Fluid Dynamics for Astrophysics
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Lecture - 29
Shock thickness recap, shock jump conditions

So, hi we will get on with our discussion of Shocks now, but before that I thought I should very briefly review our discussion of Shock thickness. A, because it is an important topic and b, because I am afraid I misstated just one small point there.

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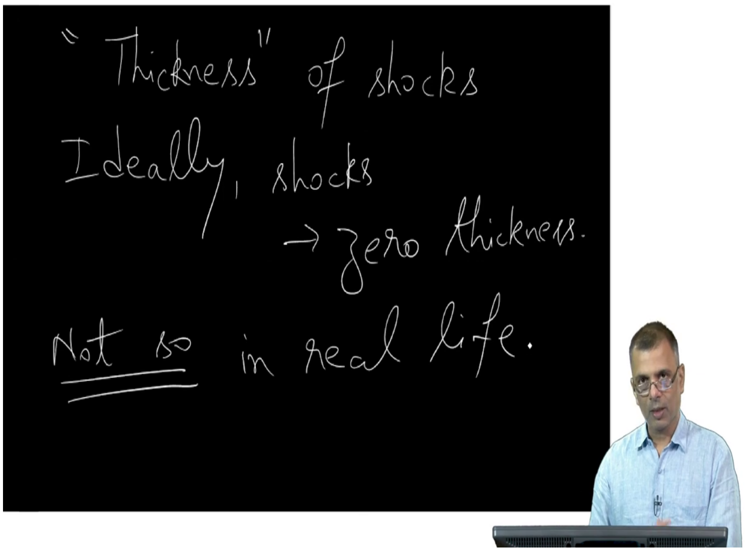

So, very quickly just to recap we are now talking about thickness of shocks. Why are we even talking about this? Because ideally shocks have zero thickness, right. But in real life of course in real life not so, in real life. In other words, in real life shocks do have finite thicknesses.

And so, the question we are now asking is you know what decides the thickness of the shock. And we found out during the discussion last time that it is the, viscosity of the fluid, which decides the thickness of the shock. Specifically we also found out that we found out. Let me just say this.

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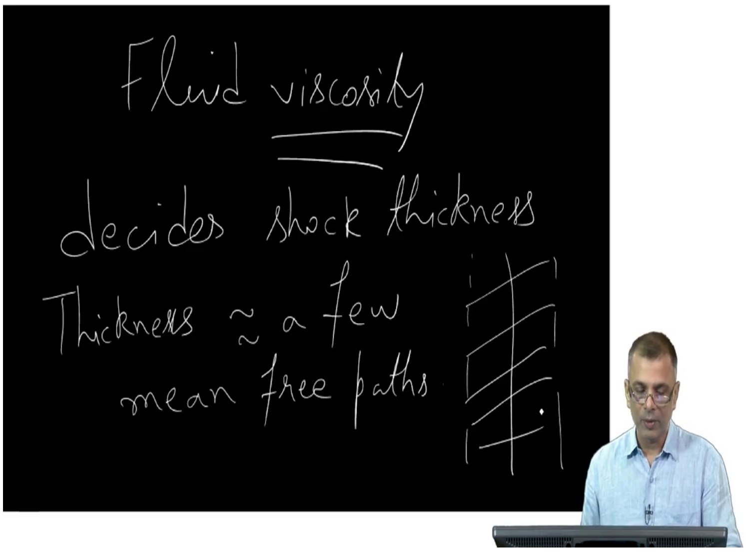

Fluid viscosity decides shock thickness. So, this were the bottom lines of you know our discussion last time and we also said that the thickness is something like a few mean free paths. Now, immediately you should jump and say well you know when you the moment you say mean free path it is no longer conforming to a fluid picture.

It is true. You know you are right about that. But, the thing is you know you have a mathematically ideal discontinuity and you are really talking about you know a thickness that is very I mean I am expanding. You know this would be roughly the shock thickness and this is a very expanded view really this is very very small.

So, when it comes to really small scales ok, you are in any case as you know I mean the fluid approximation holds for only large enough scales. Scales that are large enough so that they are not microscopic they are mesoscopic they are between microscopic and macroscopic, right.

