Fluid Dynamics for Astrophysics
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Lecture - 31
Coverging/diverging channels, de Laval nozzle and its application to astrophysical jets

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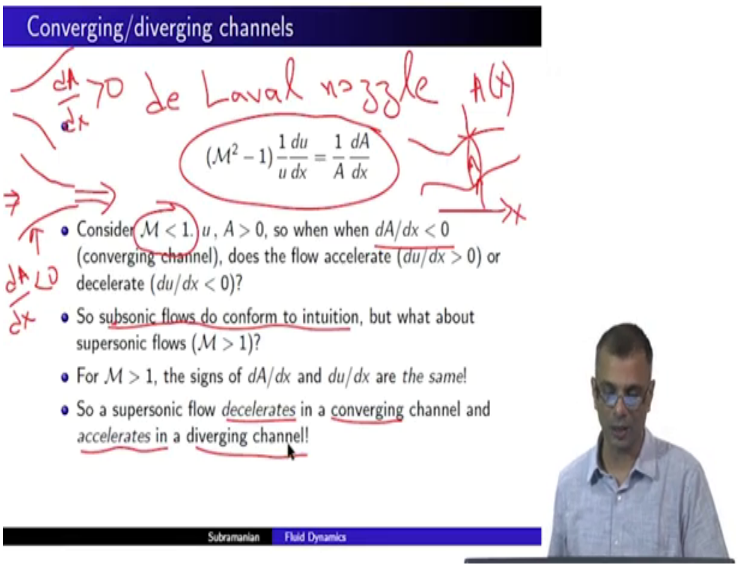

So, you remember that we were discussing this thing called the de Laval nozzle, which is really you know something that was initially thought of in engineering circles and in doing so, we essentially came up with this equation which is, which relates the; so, this is a purely one-dimensional flow ok.

So, this equation essentially relates the velocity and the derivative of the velocity with the area and the derivative of the area and area of what? Well, area of the pipe right. The area of the pipe can be you know varying like this right; the cross-sectional area right. So, this is what we are talking about and it can vary as a function of x.

So, A can vary as a function of x and the question we were asking is how and this is the Mach number of course right. So, the question we were asking is how does the flow behave with the change in area as it as it passes and does it conform with intuition right. And what we found out was well, let us first consider subsonic flows for in other words, the case where the Mach number is less than 1 right.

So, and we find you know the area itself is always larger than 0; is not it? So, when for instance, when d A dx is less than 0; in other words, the channel the area is pinched like this. It is somewhat like your pinching you know a garden hose, where you know so the area is decreasing which is what this is. When the area is decreasing you expect that the fluid flow, the fluid squirts out at a higher speed here as compared to the speed here right and that is really what happens here.

You see when M square is less than 1, M is less than 1 this quantity is negative. And what happens is this quantity is negative. Therefore, when this quantity is negative, then this has to be positive ok. So, the flow accelerates, when d A dx is less than 0 and the flow decelerates, when d A dx is greater than 0 for a diverging nozzle. This is also something.

So, in this case, d A dx in this case and in this case for this and in this case d A dx is greater than 0. And so, we normally you know expect that flows when the cross-sectional area is increased, the flow decelerates and then, when the cross sectional area is decreased; in other words, for instance, if you pinch the pipe, the flow accelerates.

And that and that its ok. I mean as far as long as you know the Mach number is less than 1, that behavior is indeed reproduced. The thing is though, so in subsonic flows do conform to intuition, but what about supersonic flows? Right and we find some strange behavior in supersonic flows, we find that because M is greater than 1 and this is always positive right. So, the signs of du dx and d A dx are the same. Because you know A and u are all positive.

So, there is no there is no scope that this can you know change signs. So, if d A dx is negative; in other words, in a situation like this, where the channel is being squeezed, you find that the flow decelerates. So, the. So, d A dx is negative and du du dx is also negative. So, the flow decelerates in a converging channel which is opposite to our intuition about subsonic flows.

So, for supersonic flows, its strange right and if d A dx is positive; in other words, like this. If d A dx is positive, du dx is also positive. In other words, the flow supersonic flow accelerates in a diverging channel right. So, this is the strange thing about supersonic flows and we will see what connection this has, we will explore this a little more and then, we will go on to see what connection this has to astrophysical jets alright ok.

