuss one particular kind of solution to the supernova shocks which are called Sedov-Taylor self-similar solutions. We will explain all these all this terminology when we get to it, right.

So, the important distinction I want to make between these two applications and the third one is that the first two applications, these two involve smooth i.e, shock-less flows whereas, the third one obviously, involves shocks, right.

So, there is a conceptual difference between these 3, but we are now having studied compressibility and the speed of sound, and in particular trans-sonic flows we are in a and also shocks as we you know along the way, we are in a position to handle all these 3 applications, ok. And time permitting we will also discuss accretion disks onto.

So, this would be spherical accretion. There is also something accretion also happens in the form of a disk and so, we will that also involves a fair amount of fluid dynamics concepts and so, we will also discuss accretion disks as we go along. So, let us first consider this well we have already considered jets from compact objects we are done, the number 2 is done. Now, let us go on to number 1 here, yeah.

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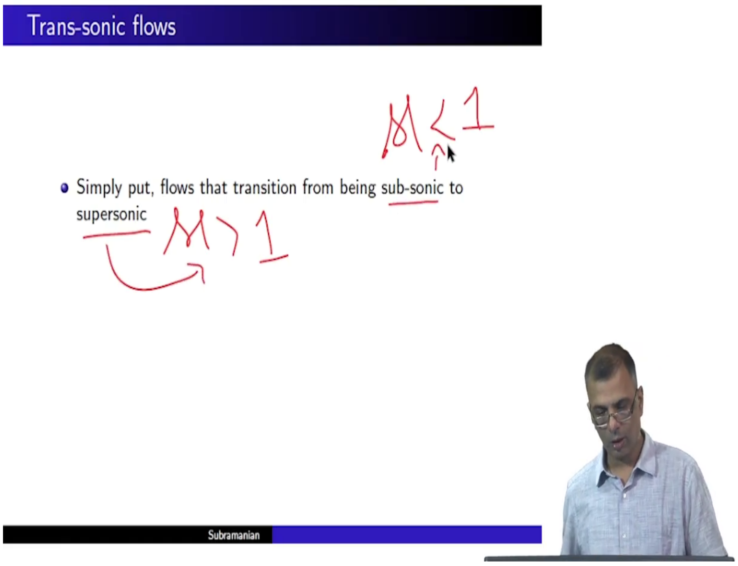

So, what are trans-sonic flows? We already said this, but no harm in emphasizing this a little more. Simply put flows that transition from being subsonic, in other words from Mach number less than 1 to supersonic, Mach number greater than 1 supersonic would be this and subsonic would be this. Or for that matter supersonic to subsonic either way, ok. And this transition can be either smooth or via a shock in particular generally shocks tend to address this, this kind of situation.

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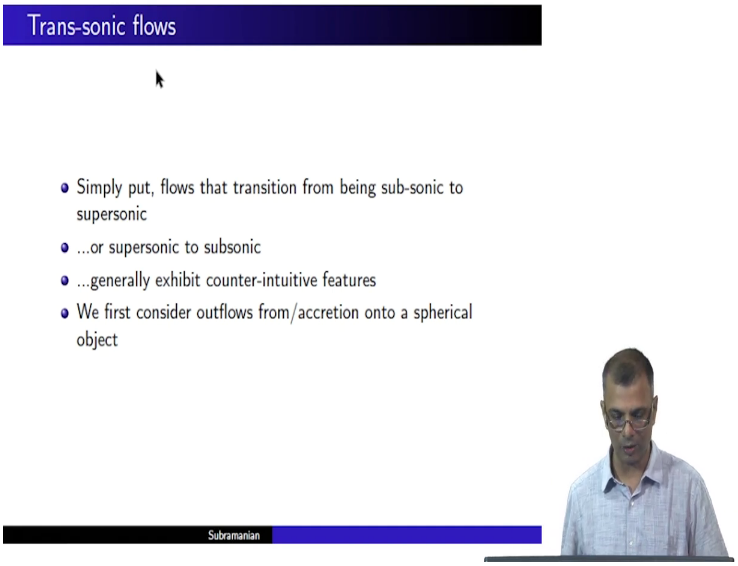

So, you realize how shocked flows as well as smooth flows, smooth transition flows both of them fall under this under this trans-sonic category, ok. They generally exhibit counterintuitive features, ok. We have seen for instance in our discussion of the Laval nozzle how some of these flows can be quite strange, ok.