But, they are still large enough that you have many many many mean free paths inside them. And this kind of thickness is such that, that assumption is on thin ice. In other words the fact there might not be many many many mean free paths in between them. In fact, the shock thickness is only about one mean free path or maybe two mean free paths, ok.

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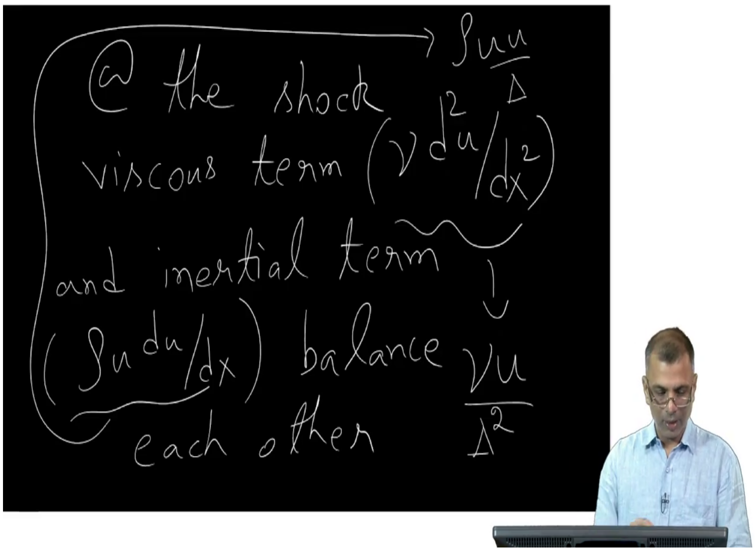

And so, here this, so, the way we arrived at that was to show that, at the shock at the shock the viscous force the viscous term rather viscous term, which is something like nu and the inertial term of both of these in the Navier Stokes equation right, term which is rho u du dx, right. And when we write du dx we are just considering one dimension for simplicity balance each other.

Why is this? Because in the bulk of the fluid the bulk of the fluid is essentially inviscid ok, viscosity does not have any role to play at all. But, when we start talking about you know large velocity jumps within very very small lengths like this then what happens is du dx becomes large. In other words, there is a substantial you know change in the velocity over a very small length scale ok, a very small length scale like this.

So, du dx becomes appreciable and d square u dx squared becomes even more so, even more appreciable. So, simply because this term is large even though nu my itself might be small the combination is appreciable. And really right at the shock we are considering a situation where the viscous term pretty much balances the inertial term.

And if you know write this in order of magnitude terms then in order of magnitude language then this would be something like nu u over delta squared, where delta is a shock thickness.

And this would be something like I do not have space on this slide, but it is important to write it write on this slide that is why I am doing it. This would be something like rho u times u over delta, something like this ok. One u cancels each other. So, this nu u over delta squared it should equal rho u squared over delta ok and doing that yields the following important formula.

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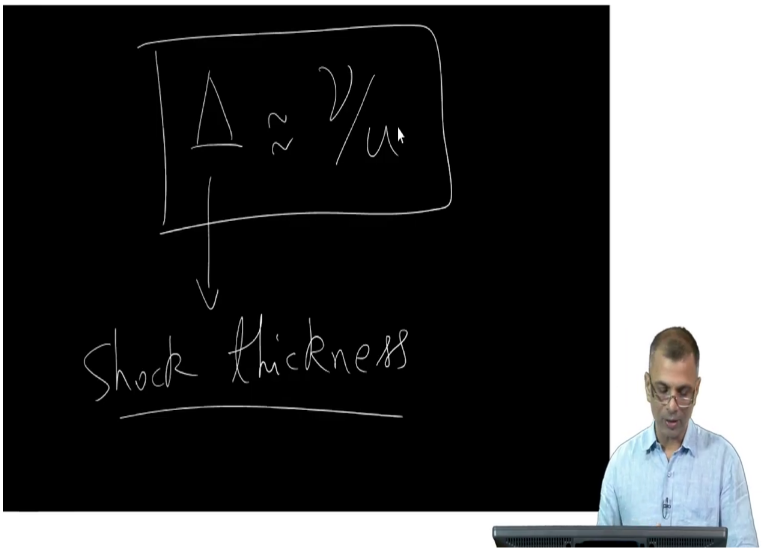

The shock thickness is approximately equal to nu over u. It is very very important and this is the shock thickness. Up until this we did not up until this it is simple enough, right. So, this essentially in support of this statement the fluid viscosity decides the shock thickness. Of course, right it is a viscosity here which decides a shock thickness, also of course, it is the fluid velocity, right.