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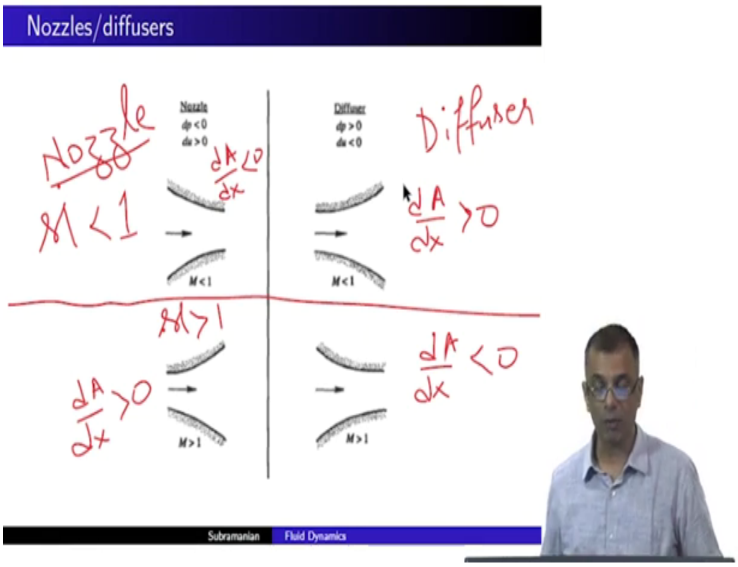

This is what one would call a nozzle on the left hand side, these are all and these are what are called. A nozzle is something that as a name implies it increases the speed of the flow right and the diffuser is something as the name implies something that decreases the speed of the flow and spreads the flow around. For instance, if you were trying to spray paint something you do not want a nozzle right.

If you are trying to spray paint, what you do is you inject the paint at high velocity and then, you want the paint to diffuse when it flows out so that the droplets nicely you know diffuse and they cover a large area.

So, that is what a diffuser is right. So, for Mach number less than 1, this would be a nozzle. What would be a nozzle for Mach number less than 1? This kind of situation, where d A dx less than 0 right. You see the d A dx is indeed the area is decreasing as a function of x. So, this would be a nozzle and this would be a diffuser.

In this case, the cross-sectional area is increasing in other words right. So, this would be a diffuser. Why is this a diffuser? Because subsonic flows pass through you know a diverging channel, their speed decreases and the flow diffuses right. So, this conforms to intuition. For supersonic flows, however, it is this kind of a thing, where d A dx is actually greater than 0. It is a diverging channel ok. It is this kind of a diverging channel that acts as a nozzle.

In other words, supersonic flows accelerate when they pass through a diverging channel. Therefore, it you know a diverging channel acts as a nozzle for supersonic flows. Here ok, like this. The top half is for subsonic flows, where Mach number is less than 1 and the bottom half is for supersonic flows or the Mach number is larger than 1 right. So, when the Mach number is larger than 1, contrary to intuition, it is this kind of a diverging channel.

In other words, a situation where d A dx is greater than 0 that acts as a nozzle. And in this kind it is this kind of a converging channel where right, the area decreases as a function of x. This situation acts as a diffuser; it is exact opposite of the situation for subsonic flows right. So, just wanted to emphasize that a little more before we proceed.

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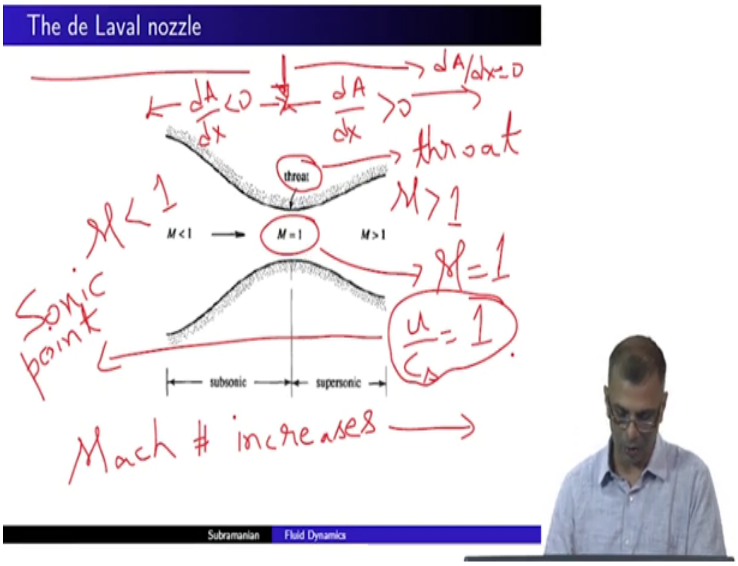

And now, let us go ahead. Finally, putting these two things together, this is what is; finally, this is what is normally called the de Laval nozzle ok. So, what you have is a situation is a pipe whose cross-sectional area is decreasing here until it reaches a throat and then, it increases ok.

And so, in this region up until here ok and from here to here, here onwards right and right here, exactly here is equal to 0. It is less than 0 to the left; it is equal to 0 exactly at this point called the throat right. So, this is called the throat right.

So, exactly at the throat it is equal to 0, that the derivative of d A dx is equal to 0 and to the right of the throat, d A dx is larger than 0. So, this is the situation. So, imagine you have a subsonic flow right.

