# Fluid Dynamics for Astrophysics
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## Lecture - 30
## Shock jump conditions (contd), transonic 1D flows, converging/diverging channels

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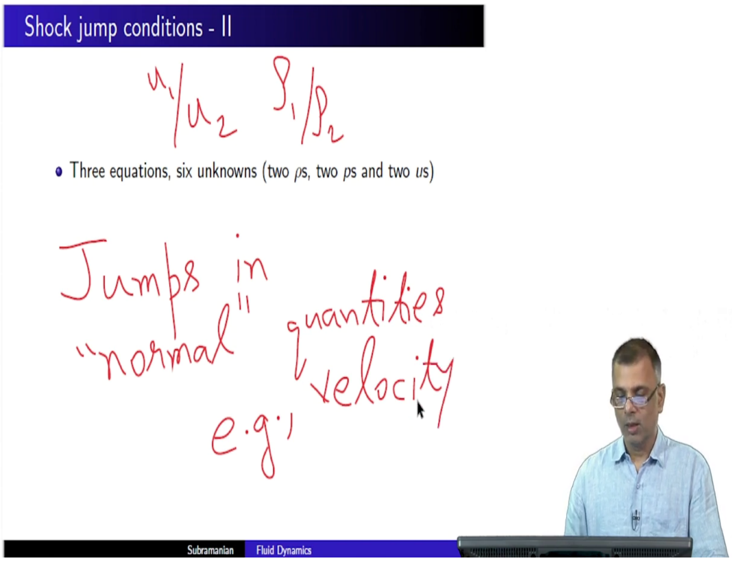

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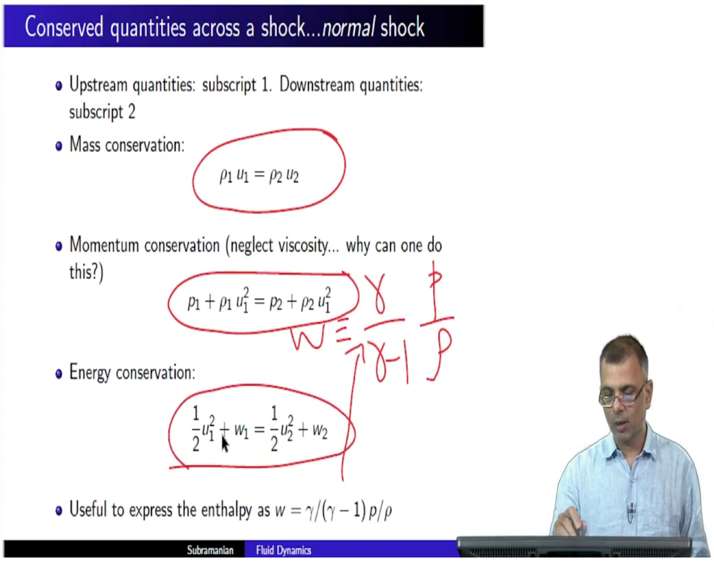


Yes. So, as we remarked, we have three equations; the equation for the conservation of mass, mass flux, the equation for the conservation of momentum flux, this one and the equation for the conservation of energy flux which is this one. We have three equations and six unknowns; two densities, one upstream, one downstream; two pressures, one upstream, one downstream and two velocities; one upstream and one downstream.

And if you think that we are introducing a new quantity here enthalpy, it is not really new, it can be quickly related to the pressure and density like this right. So, and enthalpy relates to the internal energy per unit gram of course, the internal energy of the fluid; whereas, this half u squared relates to the energy of the fluid due to the bulk motion ok. So, and it is important to add them together and so, this represents energy conservation.

So, we are now ready to start talking about things like u 1, u 1 over u 2 or rho 1 over rho 2 and these are the jumps or rho 2 over rho 1 as the case might be. But remember these refer to jumps in normal quantities, normal to the shock surface, not tangential. For instance, normal velocity this, there is no you know sensing talking about normal I mean this is just you know density so, normal quantities e.g., velocity; normal velocity ok.

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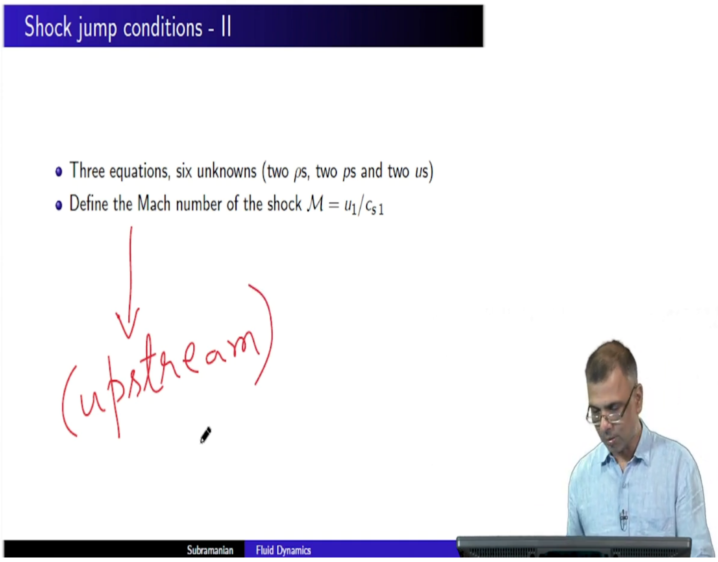


So, let us jump right ahead and let us define the Mach number of the shock as u 1 over c s 1 ok. So, this is just by definition, we could even have defined another Mach number which would be u 2 over c sub s 2 with respect to the downstream quantity. This is just convention ok. It is easy I mean it is conventionally one speaks of the upstream Mach number. You should more correctly, it should really be talking; you should really be saying upstream ok. So, this is the upstream Mach number of the shock and so, that is the definition.

