Fluid Dynamics for Astrophysics
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Lecture – 03
Continuum hypothesis, distribution function and stress-viscosity relation - Recap

Hello welcome back; so to our course on Fluid Dynamics; specifically Fluid Dynamics for Astrophysics. So, I figured out take few minutes to recap what we did yesterday; I felt I might have gone a little fast. And so and also you know it is important to recap things; the overall philosophy of what we are going to be talking about at the beginning of a course so that you know the basics are well set in your mind.

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So, one of the first things we said yesterday is this word called continuum mechanics as opposed to the mechanics of a point parcel you know like a rock or something. And so you know the forces acting on a rock say an F equals m a for a rock right. So, you idealize you know the motion of this by considering its center of mass something like that and you solve F equals m a for that. This would be essentially the momentum equation for such a discrete particle.

However, when we are talking about a continuum; we are not talking about such a situation, we are talking about the dynamics of an aggregate of particles right. So, suppose you have lots and lots of these particles; yeah lots and lots of these particles and you were talking about the dynamics of the aggregate of everything within this box.

What is its F equals m a? And we are not talking about you know putting a center of mass to this and following only the center of mass. No, we are not and that is not what we are talking about, we are talking about the dynamics of the aggregate of these particles.

And so that is really what you know talking about continuum is all about and as a visual aid to understanding this you know think about holding a microscope and going really close to say a box containing molecules of gas think that you know; think of the molecules as being coloured so that you can observe them easily.

Now, in which case you are able to; when you are holding a very powerful microscope to the molecules, you are able to see the individual molecules say zipping around. And you could you know follow each molecule around, write an F equals ma you know for each of those particles and follow the dynamics of them. Now, either you move far away or you remove the microscope so that what you are really seeing is a bit of a fuzz yeah; so you really seeing a bit of a fuzz.

So, the individual nature of each of these molecules is lost and you really want to know how you know a and a small portion of this fuzz, something like this how this is behaving. So, this is what one means by studying the continuum; its the dynamics or studying the properties or studying the dynamics say of an aggregate. This is what I mean by dealing with continuum mechanics right.

So, before doing that we want to; I want to say a little bit about how; like we said we are not following the dynamics of each individual particle right, we are following the dynamics of the continuum.

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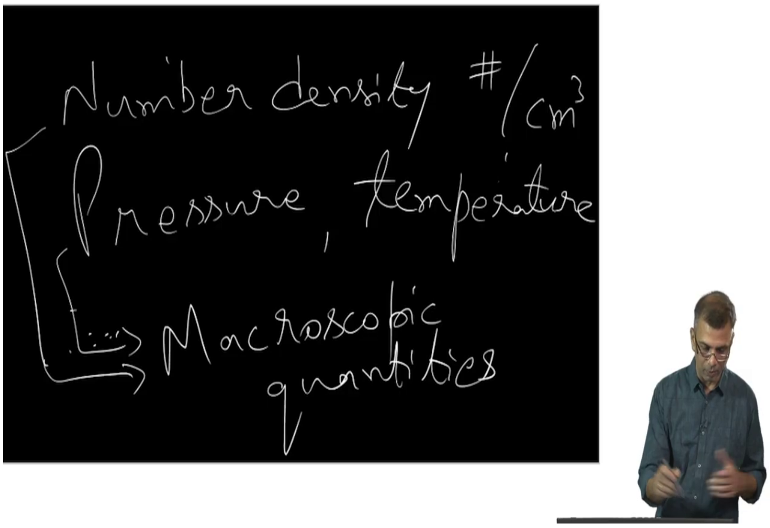

But you know things like number density which would be a number per say centimeter cubed or meter cubed; depending upon what your favourite convention is or things like pressure; temperature; these are familiar things to you when you talk about you know fluids. I mean it makes sense; for instance to talk about the temperature of gas in this room, to talk about the number density of gas in this room.

Either its so many particles per cc or something else in which case you make a comment about whether the gas is dense or not so dense, whether it is hot or not so hot. So, in order to go from; so it is important before studying continuum mechanics to at least which is what fluid mechanics is, to have at least some appreciation of how these macroscopic quantities, both of these are macroscopic quantities right.