So, a subsonic flow would have Mach number less than 1 right. So, a subsonic flow is passing through a nozzle right, it accelerates. In other words, the Mach number was initially less than 1, it becomes larger and larger because its accelerating. We are assuming that the sound speed remains constant of course ok; the sound speed does not change, the sound speed, the speed of sound is the same.

So, if the flow is accelerating, well then in any of the sound speed is the same, it remains constant; then naturally, the Mach number is increasing. It is increasing from Mach from less than 1 and it comes to a point, where at the throat the Mach number is exactly equal to 1 ok. And beyond that what happens is you know the flow continues to accelerate.

So, beyond here what happens is anything beyond here, anything beyond Mach number equal to 1 becomes Mach number larger than 1 of course. And so, and we know that super for supersonic flows a diverging channel is a nozzle; is not it? So, to the right of this, you have supersonic flows and for supersonic flows, the diverging channel is a nozzle.

So, the flow continues to accelerate and becomes and the Mach number becomes larger and larger. So, all the way, from to left to the right, the Mach number is increasing ok. The Mach number is was initially less than 1, it passes through 1 and then, it becomes greater than 1. All through the Mach number is increasing ok, all the way from left to right the Mach number keeps increasing.

But the shape of the nozzle is different on the left and the right, why is that? Because on the left, the flow is subsonic and therefore, in order to increase the Mach number or in order to more accurately in order to accelerate the flow.

And therefore, to increase the Mach number you have to have a, you know converging channel ok; whereas, on the right, once you pass what is called the sonic point. So, this is in other words, u over c sub s is equal to 1 right and so, this is called the sonic point for obvious reasons right. The flow speed is equal to the sound speed.

So, this is you right at the throat, you have what is called the sonic point right and to the right of this, the flow becomes starts becoming supersonic. So, if you have to ensure that the flow keeps accelerating and therefore, the Mach number keeps increasing to the right, also you need to have a diverging channel now.

Because we know that for supersonic flows, a diverging channel acts as a nozzle right. So, this is this clever you know situation, where you have you know de Laval nozzle is this clever situation, where you ensure that you know.

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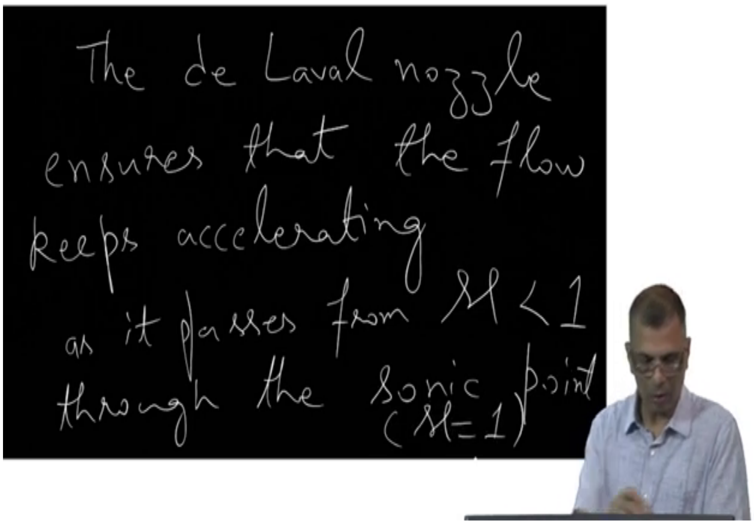

So, essentially the de Laval nozzle ensures that the flow keeps accelerating as it passes from subsonic through the sonic point which is right to supersonic, which is right.

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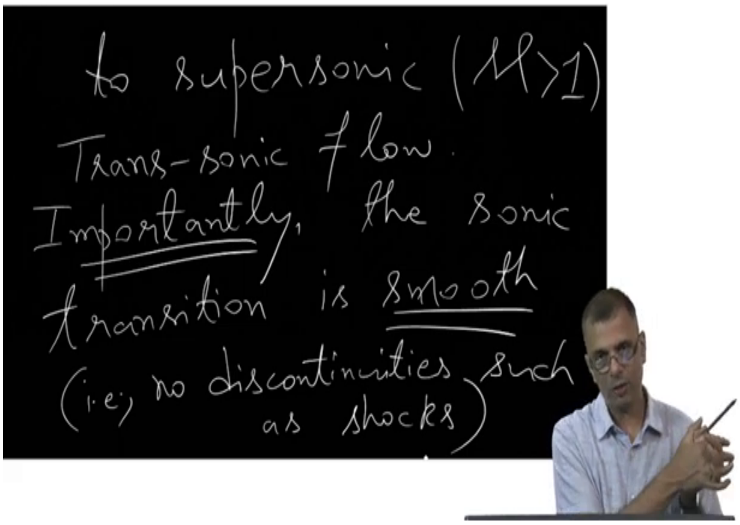

So, let me do this one. Let me show you this once again. So, the de Laval nozzle is a device which ensures that the flow keeps accelerating as it passes from a subsonic flow through the sonic point, where M equals the Mach number equal to 1 to supersonic. So, this is what a de Laval nozzle does. Importantly, so this is an example of a trans sonic flow. Why transonic flow? Because it is something that transitions, this is a flow that transitions from being subsonic through the sonic point into supersonic ok.