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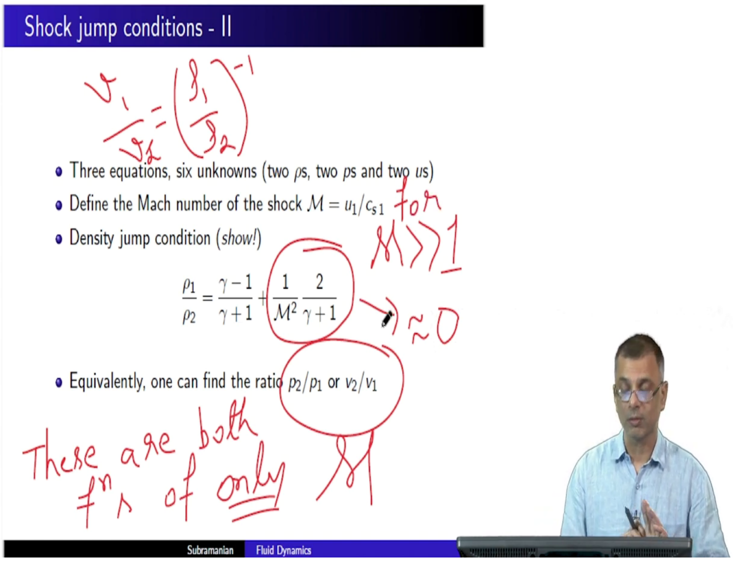


And using this and using those three equations that we showed in the slide, we can show that and I strongly urge you to do the little bit of algebra that is needed to show this. You can show that rho 1 over rho 2 is this quantity. Gamma is just the adiabatic index right and so, it can be you know five-thirds or one as the case might be and so, all as far as the fluid is concerned, so gamma is a constant.

That is what I want to emphasize, gamma is simply a constant. So, as far as the fluid is concerned, what is the quantity that characterizes the fluid flow? There is only one quantity and that is the Mach number ok. This is the only thing that appears in the jump condition and it is inversely proportional to the Mach number.

So, the larger the Mach number, the smaller the density jump right; the smaller the Mach number, the larger the density jump that is what this equation is telling you. So, this is the density jump condition. Equivalently, you can also once you have the density jump condition, you can immediately find the pressure jump condition or this the jump in velocity in normal velocity mind you ok. It is very easy to show this right and this is the plot this is a general.

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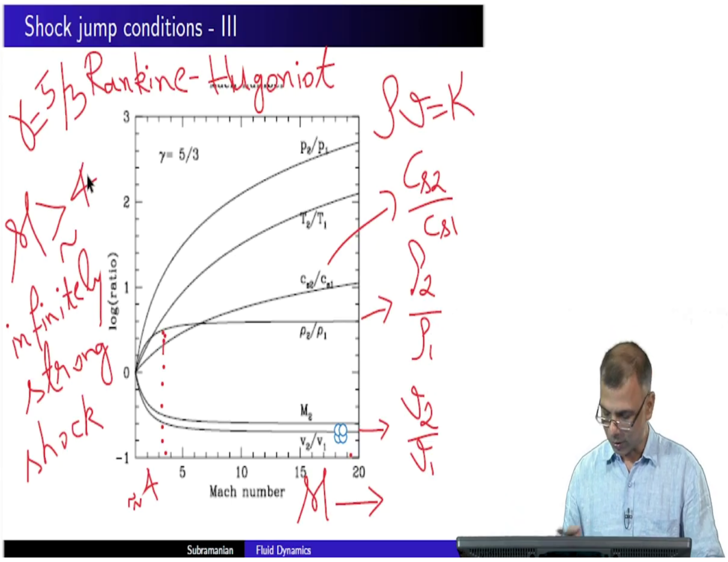


So, everything and all of these all of these functions of only the Mach number M; the Mach number M is the only thing that matters ok. The Mach number does not explicitly appear in these equations; but you can play a little bit and recast these equations in terms of the Mach number ok and you will find that this is how it appears ok and this is the plot.

So, since these ratios are functions only of the Mach number and so, the Mach number M. So, the Mach number is plotted on the x axis and the ratios are plotted on the y axis.

The ratio the pressure jump ratio, the temperature ratio or the jump in the sound speed or the jump in the density, all of these are plotted on the y axis ok. Specifically, the log of the ratio is plotted ok. So, here it would be the ratio would be 1, here it would be 10 here, would be 100 here, would be 10 raised to minus 1 and so on so forth.

Yeah. So, what are the main things to be noted here? The first curve to this is a rather busy graph and in this particular case, we have taken you know gamma to be equal to 5-3rds.

Need not be the case, you can take gamma to be any other number. But this particular graph is you know plotted with gamma equals 5-3rds. So, let us first look at the density jump. This curve which represents rho 2 over rho 1, this one, what does this show? This shows that the ratio of the downstream density to the upstream density increases with Mach number, it increases a little bit right.

So, and then it plateaus off. Beyond a Mach number of about shall we say this is something like 4 here, this one ok. This is a Mach number of about 4. So, beyond this ok, the density jump pretty much plateaus off ok, maybe not 4 ok. So, that is about 5. So, yeah something like the 4 ok.