The diverging nozzles lead to accelerating flows when they are supersonic and so on so forth, yeah. So, we first consider outflows from or accretion onto a spherical object, ok. Probably good to sketch this a little bit. So, what do you mean by outflows from a spherical object?

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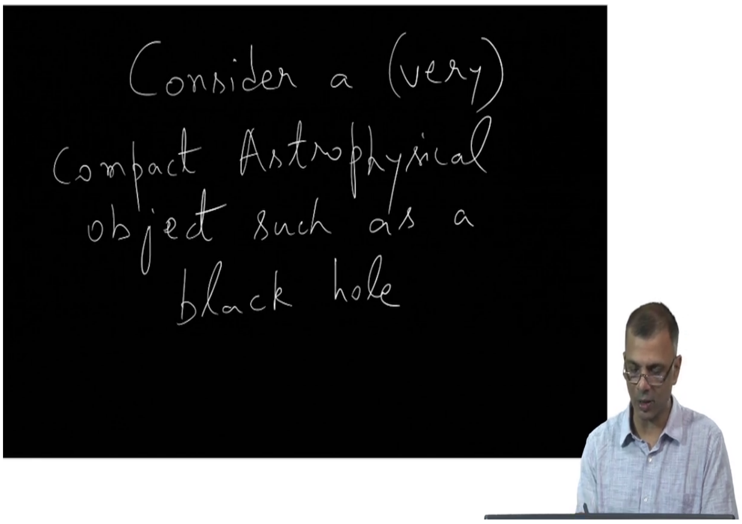

Well, consider you know consider a compact object, a very compact astrophysical object such as black hole or a neutron star for that matter. Now, what is with? This what is special about this? There is a extremely compact in other words extremely high density objects sitting here, like so.

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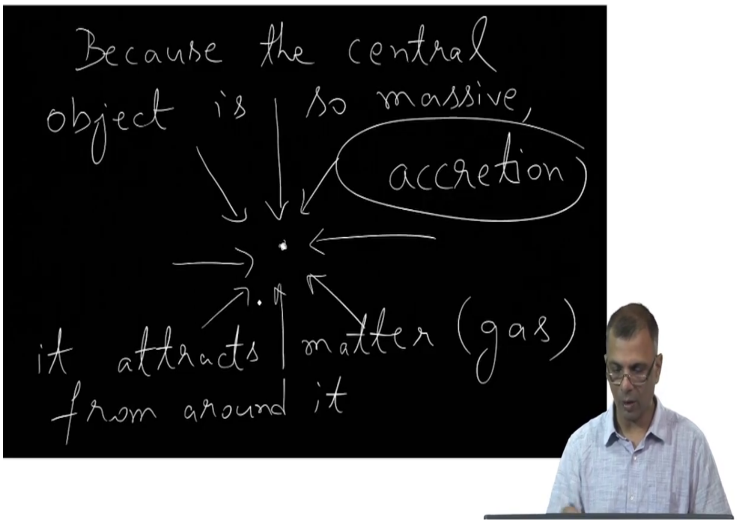

So, it is quite massive, but because it is very high density it is its almost a point object its really small, ok. Now, what does this? You put an extremely high-density object in the middle of the interstellar medium, which is not empty as we always say you know outer space is not empty, it is just very thin, very tenuous, ok.

So, on account of it is high mass it attracts the matter simple gravitational attraction, it attracts matter around it. So, matter because the central object is so massive, it attracts matter in particular gas from around it. Another way of in the matter gets attracted and it either in case of a neutron star or something it settles on the central object or in case of a black hole it gets swallowed by the black hole.

So, this process of attracting an accumulating matter onto it is called accretion. Look up the if you are not familiar with the word, you can look it up the dictionary meaning of it accrete means to accumulate. So, because the central object is so massive, it attracts matter from around it, yeah, causing it to accrete.