Importantly, the sonic transition in other words, the transition through the sonic point, where M equals 1 is smooth i.e, no discontinuities such as shocks. This is very important. It is not just that the a de Laval nozzle is not just a device that you know ensures that you pass smoothly from a subsonic flow to a supersonic flow; yes that is one thing through a sonic point; but also, it is a device that ensures that this transition happens smoothly ok.

So, you have to engineer the d A dx very carefully ok. The throat has to be you know; the throat this one, I mean its there are many ways in which you can transition from a d A dx less than 0 to d A dx larger than 0 right.

I mean this the converging nozzle can be very sharp for instance and the diverging nozzle can also be very sharp; but you have to engineer the shape of the nozzle such that you ensure that the, you know that the transition happens smoothly without any discontinuity such as shocks. This is very important.

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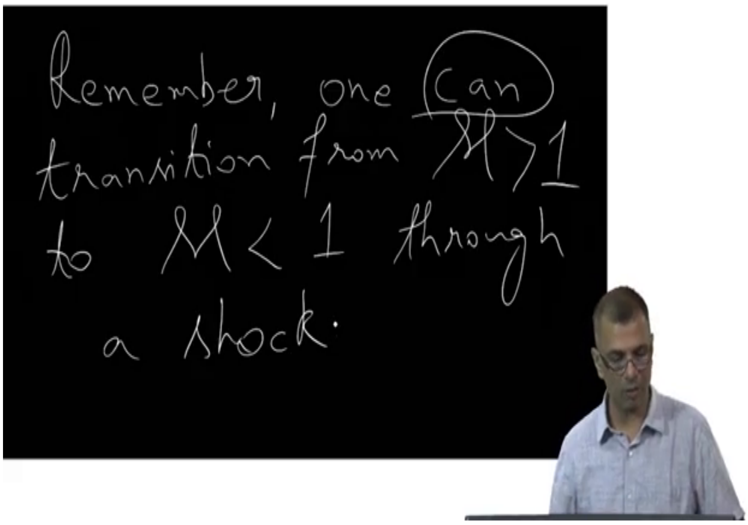

Then, you can transition from remember, one can transition from you know greater than 1 to less than 1 through a shock, one can do that ok. But a de Laval nozzle is something that does not do it this way, it ensures that the transition happens smoothly ok right.

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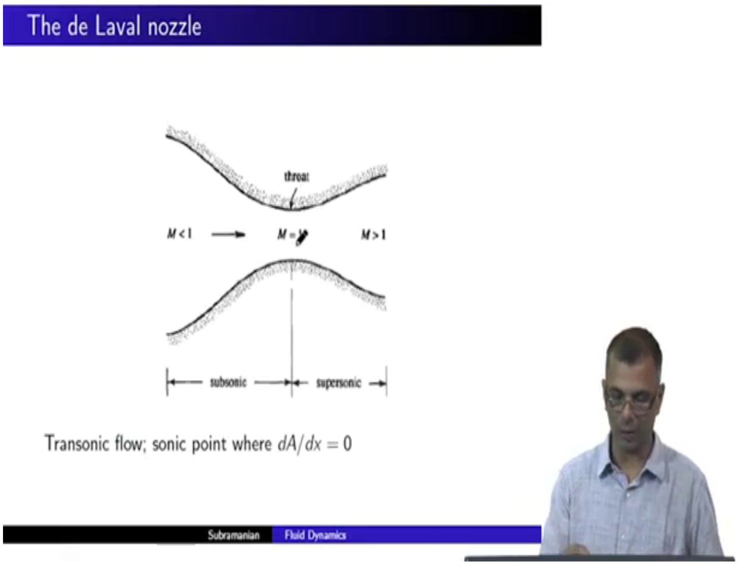

So, having emphasized this, let us go on yeah. So, the trans so, this is this the sonic point happens, where the d A dx is equal to 0; right here at the throat right ok.