So, what this is saying is that beyond a Mach number of about 4, it does not matter how much stronger the Mach number is, the density jump remains the same and the value for that is something like well, it is hard to read off a log scale ok. So, whatever this value is and you will find that the velocity jump, this one, this looks like a flip of the density jump ratio and indeed it is because remember rho v or rho u, rho v is constant.

So, in other words, rho 1 over rho 2 is equal to v 2 over v 1. So, if you are plotting v 2 over v 1, its exact opposite of rho 2 over rho 1 right. And not surprisingly, this also exhibits exact same behavior beyond a certain Mach number ok, the ratio plateaus off; except this ratio is smaller than 1. Remember this is 1.

So, this is this side is larger than 1 and this side is smaller than 1 because it is a log-log graph right. So, it plateaus off beyond about a Mach number of 4. What this is saying? I mean the way people express this term this is to say that beyond Mach number of 4, you can effectively regard this as an infinitely strong shock.

It is a bit misleading this terminology an infinitely strong shock. It is not saying that the shock strength is actually infinite. In other words, the Mach number is actually infinite. No, it is not saying that. Anything larger than 4, anything that larger than a Mach number of 4 is effectively infinite ok.

As far as the density jump and the velocity jump are concerned, it does not matter. Whether the Mach number is 5 or the Mach number is 100, it is the same result. So, that is why people normally say Mach number is an infinitely strong shock ok. So, I do not want to belabor this thing. So, this is how the you know the ratios look like for the density jump and the velocity jump.

The downstream Mach number M 2 also looks very much like the velocity curve ok. There is the downstream Mach number. What is plotted on the x-axis is the upstream Mach number ok. So, the everything is written in terms of the upstream Mach number. So, the downstream Mach number looks like this.

You increase the upstream Mach number; the downstream Mach number reduces ok. So, the flow is transitioning and they say all over here because you are lower than 0, you are basically saying that the quantity is less than 1 ok. So, this is the log of the downstream Mach number right.

So, it is less than 1. No surprise, what the shock does is it takes a supersonic flow ok and turns it into subsonic flow that is why all over here you know the M 2 is less than. The sound speed, the c s 2 over c s 1, this is the sound speed ratio keeps increasing. It keeps increasing. There is no plateauing of the sound speed ratio. But the increase is quite gentle with increasing Mach number. By Mach number, we mean upstream Mach number.

Similarly, this is the ratio of the temperatures T 2 over T 1. This is the ratio of the pressures P 2 over P 1. All of these, once you know rho 2 over rho 1 which is what we have written down here or rho 1 over rho 2 equivalently it is the same thing, you can immediately write down P 1 over P 2, T 1 over T 2, v 1 over v 2 is just the flip ok.

So, to emphasize right that is why that is why this and this graph, they look like flip versions of each other. The other thing to say is that often there is there is another name this shock, we have simply said you know shock jump conditions; but if you look in books, these are called Rankine Hugoniot jump conditions same thing ok.

This is the name, named after the two scientists who pioneered at this field ok. So, these are the Rankine Hugoniot conditions and this graph you know gives you all the information there is to need.

So, I urge you to stare at this in detail and think about it and the other thing we said was essentially any Mach number above 4 is essentially an infinitely strong shock. It is not like the strength of the shock is infinite, it is just that anything about you know as far as the density jump and the velocity jump which are the two main jumps.

You know that are you know often considered, as far as the density and the velocity jumps are considered, anything above 4 does not matter ok. The Mach number can be 4 or 100 is the same jump in density and velocity. It is in that sense that shocks with Mach numbers greater than 4 are called infinitely strong shocks. So, I just wanted to you know emphasize that a little bit.

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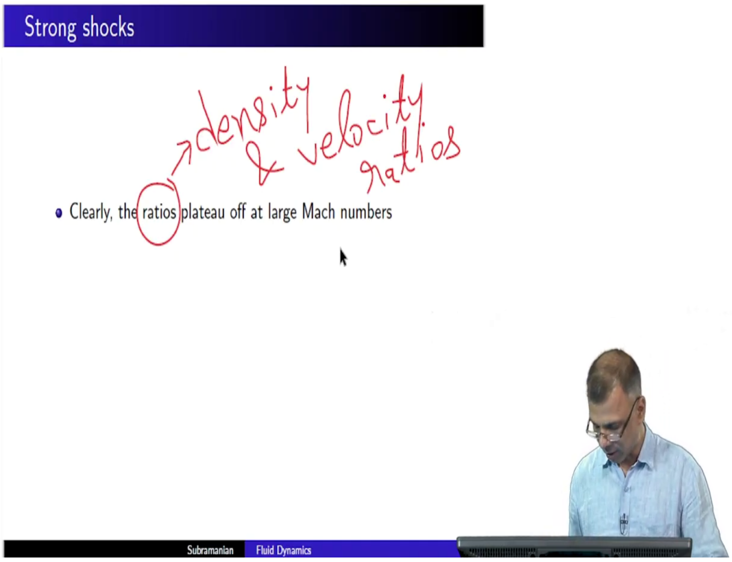


Yeah. So, this is what I was just saying. Clearly, well, not just the ratios, the density and velocity ratios. It is not the reason I said this is because you can see that the pressure and temperature ratios and other things, they do not plateau off. It is only the density and velocity ratios, they plateau off at large Mach numbers right.