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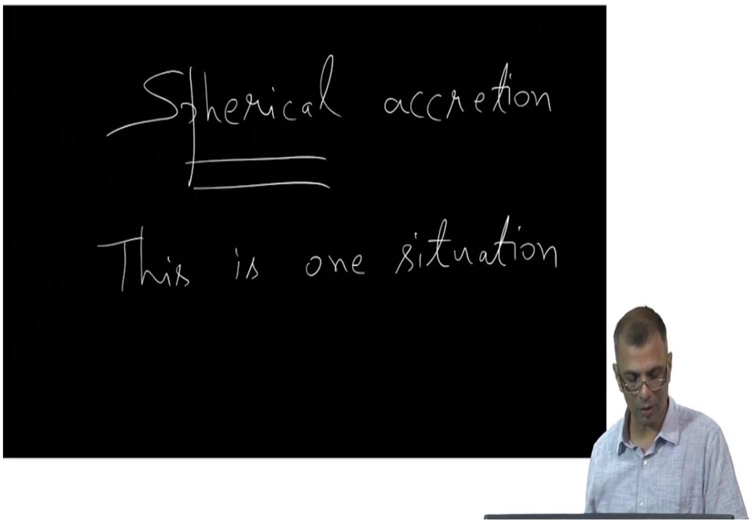

We will consider the special case of spherical accretion. Why? Well, partially because you know it is astrophysically important, there are many many situations where there is an isolated, isolated black hole just sitting there by itself and so, it accretes quasi in a quasi-spherical manner, right. And also, because you know the spherical symmetry leads to several mathematical simplifications.

So, it is simple, ok. So, this is one thing we will consider. This is one, this is the first situation, one situation we will consider, is one situation.

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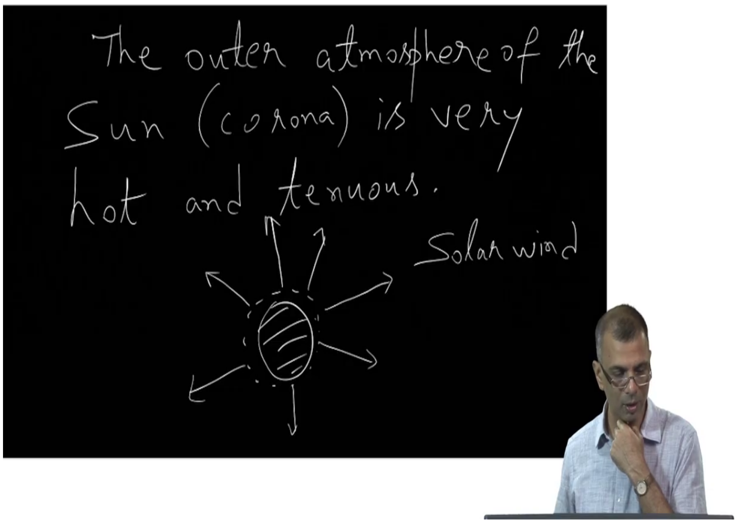

The other situation we will consider is the sun, the outer atmosphere, now the sun you realize is nothing like a black hole it is just a regular it is just a regular star. And you know it is not all that massive, it is just one solar mass, right. But, the main thing about it the outer atmosphere sorry of the sun, our sun which is called the corona which is very thin and quite hot, and it is not directly visible to us except during eclipses, right is very hot and tenuous, very thin, ok.

And what happens is the, you have a sun , yeah, and its outer atmosphere out here say, right it is so hot, that the pressure is there is enough for it the outer atmosphere is so hot that it can actually escape the pull of the sun's gravity. It is somewhat like water boiling half of the surface of a pan that you know that you keep.

The outermost you know surface of the boiling water the molecules there have enough kinetic energy, to escape and they go off, right. It is somewhat like that. It is a little more detailed than that and we will work out the mathematics, but still, ok. So, that is what happens here.

The outer atmosphere is so hot that the outer atmosphere essentially escapes, escapes in a spherically symmetric fashion, at least to first degree, it essentially escapes in a spherically symmetric fashion to form what is called the solar wind. So, there is a wind blowing out from the outer atmosphere of the sun, and as such we, it the wind permeates much of the solar system certainly it flows past the earth, and the solar wind I think it is been observed as far as the orbit of Neptune, ok.

So, now, by way of cartoons notice the similarity between this, and this cartoon. Here too there is a spherically symmetric inflow whereas; here there is a spherically symmetric outflow, ok. So, what we are going to do is we are going to study spherically, so it is simply I mean spherically symmetric inflows and outflows you just reverse the direction of the arrow one becomes the other, right.