So, its useful to have some appreciation of how these you know some of these macroscopic quantities arise from the properties of the microscopic you know molecules or the microscopic constituents that make up; you know this; that make up the continuum right. So, what I am now going to do is introduce the concept of a distribution function which I am sure you are familiar with what a distribution function is.

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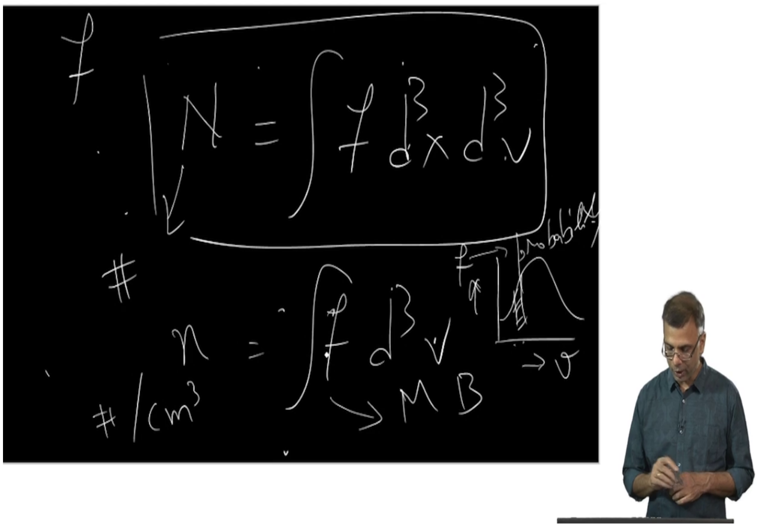

So, you know a distribution function; you can say that for instance the definition of this distribution function is this, the total number is equal to the integral of f; d cube x; d cube v. So, I could simply say and this is just total number, a dimensionless quantity. So, what is the distribution; what is the definition of the distribution function? Well, this is the definition of a distribution function right here ok.

Another kind of definition would be; if you talk about number density for instance which would be number per centimeter cube, what I do is; I integrate f only over d cube v; in which case this f, one example of such a distribution function would be the Maxwell Boltzmann distribution function which you are quite familiar with. A distribution function for which you would the velocity would be on the x axis and the distribution function would be on the y axis.

There are several interpretations and this would represent say the probability of finding the number of particles in a certain in a you are so likely to find this many particles in this particular velocity range; that would be the physical you know that would be the physical connotation of this distribution function. Now, having defined we will focus a little bit upon this particular definition; having defined a distribution function like this or for that matter like this, now this would be what is called the zeroth moment of the distribution function.

So, now what if I want to you know this ok; let me erase this and I will; what if I want to find out the average velocity, the average velocity of particles inside this kind of a you know, the average velocity of the molecules in this room right for instance.

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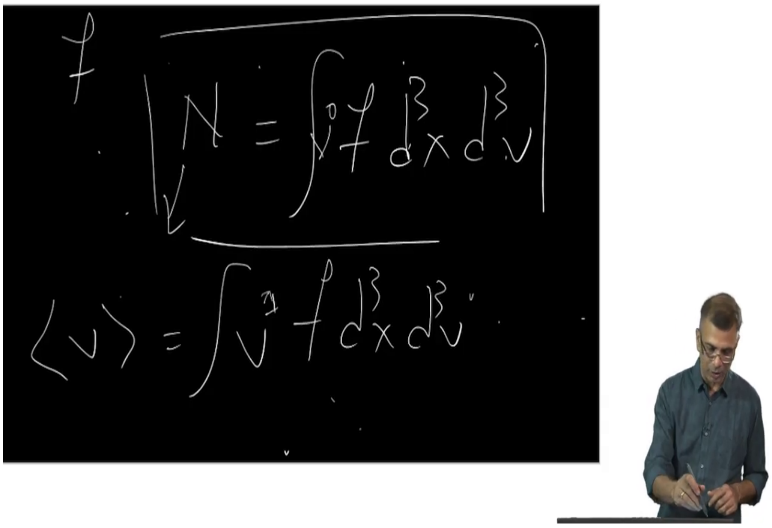

And so the average velocity would be; instead of here in particular here really you can think of having a v raised to 0 here; that is as good as 1 because you know. So, instead of 0; suppose I had, I stuck in a v raised to 1; and then I wrote f, d cube x; d cube v, this gives me the average velocity. So, this is the number 1 if you are; this is not a prime, if you wish I will just; I will just erase it just to you know not confuse you.