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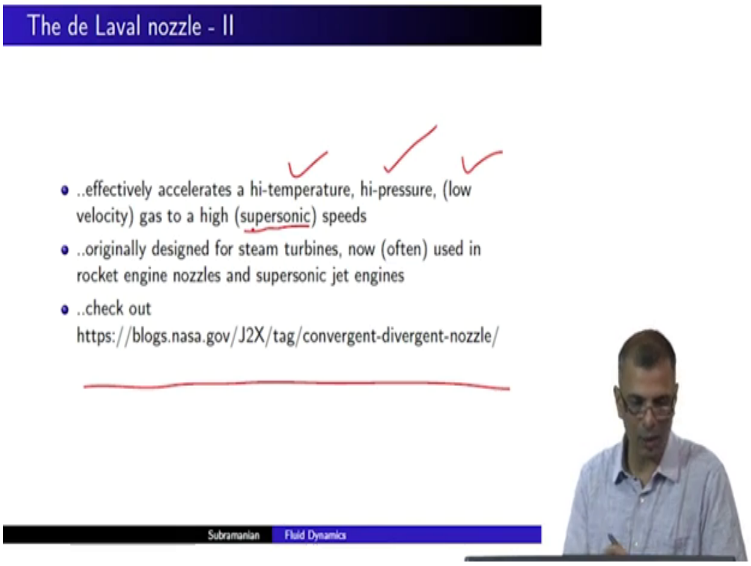

Now, what does again just to re-emphasize what we were saying. The de Laval nozzle is a device which effectively accelerates a high temperature high pressure flow. Well, high temperature, high pressure, low velocity. In fact, the high pressure and low velocity kind of go hand in hand ok.

You know it is the flow it is most of the when we say high pressure, what we are really saying is most of the energy is an internal energy of the gas ok, that is the same thing as saying high temperature ok. This and this ensure that most of the energy is in the internal energy of the gas and that it converts and the de Laval nozzle is something that converts this kind of a high temperature, high pressure, low velocity gas to high velocity gas ok.

In other words, the internal energy is converted into the kinetic energy of the bulk flow that is when you get high speeds. And in specifically, the speeds are supersonic ok. It was originally designed for steam turbines and its now, often used in rocket engine nozzles and supersonic jet engines. Unfortunately, I do not know how many of you would have heard about supersonic jet engines.

Long ago in the 80s was it yeah, the concord was a very hard thing and these were planes that could you know cross the sonic barrier and become supersonic. For various reasons, they fell out of favor and so, those would be you know one you know situation, where something like the de Laval nozzle would be used ok.

And I urge you to check out this URL. It is very very interesting. Much of what we have discussed here is discussed here ok. Let us now move over to discussing. So, remember the reason this is an astrophysical fluids course after all and the reason, we are discussing the de Laval nozzle in such detail is because we want to see how the concept of de Laval nozzle is applied to astrophysical jets.

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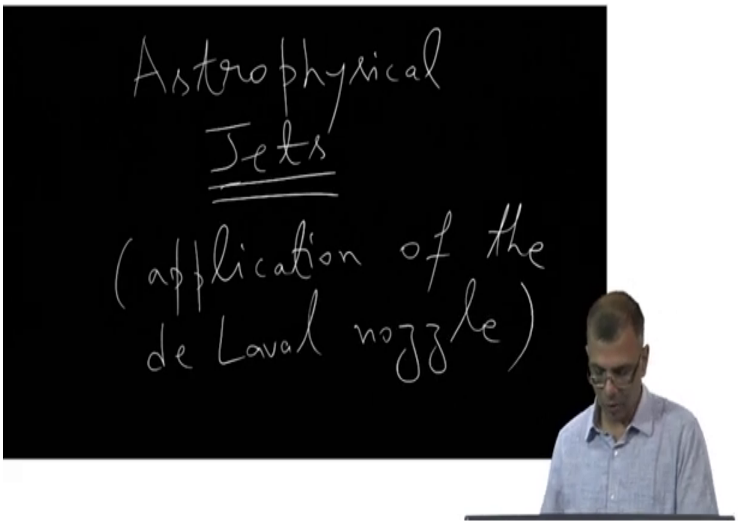

Now, what are astrophysical jets, I need to show you pictures right; and an application, I should say. It is not so much application, the concept of a de Laval nozzle is nicely applied here that is what it is ok.