If the velocity is relatively small then the shock thickness is relatively larger and vice versa. So, this is one thing. The other thing of course, is to now ask well ok this is a fluid property, but you said something about microscopics you said something about mean free path. So, what gives right?

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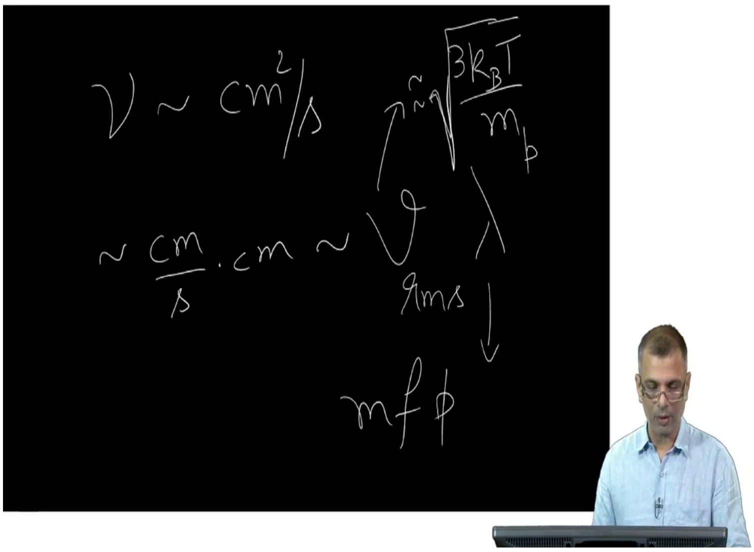

So, thing is the kinematic viscosity which is something like centimeters squared per second, which is essentially some sort of a centimeter per second times centimeter right, which is some kind of velocity times some kind of length yeah. So, what would the velocity be?

The velocity would be the rms velocity of the constituent molecules and this length is not the shock thickness it is the mean free path ok. This would be the mean free path ok. Now, what is V rms?

This is one of the mistakes I made last time. It is actually something like a square root of 3 k B T over m sub p, k B is a Boltzmann constant right, T is a temperature and then this square root extends all the way of course, right and m sub p is a proton mass this is assuming that this is the electron proton gas, ok.

If it is not an electron proton gas and this would be the effective mass of the molecules, ok. For instance, oxygen is present in that greatly overwhelms the protons. So, this would essentially be the weight of an oxygen molecule ok and this factor 3 is for you know mono atomic gas. So, this is something like the order of unity roughly. Now what else is? So, let me write this down once more and.

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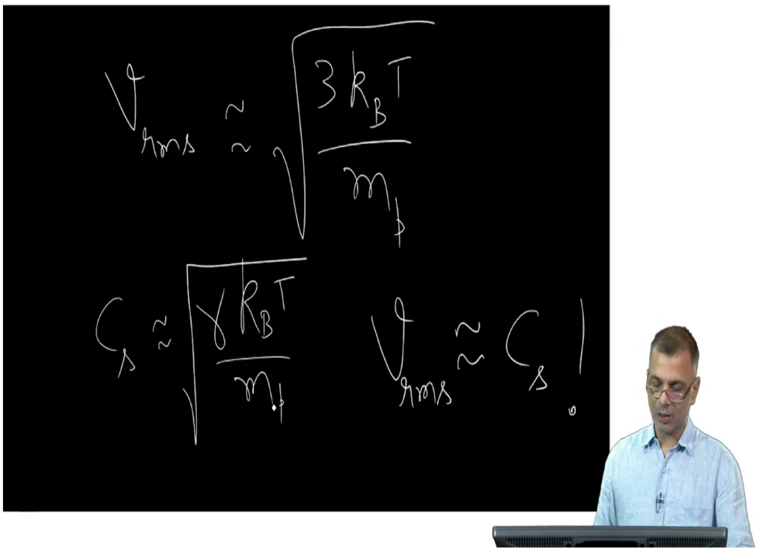

Here you see the V rms the root mean squared is something like and so, this is one thing and. So, let us try to compare this with something else that we know already, right. So, what is the speed of sound the isothermal speed of sound or for that matter any you know? The speed of sound is also something like gamma k B T over m sub p.