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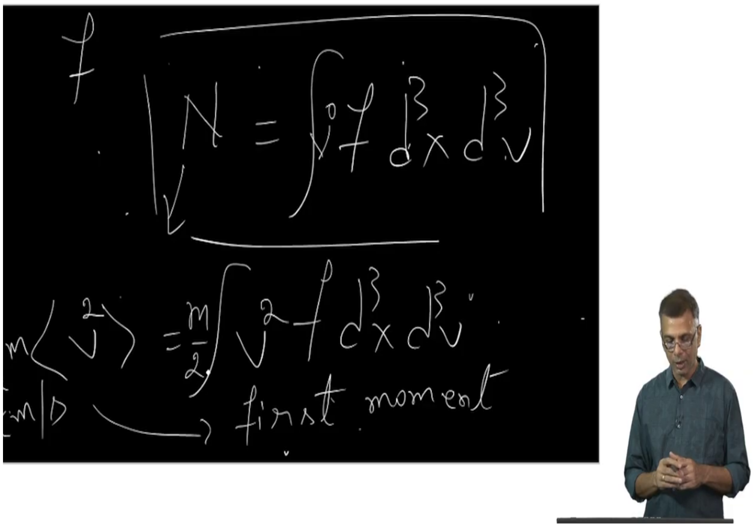

So, the average velocity would be v times this and you can think; so this is essentially the this is the first moment. I call this the first moment because you know the power here is 1 and this is the zeroth moment because the power here is 0. And so when you take the first moment, you get the average velocity and the dimensions of this are of course, centimeter per second.

If I want to take the second moment; the average of this, I just do that. And here this is a very useful quantity, this gives you the mean squared velocity and you just stick in a half; m sorry, beg your pardon. I do that and I stick in a one half m here and I get a one half m v squared and of course, I have to stick in an m by 2 here also.

So, what is this? This is the average kinetic energy. So, you notice how I am able to and this is a microscopic quantity right; this characterizes a continuum, not the; this is an average quantity ok. And what is the average kinetic energy? I know from you know from other considerations that the average kinetic energy is simply equal to this is a simply equal to right and where this is the Boltzmann constant; the k B is the Boltzmann constant.

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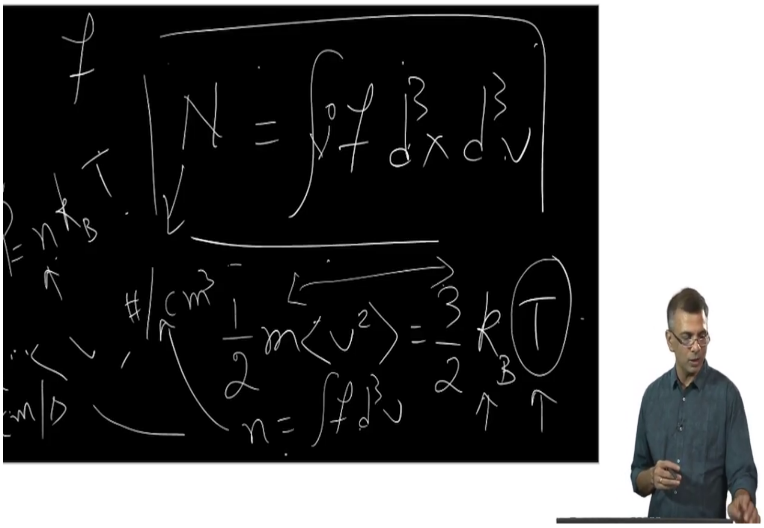

So, here you go; so here you see I am able to, so simply by altering this v raised to 0 to v squared and of course, taking in a half m on both sides, I am able to get a macroscopic quantity that you would normally associate with a continuum of gas such as temperature. And once I have the temperature; I can write, I can also get the pressure P equals n; k B; T, this is the; this is the first moment.

Well, not quite the first moment; you do not you do not integrate over velocity, you only integrate over. Sorry, you integrate over velocity; you do not integrate over, let me not make a mistake here; n would be integral of f; d cube v right. And so this would be number per centimeter cube and this is the quantity that you are talking about here.