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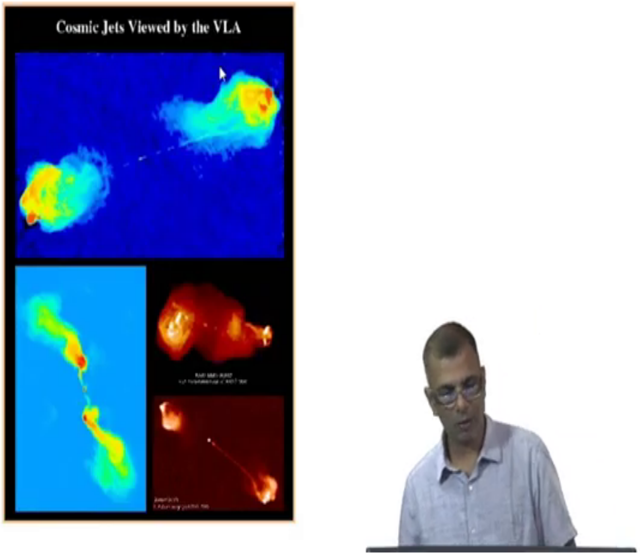

So, let me show you a picture this right. So, what is this? These are cosmic jets viewed by the very large array and these are essentially what are called radio images. These images are images that are obtained at radio frequencies ok and this is one picture, this is another picture, this is yet another, this is yet another ok. These are all different pictures. Let us just concentrate on this, this first one for instance. Let us just look at this ok. This is good enough for our purposes.

And what you have here is a nucleus ok and from which you can see there is matter even from the picture, you can see you cannot see too much here; but nonetheless, it is evident that there is matter being squirted out like this ok. Although, I must say that what you are really seeing is radiation and not the gas or the plasma itself ok.

What you are really seeing are photons, in particular radio photons that is what you are really seeing ok. So, this is something to be kept in mind. And so however, we will take the radiation to be a proxy for the matter ok. After all, if there was no if the matter here was not denser than the surroundings ok, it would not appear brighter is not it.

So, it is that is, it is in that sense that you know and radiation is always a two-body process generally. It goes that the intensity of radiation generally goes as n squared; it is a two-body process. So, the denser the matters the more the radiation. So, wherever you see excess radiation, you think that this excess matter. That is what it is. So, I just wanted to emphasize that what you are really seeing here is not matter itself as you would in a lab, what you are really seeing here are photons that are being emitted.

And there are reaching telescopes of the earth and after a lot a great deal of image processing, you obtain these spectacular images such as these and from these you surmise, you infer that matter is being ejected from the central point which is often called the nucleus in both directions ok.

And it out here what happens is I mean this is what is called the hotspot and this is called a blowback, this is these are called lobes. So, most likely what is happening is this matter is being is being you know blown out in the form of a jet, the geometry, the picture, I mean the reason we call them jets is pretty obvious from the picture. These do look like very narrow, very collimated jets right.

At this point, it is almost as if the jet is running into a wall and when the jet runs in the wall what happens? The matter splatters all over ok and so, this is really back flowing matter ok and these are radial lobes and these are hot spots. This is where the maximum dissipation takes place. All of this is not terribly important to us for the purposes of our present discussion.

Our present discussion mainly centers on the fact that there are very collimated jets of matter that are being ejected from the central nucleus, that is it and how is this happening? This is essentially what we want to discuss ok. And just to give you an idea, from here to here is often it depends I mean this is just one such jet, there are many many such jets ok.

And it is also been surmised that the speeds of these jets are often relativistic and they are relativistic and they are definitely, definitely supersonic, no doubt about it ok. Yeah. So, the other point I wanted to make is the extent of these jets, the extent of these jets is truly staggering. They are mega often they are several megaparsecs long. Huge monster jets, from here to here would often be several megaparsecs ok.

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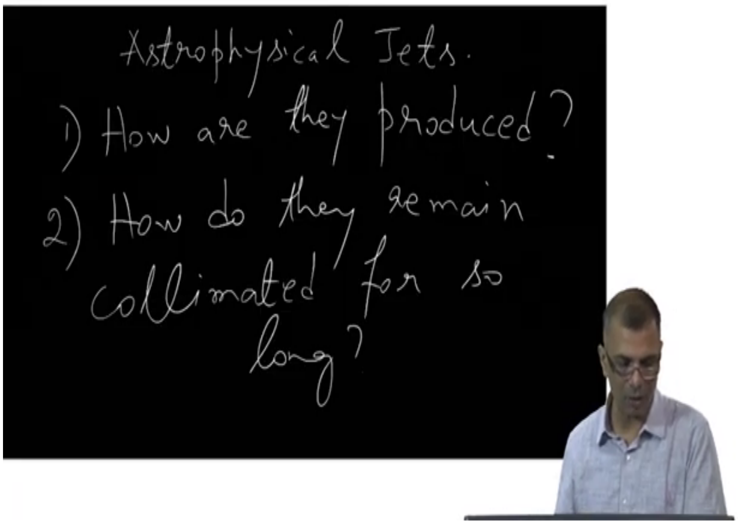

So, the question we want to address, well there are several questions that can be addressed in the context of astrophysical jets such as the one we saw ok. One is how produced? How do they remain collimated for so long right? And you can see that right. I mean this thing this jet remains incredibly narrow and collimated over really long distances; how long? You know all the way to mega parsecs right ok.