This looks very similar to this ok except this gamma can be 1 or five-thirds and here you have a 3. To the extent in the manner in which we are working we can confidently say that V rms, ok. I made a mistake in the factor of m sub p. When the last time we were doing this, that is the reason I wanted to emphasize this a little bit now, ok.

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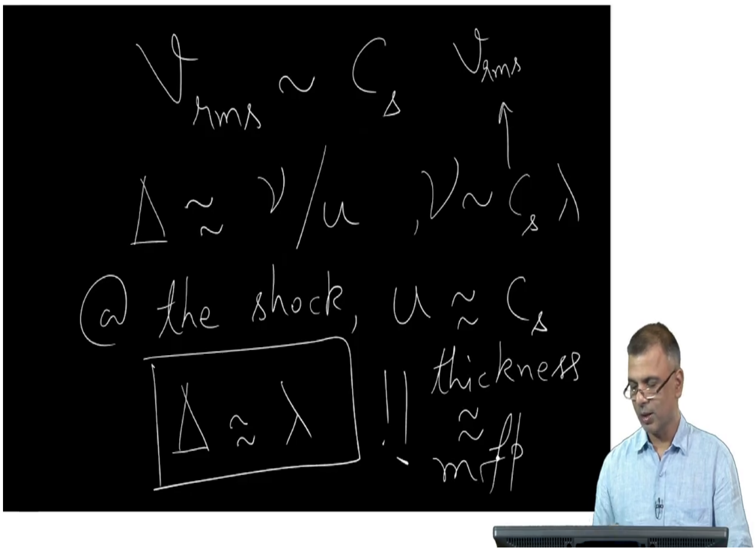

So, now, what happens is so, this is the main thing we basically said V rms is approximately the speed of sound, ok. And now so, this is one thing. And remember we also wrote down that the shock thickness, right and nu is something like V rms times lambda. In other words something like C sub s times lambda because C sub s is basically the V rms, right.

So, the nu is something like C sub s times lambda. Now, what is the speed of the fluid right at the shock ok? Is something like C sub s, right? So, the speed the u at the shock u is approximately C sub s. It might be twice C sub s. It might be 3 times C sub s depending upon what the Mach number is, but to within a factor for unity it is about C sub s.

We are talking about moderately supersonic shocks. If that is the case looking at this and this and this right, we get this important thing, right. So, shock thickness the thickness is approximately the same as the mean free path. So, the shock thickness which we sketched here you know this one the thickness is 1 mean free path or 2 or 3 mean free paths.

Why are we saying 2 or 3 mean free paths? Because you know we have played fast and lows with several of the factors of order of unity that we have here. For instance, factors of the order of unity we have neglected, alright. So, there might be a mistake of something like 1 or 2 or something like that and so, that is one place we made a significant you know. So, therefore, this derivation that we have done the shock thickness being equal to the mean free path this is only approximately true, right. It might well be of the order of a few mean free paths.

So, I just wanted to emphasize this primarily because this is not something you find discussed in several fluid dynamics you know texts or in discussions. Many of them just say that the shock is a infinitely thin you know discontinuity and then they move ahead from that, ok. So, this is then this was one important thing that I wanted to you know highlight.

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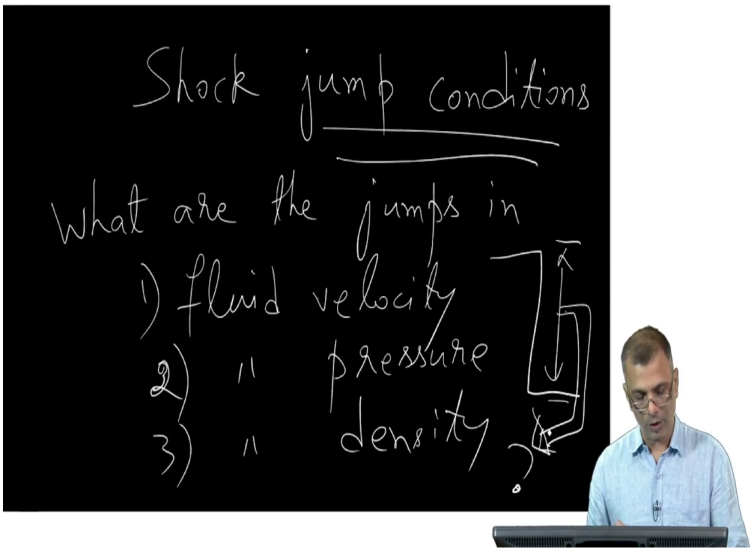