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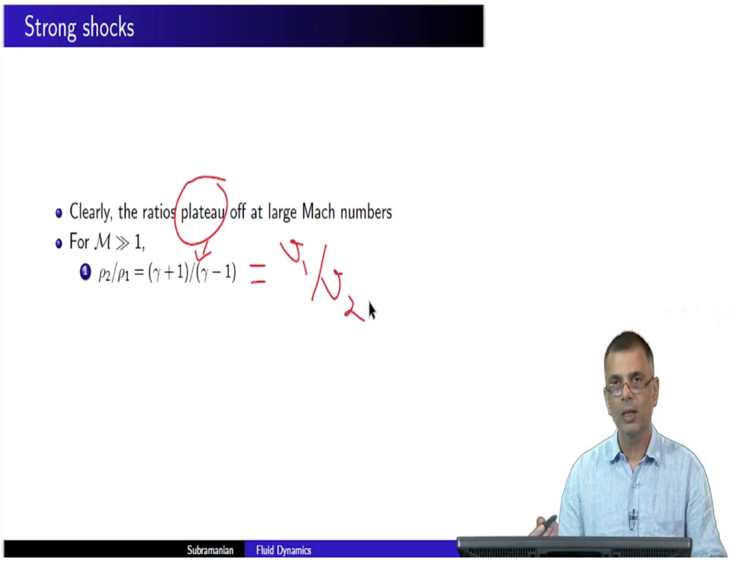


And you can see this just from the expression. For Mach numbers much much larger than 1, rho 2 over rho 1 is just a constant and this is exactly this plateau is just a constant. There is no Mach number appearing here. Why is that? You look at this. You look at the expression here. When the Mach number becomes much much larger than 1, this term is negligible. So, this goes away all you have got is gamma minus 1 over gamma plus 1, that is it.

So, that is what this is saying. There were rho 1 over rho 2; whereas, here we are writing rho 2 over rho 1, that is why this is also flipped. There is no Mach number dependence anymore and if this is rho 2 over rho 1, this will be equal to this will naturally be equal to v 1 over v 2 same thing right yeah, it is just a flip.

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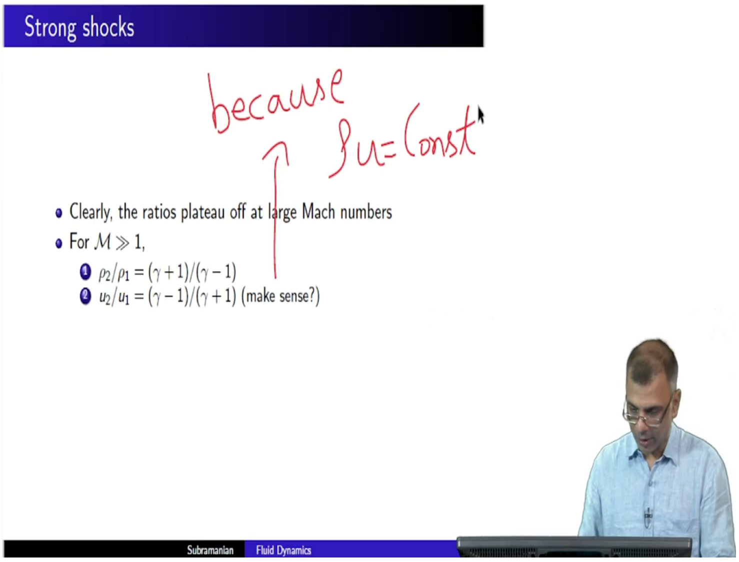


So, I beg your pardon; sometimes I write v, sometimes I write u, it is the same thing; it refers to the fluid velocity and the reason is simply just to show you once again its simply because of the way this term appears, when Mach number is much much larger than 1, this term is approximately equal to 0 right.

So, for the time being, I will just erase. This is not so important; if I can find the eraser. Anyway, I mean you know. So, this is for I should say for very large Mach numbers, this is essentially equal to 0 ok. So, that is what this is saying. Does it make sense? Of course, it makes sense right because that is why this is the flip of that.

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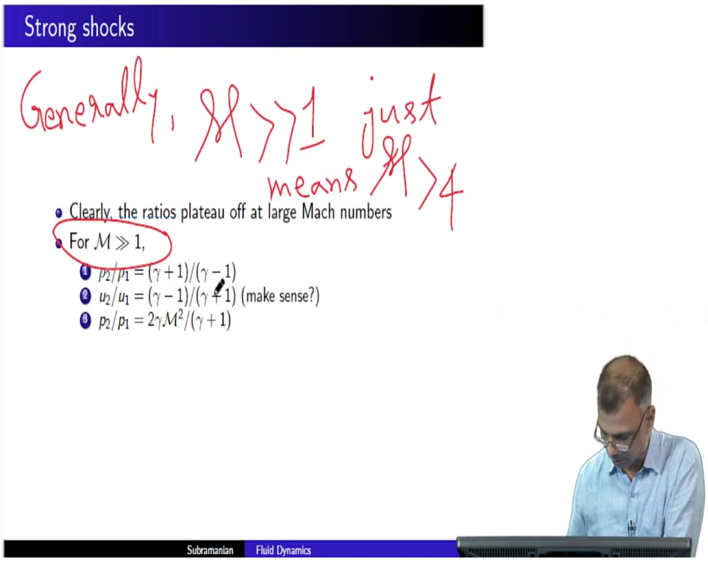


The ratio of the pressures ok is essentially you can write p equals nkt ok or you can go back to the basic conservation equations and derive this again, the ratio of the pressures looks like this. It does have a Mach number dependence that is why the pressure ratio, you remember this the pressure ratio keeps increasing with Mach number. It does have a Mach number dependence and that is that is reflected here ok.