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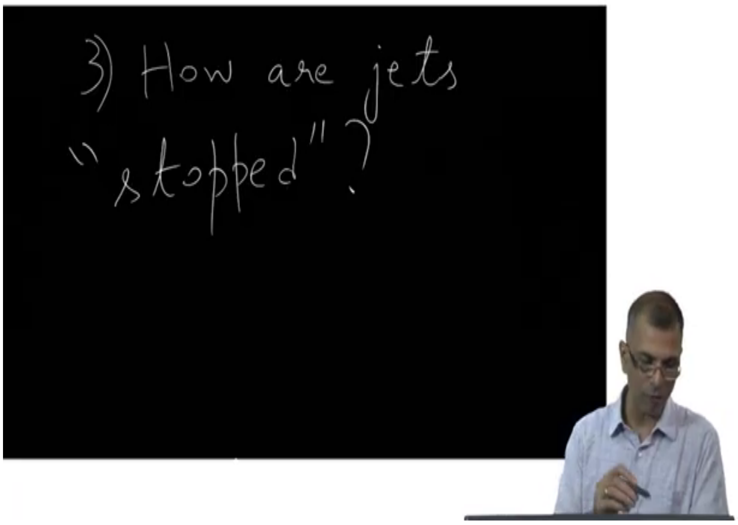

So, that is often the second question and many other questions that can be that can be addressed. For instance, are jets stopped; stopped meaning like this out here ok. It seems as if the jet has run into some kind of a wall and it splatters out there and it you know. It is essentially, you know splattering all over and that is that is what leads to this these lobes right ok. But, however, in this case, we will confine ourselves just to this first question; how are they produced and right.

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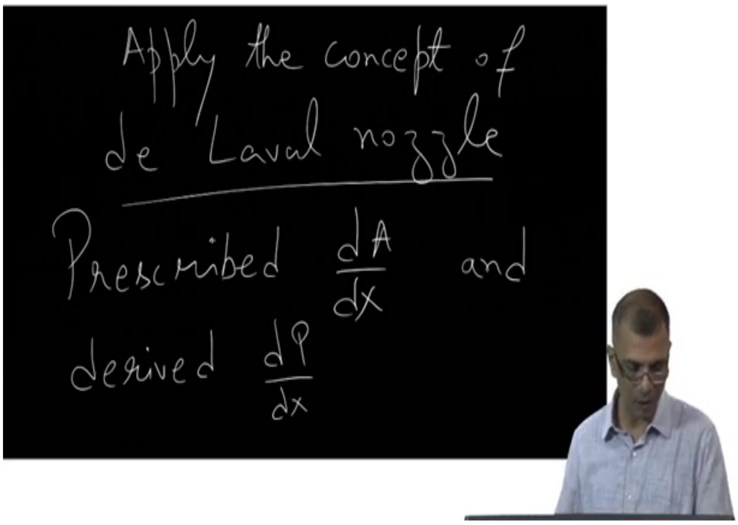

So, you see we will apply obviously, that is the reason we discussed the de Laval nozzle so much. We will apply concept of the de Laval nozzle, but you might ask how? How, I mean how is it de Laval nozzle concept applicable here? The first thing to remember is that in our discussions of the de Laval nozzle, we prescribed and from the prescription, we said here is a converging nozzle, here is a diverging nozzle.

In other words, we are prescribing exactly how the area varies as a function of x right and from this prescription, we are deriving how the Mach number varies or equivalently, how the pressure ok, that is how we have been doing this. For astrophysical jets, we will do the opposite.

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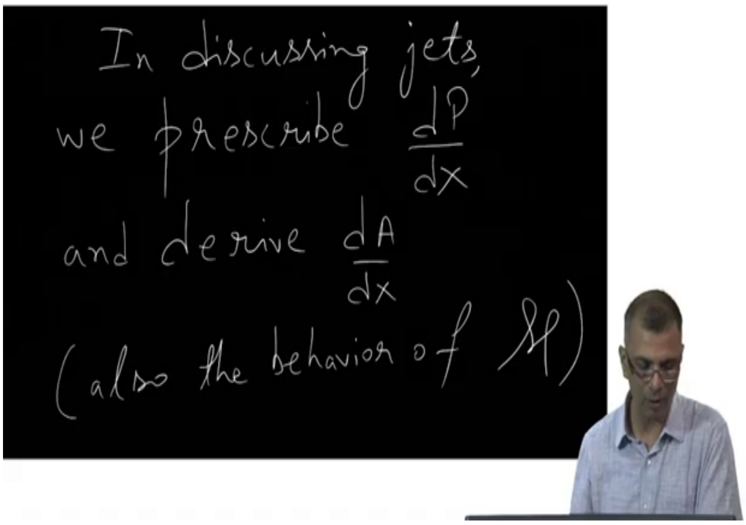

We will prescribe in our discussion in discussing jets, we will do the opposite we prescribe the pressure gradient and derive d A dx ok and also, the behavior of the Mach number of coursethat is the whole point, is not it? But the main thing is you are prescribing the pressure gradient and from that it is as though the pressure gradient is equivalent to a d A dx, that is what it is. There is no physical nozzle as such.