The other thing as we go on the other thing that I want to the next topic I want to cover is the issue of shock jump conditions. What do I mean by jump conditions? I mean what are in other words, what are jumps in say fluid velocity, fluid pressure, fluid density and things like that across a shock.

Why is this important? This is next natural thing to ask, right. We have been repeatedly saying that shocks are essential discontinuities, ok. There are discontinuities there are jumps right, they are essentially jumps right. So, what is; so, suppose this was a jump in suppose this was a jump in density.

Well, what how much is this jump? That is the next question to ask right. So, how much is the jump in density, how large is this jump? So, that is the thing that we will be trying to answer and we will find that we will take recourse the usual things to the usual conservation laws, conservation of mass, momentum and you know such things that we are already familiar with I should. This is not yeah ok. And the other thing I want to emphasize is that we will be deriving jump conditions, but what we will be what we will be deriving is for what is called a normal shock.

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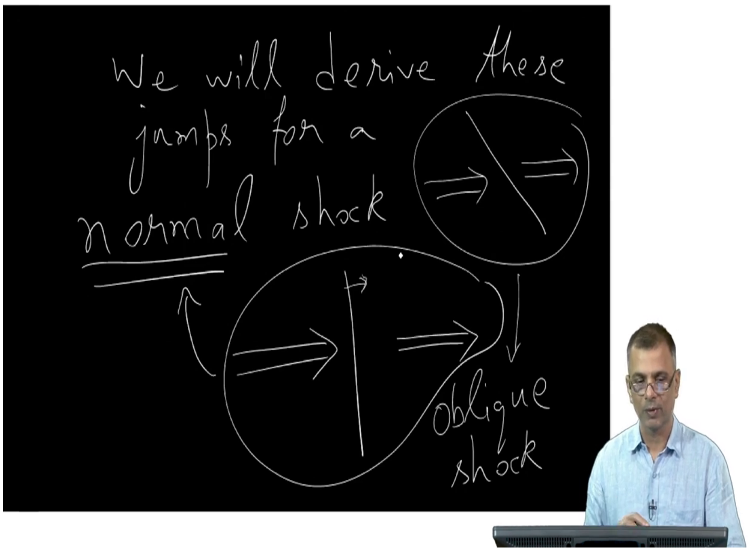

We will derive these jumps for a normal shock. In other words, if this is a shock and this is the fluid velocity they are normal to each other. Or the other way put it is the shock itself is like this and the shock normal is pointing in this direction right. So, and so, the shock normal and the direction of the fluid flow are in the same direction. The shock is not oblique like this, ok.

This would be an example of a oblique shock. And this would be an example of a normal shock ok. For simplicity we will simply be discussing this case that of a normal shock. The jump conditions for an oblique shock are very similar except they are a little more involved. I mean you know it is just one more step it does not I mean it follows.

If you understand how to derive the jump conditions for a normal shock if you understand that thoroughly, it is not a big deal. The I mean the other thing to realize is that the jump conditions we will be deriving will only be the jump conditions on the normal component of the velocity on this component of the velocity and the normal component of the momentum.

Mass does not matter mass is a scalar, so, only velocity. So, that the tangential components this you know those we assumed to be continuous. However, there can be situations where there is not a shock at all there is no shock, ok. In other words the normal components of the mass flux, the velocity flux, the momentum flux these are all there is no jump in these quantities as you pass this kind new kind of discontinuity.

However, there is a jump in the tangential component, ok. So, these are what are called tangential discontinuities and these are not shocks ok. We will not be discussing this in any detail as we go ahead that is not our main focus at this point. But, I just wanted to sort of alert you to the fact that lest you go on thinking that the only kinds of discontinuities there are a shocks no that is not true ok.