And all these are you know all these hold only for Mach numbers much much larger than 1 ok and generally, generally, just means as long as you are above 4, you are essentially an infinitely strong shock. Generally, this that is what it means ok.

So, I wanted to emphasize this because generally, you would say well Mach number, any quantity in physics if you want it to be much larger than 1, it has to be larger than at least 10 ok. It should be something like 100 or 1000, only then this kind of thing.

In this particular case that is not so, that is why I am saying it. If you are above 4, above something like 4 or 5 or something, you are already in this much much larger than 1 regime ok. I just wanted to emphasize that and you can see that from the plateauing off of the density ratio and the velocity ratio ok.

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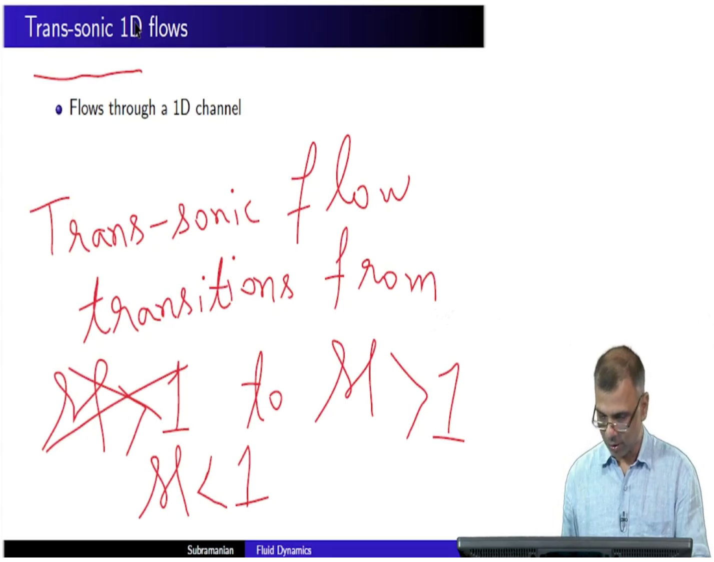


So, now having talked about shocks and these interesting things, we now start talking about what are called transonic flows. Transonic flows essentially means a transonic flow is one that transitions from Mach number less than 1 to Mach number greater than 1 ok. Of course, through Mach number equal to 1 naturally. Your transitioning from a subsonic flow which is

Mach number sorry very very sorry, it is not this. Yeah, Mach number less than 1 to Mach number greater than 1.

So, this would be a subsonic flow right. So, a transonic flow is one which transitions from a subsonic flow which is Mach number less than 1 to a supersonic flow which is Mach number greater than 1 through Mach number equal to 1 naturally. You will encounter Mach number equal to 1 while transitioning from a subsonic flow to a supersonic flow.

Yeah, and these are called transonic flows. The reason, we pay attention to this is because we as we have seen the character of subsonic flows and the character of supersonic flow is very very different, very subsonic flows are essentially quasi hydrostatic; pressure gradients and pressure differences and boundary conditions play a very vital part.

Supersonic flows on the other hand, are essentially ballistic. Boundary conditions really, they do not care about boundary conditions and so, that the character is very different. So, while transitioning one has to pay special attentions, I mean it is one thing when we are dealing with entirely subsonic flows or entirely supersonic flows, certain terms can be neglected and you can go on with the analysis.

However, as with any you know thing in physics, when you are dealing with trans sonic flows or flows that straddle these two asymptotic limits of subsonic and supersonic, when you are straddling these two limits you have to be a lot more careful ok right. So, these are what transonic and in particular, we will pay attention to transonic one-dimensional flows in other words say just the x-dimension, y and z are not important.

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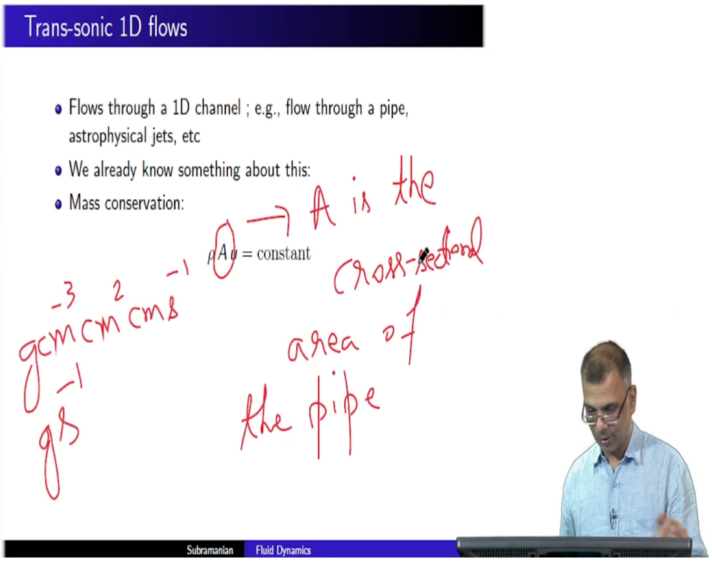


Yeah. So, why are we discussing this? Because why are we discussing this in an astrophysical fluid dynamics class, because transonic one-dimensional flows are of essentially of great importance in understanding things like astrophysical jets. But before talking about astrophysical jets, let us talk about an engineering application because these things are much better established in the lab and then, we will go on and apply our understanding to astrophysical jets ok right. So, we already know something about.

So, let us talk about flow through a pipe. A transonic flow through a pipe ok, we already know something about this in fact. We use the same you know the same kinds of equations. We write mass conservation in the following form rho A u equals constant; where, A is the cross section.