So, again, let us be a little more specific. So, what we are saying is that here is I do not have a very good cartoon of this. So, here is a central object ok, this thing, this thing; the central object right.

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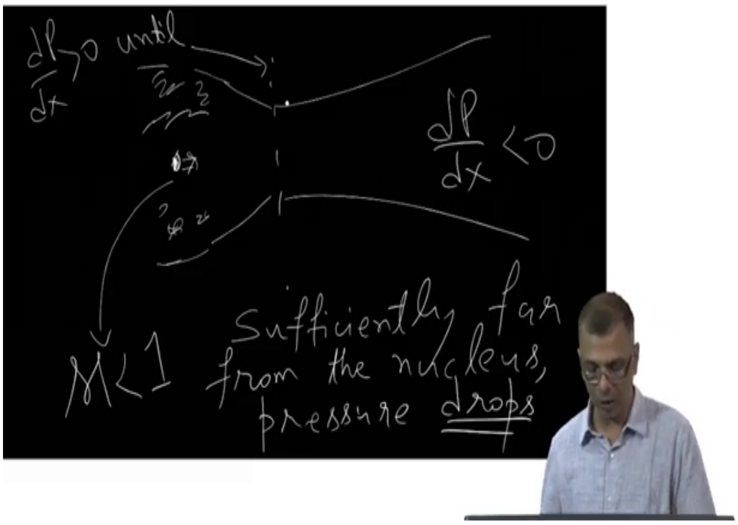

And around the central object, there is really you know that there is very dense medium ok. So, what I am there is really dense medium, very high-pressure medium ok. So, there is matter being squirted out somehow ok and there is very high pressure medium ok.

And when its squirted out, when its initially being squirted out, the Mach number is less than 1 ok and there is very high pressure medium and it is squeezing you know and in this is subsonic flow and you are however, why is there very dense medium around?

Because this is a very compact object and compact objects always like to accrete matter, stuff is accreting onto the compact object. So, in the vicinity of the compact object, you would expect the density to be large and therefore, the pressure also to be large. But as you move away, as you move away, you would expect that the density falls off.

And therefore, the pressure falls off ok and it is that is essentially equivalent to a nozzle that is converging. Rather, I am sorry in the very vicinity, in the very vicinity ok, as you transition from here to here, the density still keeping keeps increasing up until some point. So, that would be.

So, d P dx is greater than 0 up until say up until somewhere here. So, this would be equivalent to a converging nozzle until somewhere here ok. So, this would be something like a converging nozzle and we know that converging nozzles for subsonic flows keep accelerating the flow; but as you move, so all this is very much in the vicinity of the central object right.

But as you move away from the central object ok, away meaning sufficiently far away ok, sufficiently from the nucleus pressure drops. In other words, d P dx becomes less than 0 right. So, and d P dx becoming less than 0 is as good as a diverging nozzle. What you have to engineer is that it there is the flow used to be subsonic until and so, so the other thing is this is not to scale, this drawing is not to scale ok.

This is very very close to the object and all of these are quite far from the object, from the central object ok. So, very close what is happening is the flow is subsonic and you have to you have to engineer in your mind the concept is that around the time that d P dx changes sign from greater than 0 to less than 0, you have reached the sonic point. And so, to the right of the sonic point, you know you know the flow now becomes supersonic.

And once the flow becomes supersonic, you the d P dx becomes less than 0 and that is as good as a diverging nozzle and we know that a diverging nozzle serves to accelerate a supersonic flow.

This is so this is exactly like that de Laval and that is what we want. We want the flow to keep accelerating and becoming more and more supersonic ok, more and more supersonic so that it can travel to you know it can accelerate so that you know; essentially, the bulk kinetic energy of the flow is increases keeps increasing so that you know it can go on for long distances as long as a megaparsec ok.

Up until the point, where something else happens, the density of the interstellar medium increases so much that you know the flow cannot push against the density, against the interstellar medium anymore and it comes to kind of a halt and it splatters all over. But that is not our concern right now. Our concern is trying to figure out how the flow becomes supersonic and how it keeps accelerating and this is how.

So, you can see that in prescribing the pressure variation, very close to the central object and very far from the central object, we are essentially creating a de facto de Laval nozzle and so, this is how the concept of a de Laval nozzle is very very closely intertwined to this question, how astrophysical jets are produced ok. So, that is all we have to say.

Thank you.