So, you get this from the microscopic distribution function; you get this from the microscopic distribution function like this. And at the end of the day, you are able to construct a macroscopic you know quantity like pressure.

And this is valid for the continuum right, not for the individual particles; it will be very expensive computationally to be following the individual particles around, that is not what we are interested in. We are interested in the dynamics of a continuum and for that we need to know you know average quantities such as the average velocity.

Is there an average velocity of gases in this room? In other words, is there a breeze flowing through this room or not? Yes or no? The answer can be either way. What is the temperature of this room? How can I figure that out? From the you know from the properties; from the kinetics, this is how you figure it out. So, I wanted to at least you know bring this to bear; a little bit before going on to actually studying fluid mechanics because you know; we will be talking exclusively in terms of quantities like the density, temperature, pressure which are all average quantities, but it is important to know where these came from.

Physically, as you know the pressure if you want to relate pressure to kinetics; the pressure is essentially a statistical average due to lots of molecules of pressure on a wall of a container; for instance is the result of lots and lots of molecules striking the surface of the wall right.

So, that is the physical picture you have, but nonetheless; how do you construct the pressure? This is how you construct the pressure. So, I thought I would talk a little bit about you know; reinforce this concept of how to derive macroscopic quantities from the microscopic kinetics; that was the first thing we did yesterday.

And the next thing we did; the next before moving on to actually studying fluids, I figured I would say something about how a fluid which you normally think of as either a liquid or a gas.

Most of the time in this course, we will be implicitly thinking about a gas. But how, for instance have you have not ever wondered how a solid is fundamentally different from a fluid? The answer there is the most simple minded answer and there are lots of complications to this, but it is important to stick to the most simple minded answer.

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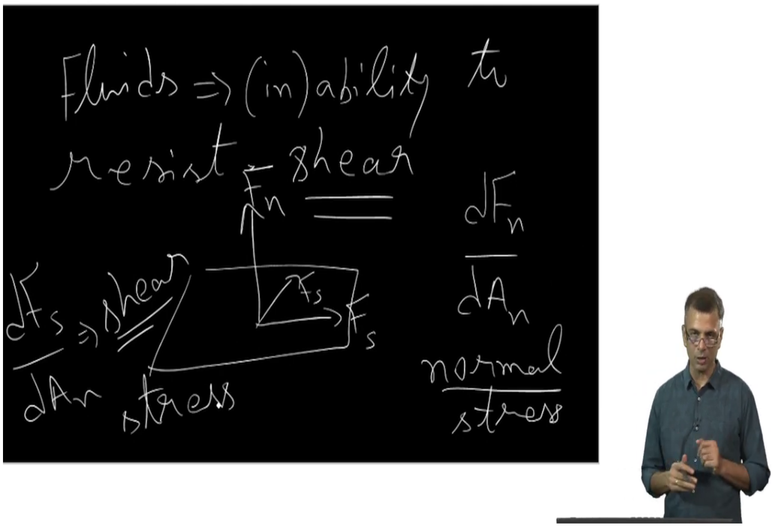

And the answer there is the ability fluid; sorry, their in ability to resist shear; under the influence of shear even an infinitesimal shear, they flow. And this is what fundamentally differentiates fluids from solids and we all we talked a little bit about what shear means.

Essentially, you think of a surface and if there are two kinds of stresses, two kinds of force per unit area you can think of. One is a normal, so there is the area element; so the normal is like so. So, one is a normal force like this and so you are applying a normal force and the force per unit area is the would be simply the result of this normal force.

The other kind of force you can think about is when the force is tangential to the area; to the surface element like so or like so and these are called shear stresses. This is called a normal stress and this or that is called the shear stress.

So, if you have an area element like this, this would be a normal force. So, you know a d F n; over d A n; this would be a normal stress, whereas, if you have a this or this; F you know, this is also F s and this is also F s. So, a d F s; d A n would be a shear stress, this is the difference between a normal stress and a shear stress.