Sorry, A is the area of the pipe which we allow to vary. A can vary with x, you can transition from a thin pipe to a thick pipe, this is the main thing right. And what does what exactly does this represent you see again dimensional analysis rho is something like grams per centimeter cube and A is something, it is an area therefore, it is something like centimeter squared and u is centimeter per second.

So, what this is essentially saying is that this is expressing not the conservation of mass flux; but the conservation of a quantity which is grams per second.

We call, both mass conservation; we call in earlier when we were talking about shocks, we call conservation of mass flux just mass conservation. In this case, we are still calling it mass conservation, but we are really conserving grams per second that is what we are doing here ok. Rho A u equals constant; where, A is a cross- sectional area of the pipe which is allowed to vary with x ok.

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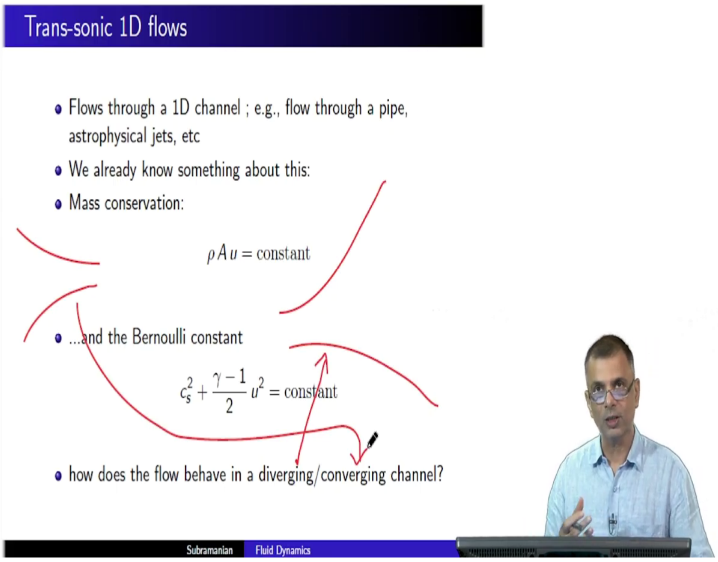


The other thing is the Bernoulli constant which is essentially an energy conservation equation and that can be written as this. This is slightly different from the way we were writing it earlier, but it is essentially the same thing; where, c s is of course the sound speed which is allowed to vary, which can vary ok. Yeah. So, now, the question we ask just based on these two equations, how does the flow behave in a diverging or a converging channel?

A diverging channel would be one that that looks like this. This would be a diverging channel. A converging channel would be one that looks like this; a nozzle. This would be a converging channel. And of course, you know a clever people would put these two together.

You would have a channel that converges, goes through a throat and then diverges. But before that, let us try to understand how a transonic flow would behave in a diverging channel, how would behave in a converging channel right.

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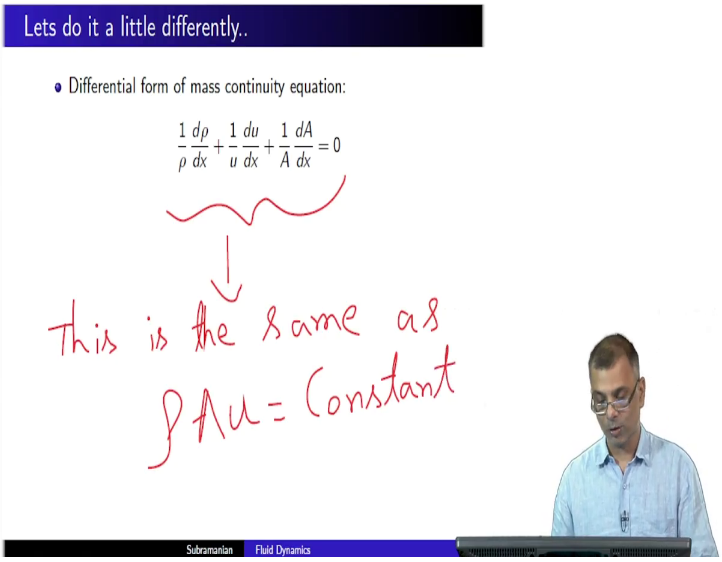


So, let us try to understand this. Instead of writing instead of writing rho A u equals constant, how about writing it down like this. This I emphasize is the same as. Can you see how this is so?

What you do is you know you differentiate both sides with respect to x right and so, this is just the chain rule and you divide everything by rho A u, both sides. When you differentiate a constant with respect to x you get 0. So, that is how you get this ok. It is the same thing. Rho

A u is equals constant is the integral form and this is the differential form. The differential form is more useful to us ok right.

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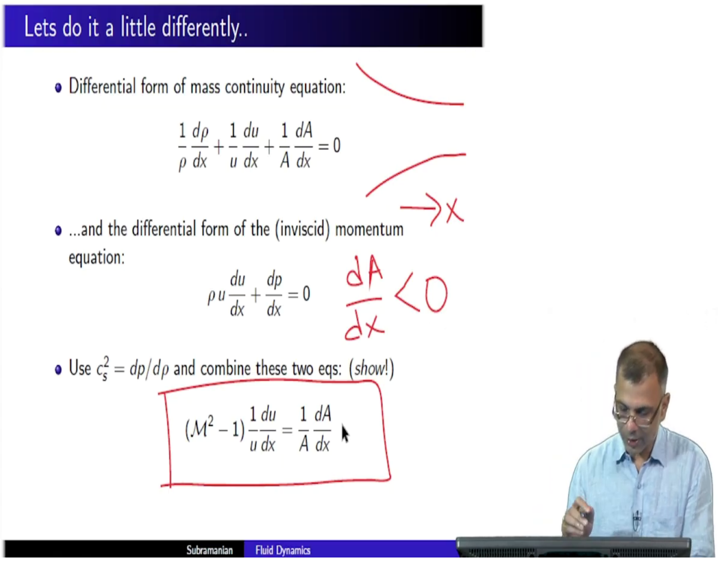


And the differential form of the inviscid momentum equation, we do not bother about. Viscosity, we are not concerned, even with shock thicknesses or anything ok; we are concerned about flows that transition from subsonic to supersonic not via a discontinuity not via a shock, but smoothly ok.