There are these other kinds of discontinuities which are called tangential discontinuities and one important tangential discontinuity is called a contact discontinuity. And what happens in a contact discontinuity is that only the density jumps across this discontinuity the other quantities are all conserved.

Conserved is not the right word, the other quantities are all continuous across a contact discontinuity. There is only a density jump and so, this contact discontinuity is also a bona fide discontinuity. In the sense that, you know there is a mathematical problem here ok, the density is discontinuous it jumps ok.

But, the other quantities do not jump and a contact discontinuity is an example of a tangential discontinuity and it is not a shock, ok. I just wanted to say this. We will not discuss this anymore. We will confine our attention to jumps ok for a normal shock not even an oblique shock. We will confine our attention only to jumps for a normal shock, ok.

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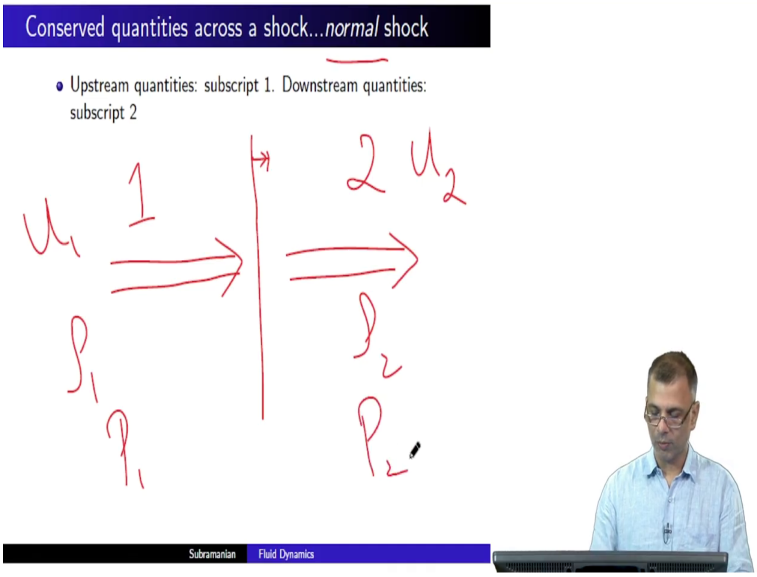

So, as promised we will now start talking about jump conditions right. Jump conditions for a normal shock right, a shock where you know this would be the shock for instance and the shock normal is pointing in this direction. And the fluid velocity is also pointing in the same direction as the shock normal, ok.

So, this is what I mean by a normal shock ok. So, before going into details if this is the direction of the flow, right, upstream quantities, which, would be this. These would have this would be denoted by a subscript 1 and downstream quantities, which would be this which would be denoted by subscript 2. In other words, assume that you are in a boat yeah going along with the stream like this yeah like this.

So, naturally yeah, so, you are down the stream here quantity 2 and up the stream here quantity 1. So, suppose some we are talking about something like u 1 velocity. So, the u sub 1 would be the velocity here and u sub 2 would be the velocity here. Similarly, you know the density rho 1, rho 2, so on so forth; P 1, P 2, this is what I mean by subscripts, ok.

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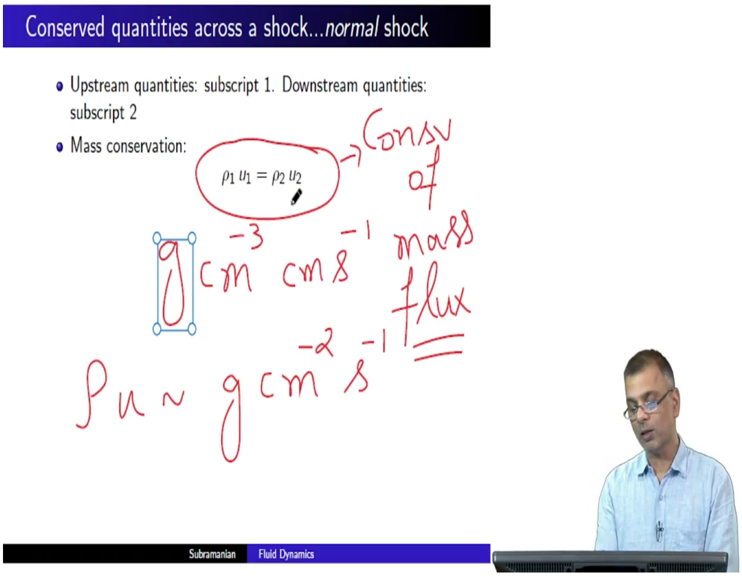