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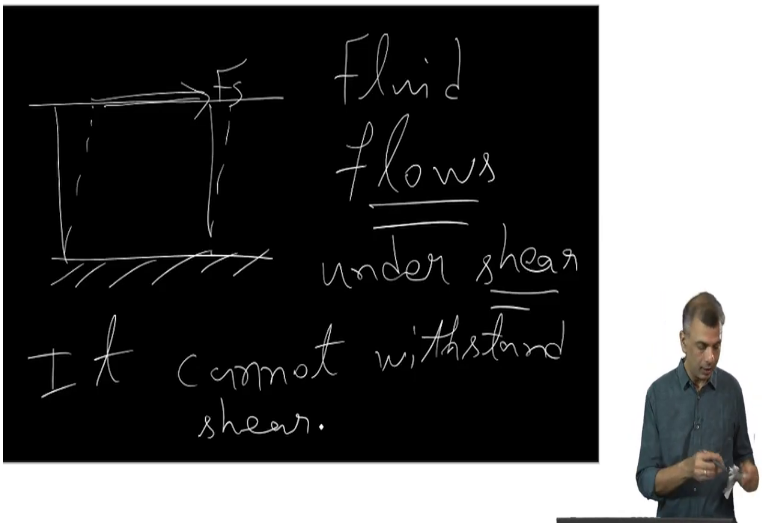

And imagine a container of a fluid with a surface like this, either a solid or a fluid this is how you differentiate and you apply a stress that is along the surface; in other words like this, you apply a shear stress like this. Now, a solid you know as you would imagine would be distorted, but not that much; it would kind of distort a little bit like this, but it would tend to snap back to its original position, it would snap back right.

Whereas, a fluid on the other hand will not be able to withstand this shear, it will flow; a fluid flows, it cannot withstand shear. This is what characterizes, this is the fundamental thing that characterizes a fluid right.

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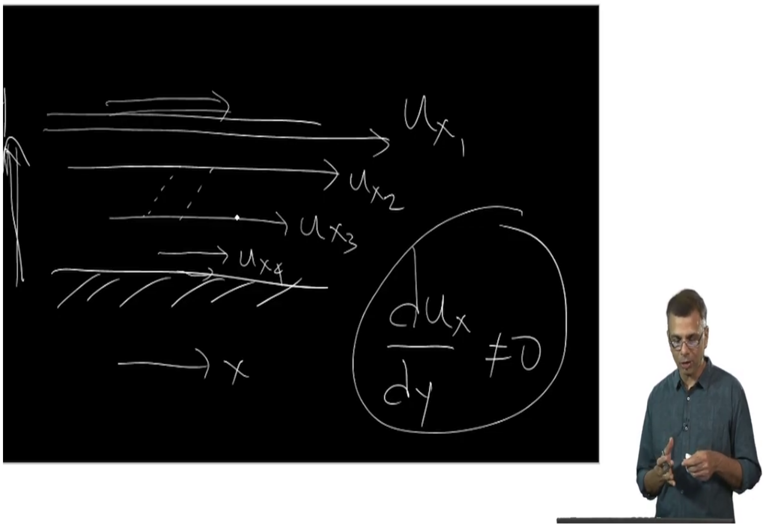

So, it cannot withstand shear and more specifically when you; in the same kind of situation when you have you know like so and a fluid element a fluid surface like this and you apply some kind of a say wind blowing over the surface of a lake or some kind of you know situation like that; you know.

Suppose, this was a container filled with honey; so what would happen is if you apply a force on the surface? The top surface, you can think of the top surface flowing with some kind of velocity. Suppose this would this was the y coordinate and this is the x coordinate and you have you know an x directed velocity like that. So, the top surface would flow fastest and you can think of the next; the next layer flowing, but not nearly as fast flowing with a slightly reduced velocity, this also say u x 1, u x 2.

The next layer would flow slightly reduced like this; until you come to the bottom where the honey actually sticks and there is practically no velocity and if you wish you can also draw u x 4. In other words, there is a d u x, d y; that is non 0. There is a gradient that I mean you know the u x varies with y right and so there is a nonzero and this is what characterizes a fluid.

There are two things; a, it flows under the influence of shear and b typically there is a gradient in the direction that is normal to the direction in along which the forces; there is a gradient of the velocity and in other words; there is a nonzero. And this is caused by the fact that fluids have non zero viscosity; it is almost as if these layers of fluid are connected by little rubber bands, like this that refuse to let the top layer slide over the bottom layer ok.