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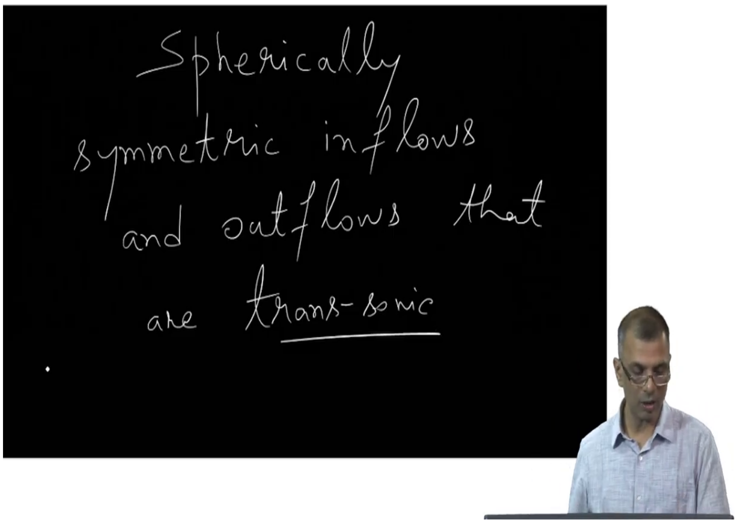

So, we are going to study spherically symmetric inflows and outflows that are trans-sonic that is our brief, ok. In case of the sun, the outflow starts out being subsonic from very close to the sun very close to the outer atmosphere of the sun, which is the corona it essentially starts from rest, ok. It has a finite velocity that is why it is boiling up, but it starts out subsonic and it accelerates and it becomes supersonic.

Whereas, here, here what happens is this is a spherically symmetric inflow and you know very far from the compact object the stuff the gas is essentially at rest.

So, it is subsonic. And as it gets attracted towards a compact object it gains speed and it becomes supersonic. Either way these are trans-sonic flows, ok. So, this is you know the subject of our, you know discussion we will first consider outflows from or accretion onto a spherical object.

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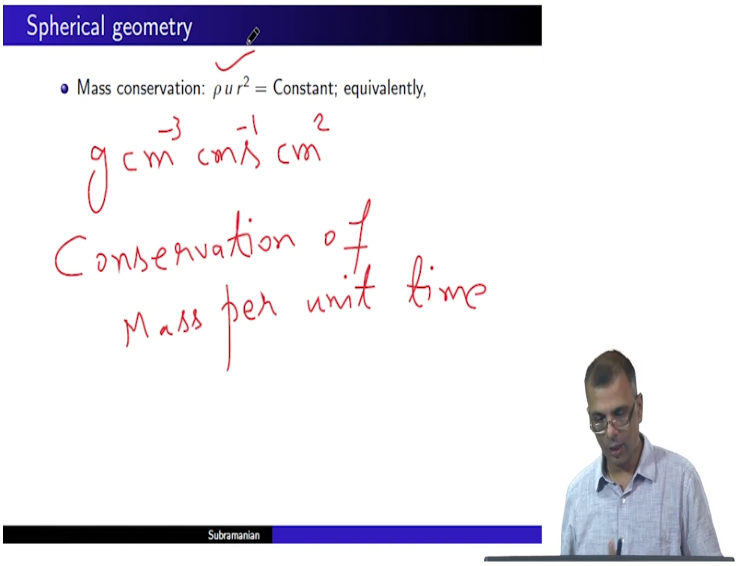

So, in spherical geometry the first thing we consider is mass conservation and we say rho is the density, u is a velocity, and r square is simply the radius in spherical coordinates and we say rho u r squared equals constant. So, as with everything we call this mass conservation. But what is it? Really conservation of what? Right. So, the rho is in grams per centimeter cube, is not it. So, that is what the rho was, right.

The u is a velocity, so you have centimeter per second, and r squared is essentially centimeters squared. So, you can see that we are really discussing conservation of mass per unit time, grams per second sorry, grams per second the centimeters. So, where did this r squared come from? Well, you know what you do is you consider the mass flux into a sphere. And the mass flux you remember is grams per centimeter squared per second, ok.

And what is the area of a sphere? It is 4 pi r squared, right. So, you multiply the area with the mass flux and you get, so that is where the r squared comes from and you get mass per unit time, and the constant the 4 pi is simply absorbed into the constant, ok. So, that is where this thing comes from, ok. But as with our discussion of you know the Laval nozzle it that this integral statement is not as useful to us as the differential statement, ok.