Even when we were discussing a shocks, we neglected viscosity. So, we neglect viscosity here too; but it is important to realize that we are considering shock less flows. Flows that transition from subsonic to supersonic without a shock.

This is also something that you should keep in mind. Not all transitions from subsonic to supersonic in other words from Mach number less than 1 to Mach number greater than 1, you need to go through a discontinuity (Refer Time: 27:00) a shock. They can be smooth and that is the kind of thing that we are talking about here. So, that is the differential form of the inviscid momentum equation. We combine these two using the familiar sound speed, we combine this.

Again, I strongly urge you to show the following result, this result ok and this is a very important result ok. So, this would be the way the velocity changes; this would be the way the area changes, the area of the tube. So, dA dx, you see for instance, consider a converging channel like this.

So, and if this is x, in this case you see the cross-sectional area of the tube is large here and small here. Is not it? It is its large here and its small here. So, in other words, in this case dA dx is less than 0 and its opposite dA dx would be larger than 0 for a diverging channel. In this case for a converging channel, dA dx is less than 0, that is what it would be right and yeah.

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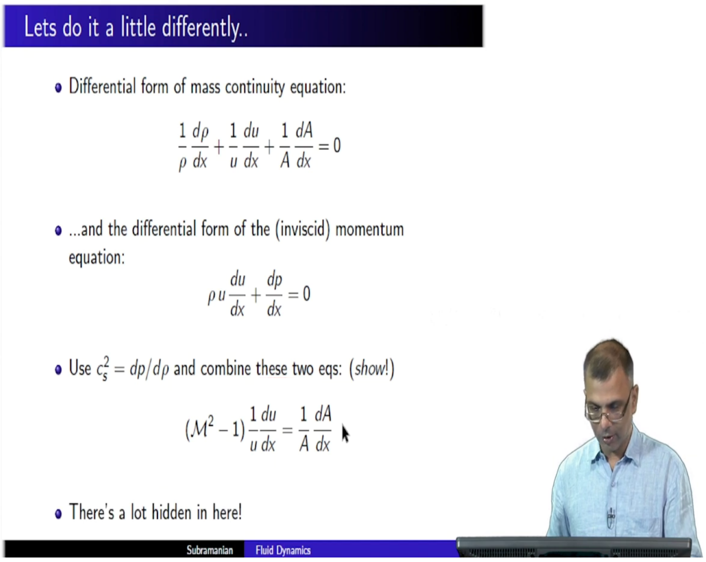


So, let us sort of look at this equation and try to see there is a lot hidden here ok and this is a very important equation and there is a lot hidden here.

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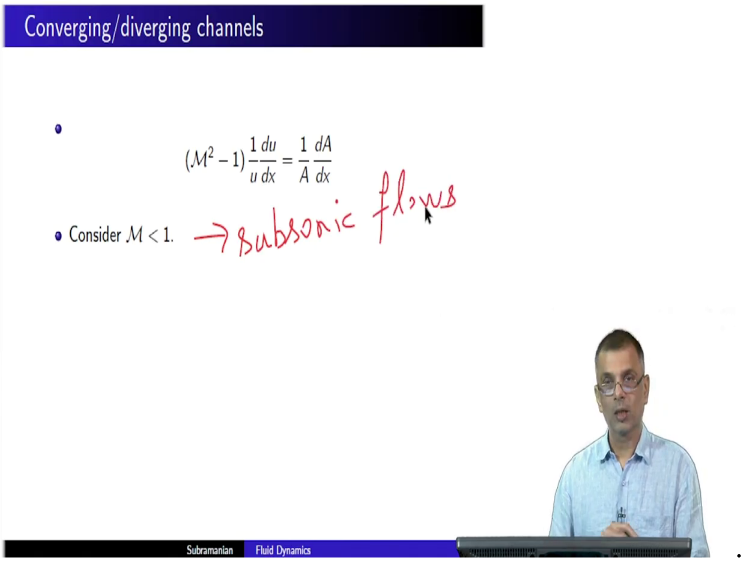


So, repeated that equation here. Consider subsonic flows. Consider flows that have Mach number less than 1 in other words right consider this.