So, the way we go about deriving these jump conditions is we write down things like mass conservation. The fact that mass cannot be created or destroyed across a shock and we the way we write this down is say rho 1 u 1 equals rho 2 u 2. Now, what are we really conserving? Are we consuming mass itself? Are we conserving the mass flux? What are we conserving?

In order to, in order to answer that you will simply look at the dimensions. The dimensions of rho are you know grams per centimeter cubed. You agree? And the dimensions of u are centimeter per second. So, the quantity rho u has dimensions grams per, right.

So, and this is what the mass flux is. So, really this a more accurate way of saying this is this represents conservation of mass flux that is what this represents. We say mass conservation that is a bit of a lose term, but strictly speaking what we are saying is the mass flux across a shock is conserved. In other words rho 1 u 1 which would you know the mass flux in the upstream is equal to the mass flux in the downstream. So, this is one important conservation quantity.

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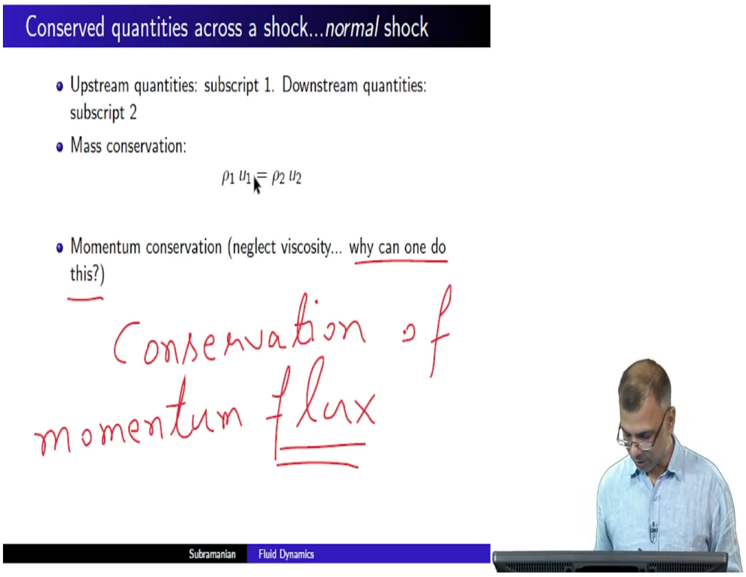

The other thing is momentum, again momentum flux ok. We neglect viscosity. Why can we do this? This is very important why can we simply neglect viscosity why is it that we you know said we talked a lot right. I mean we are essentially now going to be using just our Euler's equation in. So, again we are of course, talking about conservation of momentum flux.

Just like we talked about conservation of mass flux here we are now talking about conservation of momentum flux. And in doing so, we are going to be neglecting the terms that involve viscosity. Why is this? Because as we you know as we have remarked earlier we are only when we talk about the mass flux here we are talking about the mass flux upstream or downstream not near the shock ok; far upstream from the shock or far downstream from the shock.

We are you know similarly when we do when we write down the momentum flux in the upstream region or we write down the momentum flux in the downstream region. Both of these quantities refer to the fluid far away from the shock either far upstream from the shock or far downstream from the shock.

And we know that in these regions you know the viscosity the viscous terms are of no importance. Viscous terms assume importance only near the shock, right. This is how we derive the shock thickness remember. Viscous terms are important only very close to the shock. Away from the shock you know is essentially an in-viscid fluid. So, is why we neglect viscosity.