The kind of like solids in that; they also try to snap back, but they cannot ok. So, the stiffness of this rubber bands kind of define what the viscosity is that it characterizes the viscosity rather.

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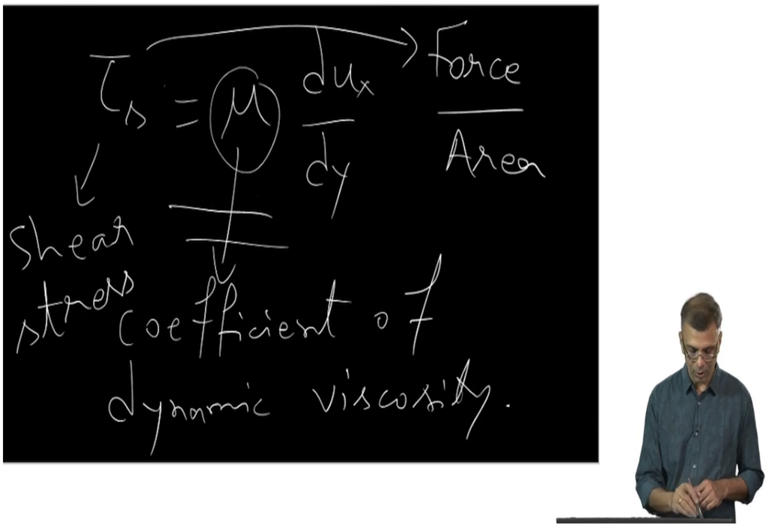

And so this was another thing that I wanted to you know bring forth before plunging into specifics and in particular, if you write down this kind of a quantity; tau s on the left hand side and d u x; d y which we saw on the previous slide, there is a constant of proportionality and this is called the coefficient of dynamic viscosity.

And this is the shear stress; it is essentially force by area; just like pressure. And that brings me to my point, there is really no need to draw a distinction between shear stress and pressure; it is the same thing what you are really what you are; suppose, we draw a little a little imaginary cube like that.

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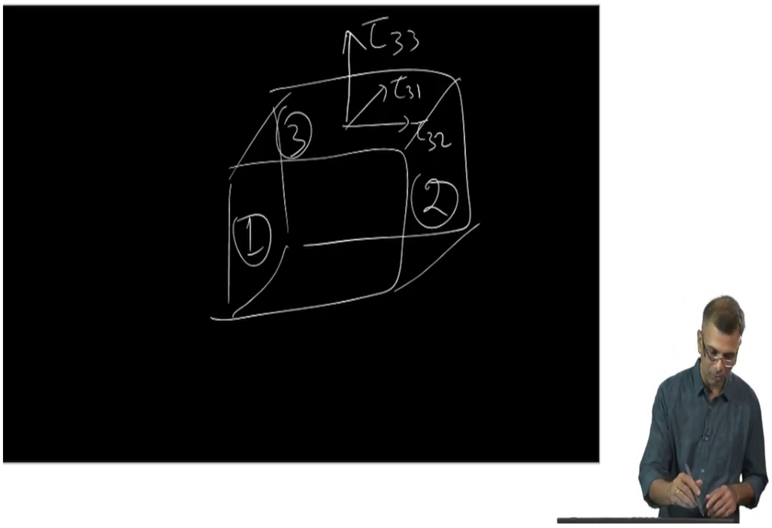

There is really no need to draw a distinction between pressure and shear stress is really the same thing. Pressure would be force per unit area in this direction shear stress would be force per unit area like this and like that. So, if we for instance; if we call this tau; if I call this phase 3, if I call this tau 3 3 and if I call; if I call this tau 3 1 and if I call this tau 3 2 simply because this is phase 3, this is phase 2 and this is phase 1 ok.

So, the tau 3 3 would be the force per unit area acting on phase 3 ok, along the normal to the phase. Whereas, tau 3 1 would be the force per unit area acting on phase 3, well its another component and tau 3 2 is the force per unit area acting on phase 3, along the normal to phase 2 right. So, this is by the way I think I made a mistake here, this is really not phase 1, I should not let me simply label phase 3 and phase 2 and that will be that is good enough right.