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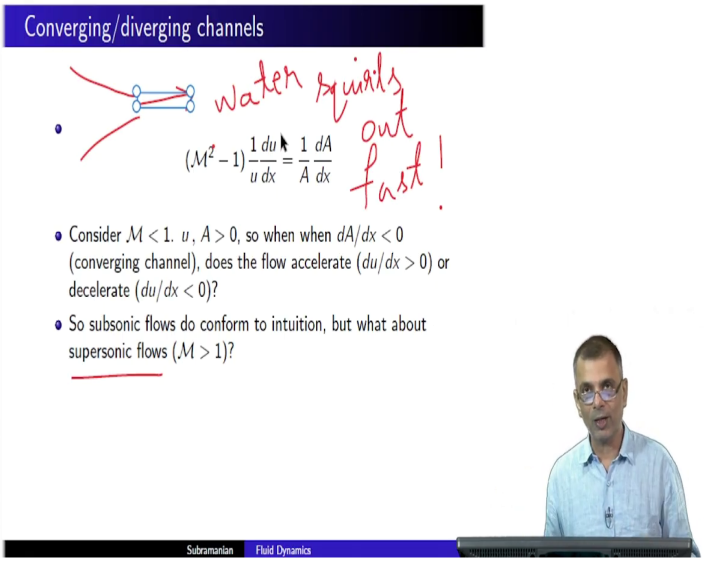


So, when dA dx is less than 0; in other words, a converging channel as we you know sketched in the last slide, does is du dx larger than 0 or less than 0? Let us just look at this equation right. So, Mach number is less than 1 which means that this is negative M square minus 1 is yeah.

So, and then the a itself is always a positive quantity, the area itself is always a positive quantity; is not it? So, and the dA dx is less than 0 right. So, say the Mach number is 0.1 right. 0.1 times 0.1 is 0.01. So, 0.01 minus 1 is always negative and what we are saying is that dA dx is also negative; everything else is positive. So, the negatives cancel each other.

So, is du dx going to be greater than 0 or less than 0 right? Obviously, it is going to be greater than 0 and this conforms to intuition. You pinch, you take a garden hose pipe right and you pinch it; in other words, you decrease the area and you find that the speed of water coming out from the nozzle ok is larger right.

You take a garden hose and you pinch it; in other words, you make it a converging channel like this and you find that pinching it, makes the water squirt out; water squirts out fast. In other words, the flow accelerates.

So, this conforms to intuition, when the water flow is subsonic and in everyday situations. Of course, the water flow is subsonic. It is very much; the speed of the water flow is very much lower than the speed of sound ok. So, subsonic flows do conform to intuition and the opposite can be said if dA dx was larger than 1, if you if you consider a diverging channel, you have a fast flow and as soon as it encounters a diverging channel, it slows down the du dx would be less than 0. So, this is what your intuition tells you.

But what about if Mach number is larger than 1, what about supersonic flows? In that case what happens is this quantity is larger than 1 right and the entire thing gets reversed. If dA dx is less than 1, now you see now we are considering you know supersonic flows, now we are talking about supersonic flow. So, this quantity is always positive.

Once this is positive, if dA dx is negative, then du dx is also negative; it is forced to be negative. If dA dx is negative; in other words, if it is a converging flow, du dx is forced to be negative. So, for supersonic flows, what we are saying is that if the flow here was supersonic, you pinch the nozzle and the flow actually slows down, very strange; does not conform to intuition.

That is the reason, we are talking about it here ok and in a diverging nozzle, if you have a diverging nozzle and you have a supersonic flow, the flow actually accelerates; du dx is actually greater than 0.

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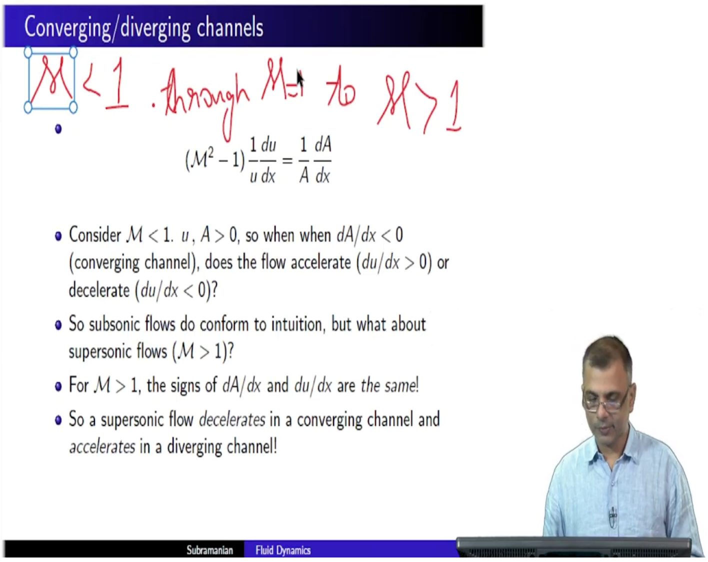


So, this is very strange. Yeah, so, for Mach number greater than 1, the signs of dA dx and du dx are the same. So, a supersonic flow decelerates in a converging channel and accelerates in diverging channel. This is really something strange and this does not conform to intuition.

And so, when you are transitioning from Mach number less than 1 to Mach number greater than 1 through you know when you are going from the subsonic regime through equal to 1 to when you are going from a subsonic regime to a supersonic regime through you know the sonic point so to speak, that is that, that is when you know Mach number equal to 1, you have to be very very careful and and this is an example of what is called de Laval nozzle.

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And so. So, you can marry these two kinds of converging and diverging channels together to get some very interesting behaviors and de Laval nozzle, the concept of the which is just this. A converging channel you know welded to a diverging channel through a throat ok and so, it really matters whether this flow is subsonic and this side or supersonic on this side.

The behaviors will be very very different and so, this was used for engineering applications in specifically, for instance, for rocket thrusters ok, where if you want the flow to accelerate. If the if you want a large momentum thrust on the rocket and you already know that the exhaust supersonic, then you want a diverging nozzle at the end of the rocket so that the flow accelerates ok.

And so, so this was really the concept of the de Laval nozzle was really thought of and studied extensively in such engineering applications. But as we will find out when we meet next, it has important applications in astrophysics as well, you know in trying to understand astrophysical jets. So, that is it for now.