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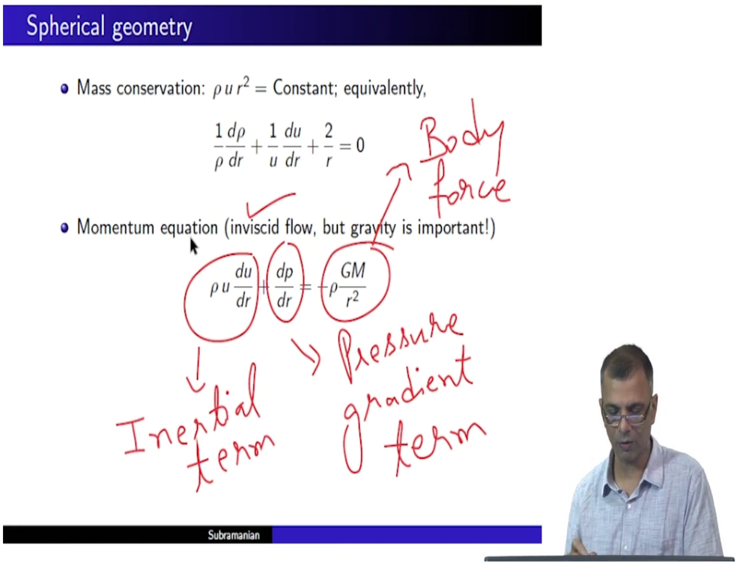

So, this and this are essentially equivalent, ok. This and this are the equivalent statements. You can; what you need to simply need to do is differentiate both of these with respect to r, and assume that everything the density as well as the velocity is a function of r, ok. So, you get this. So, differentiating a constant gives you 0. So, that is what you get here.

So, this is the first equation, first important equation. So, this is the mass conservation equation. And we next consider the momentum conservation equation. We neglect viscosity, we consider inviscid flows, but in several applications that we were considering so far in lab situations we were neglecting body forces, ok. You consider I do not know you know water in a container or so and so forth, the height of the container is so small that you know variations in gravity are negligible.

So, gravity was most of the time neglected in our discussions of lab fluid dynamics. Whereas, here of course, gravity is of utmost importance. Gravity is important. This is what drives everything really, ok. For a compact object it is a gravity of the central object that causes matter to accrete, ok.

And for the sun is the fact that you know the sun is continuously the gravity of the sun is continuously trying to hold back the outer atmosphere, but the outer atmosphere is hot enough such that it kind of evaporates and so, there is a competition between that and the gravity of the sun, ok. So, these are of at most important. So, gravity is now considered, ok, yeah.

So, the momentum equation now you recognize the dp dr, and this is the inertial term and this is the body force term, right. So, this would be inertial term pressure gradient. We do not know what the pressure is yet we will solve for it, ok, pressure gradient term and this is body forces due to gravity of course, right. And we do not consider surface forces because we are now only dealing with inviscid flows, ok. So, these are our two main equations, mass conservation and momentum conservation.

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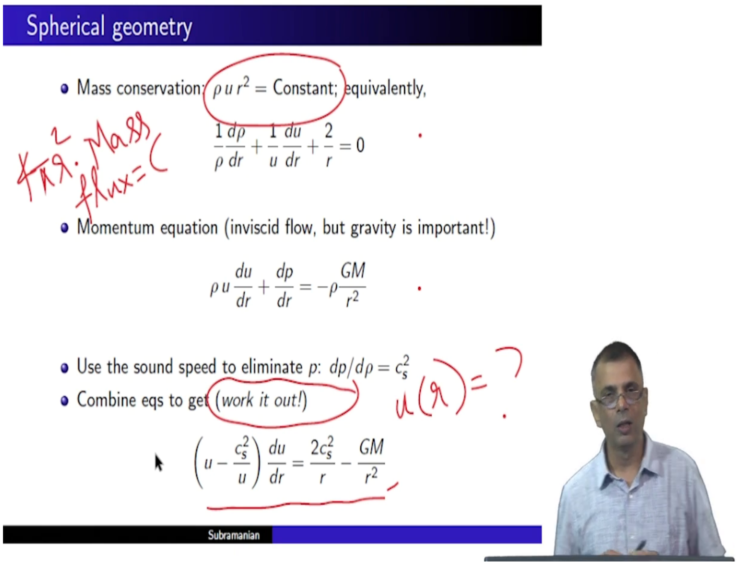

We do the usual trick. We use the speed of sound, the definition of the speed of sound this one to eliminate p in favor of rho. You see here you have the p, right. So, why do that use the speed of sound? To cast this in terms of rho like this. So, you just use a you know chain rule essentially.