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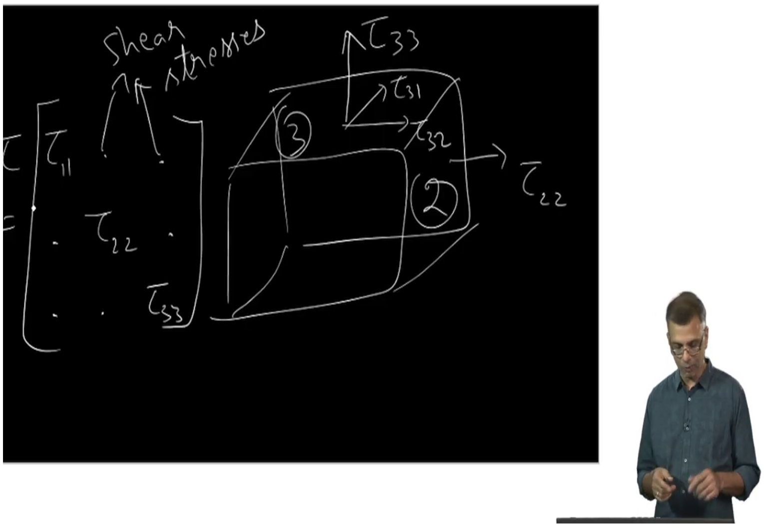

So, the point is; it is useful to think of the stress as a matrix with the elements tau 1 1, tau 2 2, tau 3 3 and then other off diagonal elements like this. So, the diagonal elements are these; you know tau 3 3, tau 2 2 and so on so forth and the off diagonal and these are what you normally think of as pressure; normal pressure and off diagonal elements are what you think of as. So, these are all shear stresses ok. It is useful; this kind of you know this kind of a description will be useful to keep in mind, when we go further; when we study the momentum equation which is simply F equals m a for the fluid ok.

So, I wanted to introduce this up front and introduce the concept of viscosity. So, in each of these case, for each of these; each of these shear stresses there is a you know this kind of; this kind of relation that is valid ok. You can think of a mu that characterizes each; each of these shear stresses ok.

So, the components of mu will also form a matrix which we call a tensor ok. In this case, that is good enough; there is really no need to go into the into more detail about what a tensor is, you can simply think of it as a matrix and it is the same way one thinks about you know elasticity for solids right.

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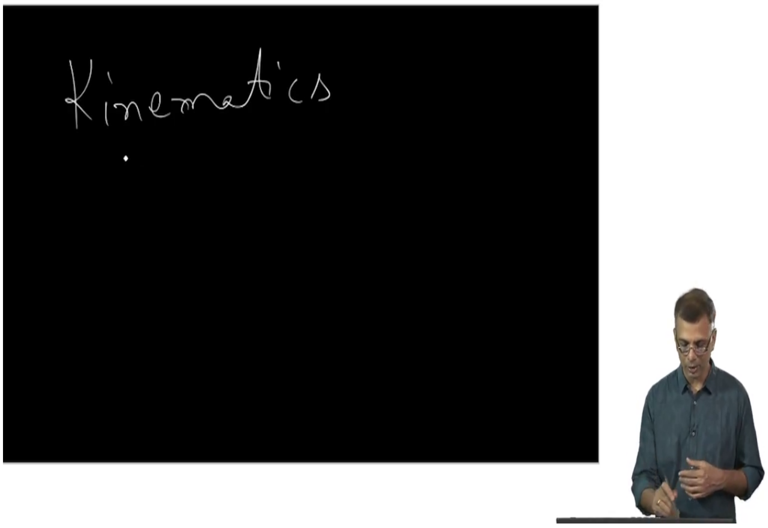

So, this was the other key thing we talked about and from now on we will start talking about the kinematics of fluids; that is mostly what we will talk about from the next session onwards which is a focus on the appearance of motion such as displacement, velocity, acceleration so on so forth of fluid elements without explicitly talking about the forces that are acting on them.

We are really not concerned about an F equals ma, a conservation law, but instead we want to; we want to see you know the manner in which the fluid is moving or not. So, we will talk about concepts like stream lines path lines and so on so forth which will be very useful when we go ahead. So, from the next session onwards, we will start talking about fluid kinematics.