And you can combine this equation and that equation, and I strongly urge you to work it out to obtain this. This, all important equation. This is all we will really need for the time being, ok. So, this is the main equation that we will be working with from now on, and, yeah.

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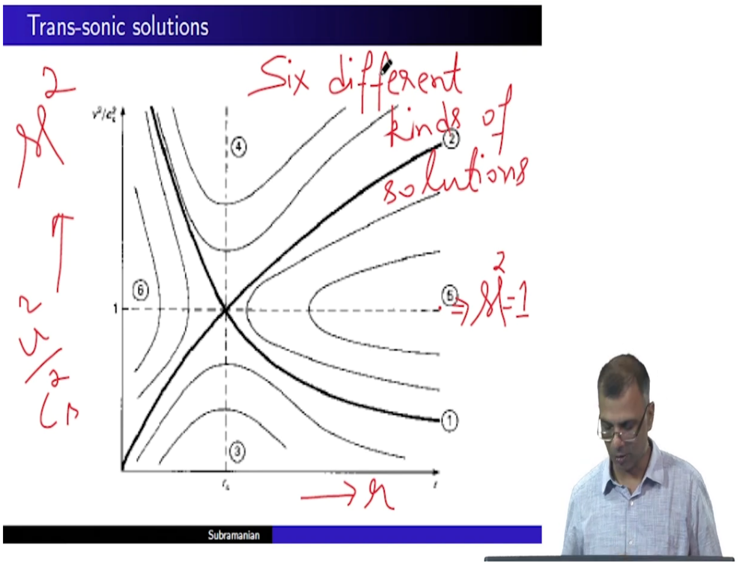

So, what we will do in our subsequent discussions is we will look at the parameter space spanned by this parameter space of allowable solutions that can be that are predicted by this equation and to investigate. So, this would be r here, and this would be the square of the Mach number or u squared over, ok.

So, u squared over C sub s squared as a function of r, ok. There are several allowable solutions that are predicted by this equation, ok. What you really need to do is of course find out u of r. In order to do that, what you do is you take this du dr you take it to the right-hand side, du dr is equal to this term divided by this term, is not it. And you integrate it and you will have a constant of integration, ok.

And depending upon what that constant of integration is you will find you will arrive at different solutions and these represent different possible solutions, you have several classes of solutions, you have a solution that can look like this, you have a solution that can look like that, you have solutions that can look like.

So, this would be solutions of nature 1, this is solution solutions of nature 2, and these are another kind of solution which are sub which is called solution 3, and these would be solutions of the type 4, these would be solutions of the type 5, and there are Six different kinds of solutions. Six different kinds of solutions that are allowed by this equation.

And what is the main thing that distinguishes one kind of solution from the other? Why do we divide it into 6 categories? Because this dotted line corresponds to M squared equals 1, in other words Mach number equal to 1. If the square of the Mach number is equal to 1, the Mach number itself is also equal to 1.

So, upwards of here you have supersonic flows, downwards of here you have subsonic flows; subsonic flows, sonic point so to speak and supersonic flows, ok.

So, this represents a parameter space of acceptable trans-sonic solutions to this equation here. Why is this spherically symmetric? Well, you know the conservation of mass this essentially arises from you know 4 pi r square times constant that is where you got this, right. And 4 pi r squared is the area of a sphere.

You are assuming that the mass flux is distributed symmetrically on the sphere, either outgoing mass flux or incoming mass flux, ok. And in the momentum equation you see you just have one coordinate r, just one coordinate r, this is, spherically symmetric gravity and you have only have; so, spherical symmetry is invoked in this equation as well as this equation which is why you know spherical geometry.

And so, this is essentially this will you know tell us the character of trend that we can expect from different kinds of trans-sonic solutions, ok. Due to either accretion or excretion or winds in spherical geometry. And we will discuss the character the nature of each of these solutions in detail, so, that is it for the time being.

Thank you.