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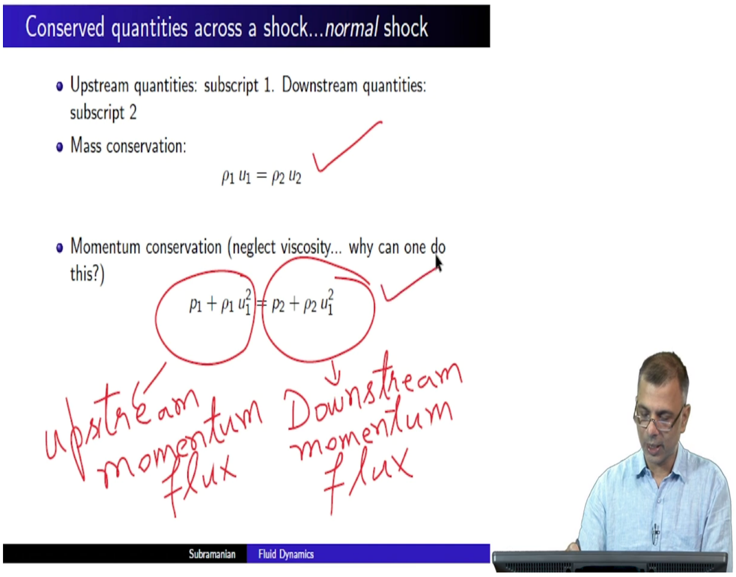

And so, if we do this what happens? The momentum flux and this would be the upstream right, and this would be the downstream momentum flux. And we equate these to each other and the upstream again like just to emphasize the upstream momentum flux refers to the momentum flux far upstream from the shock.

The downstream momentum flux refers to the momentum flux far downstream from the shock. That is why we do not need to because the fluid far upstream and far downstream are both essentially in-viscid that is why we do not need to in you know add the viscous terms here.

And if you look at the Euler equation this is exactly this is the pressure term and this is what is called the ram pressure term ok. This is associated with the kinetic energy of the fluid, kinetic energy per unit mass of course, and this is just the pressure. And dimensionally they are both on the same footing: otherwise we would not be able to add them together like we are doing now. So, this is the first important equation, this is the second important equation.

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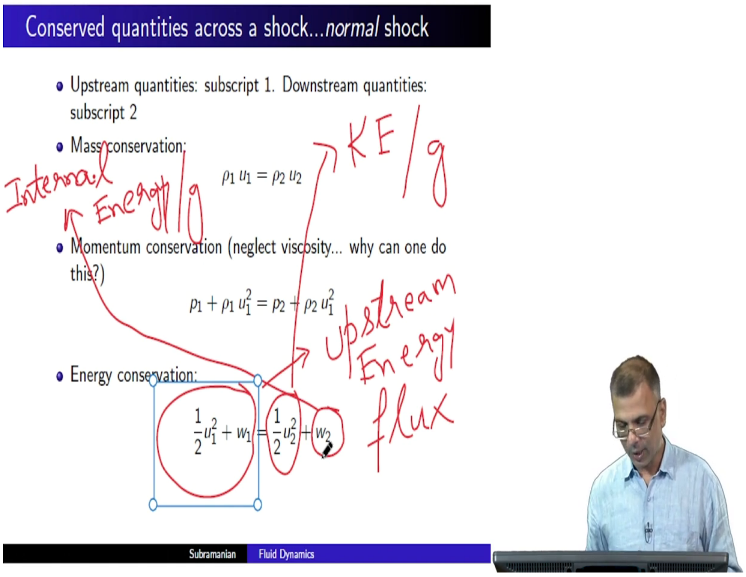

The third: of course energy conservation ok, conservation of energy far a far upstream and far downstream. Again really this really means so, this would be the upstream energy flux and this would be the downstream energy flux. Now, we have introduced a new quantity w and the w is the enthalpy, ok.

The difference being that the u squared half u squared this is essentially kinetic energy right, I mean kinetic energy per unit gram right. So, let me just write this down. Kinetic energy per unit gram of fluid and the kinetic energy is essentially the energy due to the bulk flow of the fluid.

However, this is the, so, this is the kinetic energy associated with the bulk flow of the fluid. It says nothing about whether the fluid is hot or cold or anything. This term contains yeah. So, it says it is only due to the bulk motion. Whereas, this term is the exact opposite, it has nothing to do with the bulk motion.

You can be sitting in the fluid frame ok and this would be the energy per unit gram that you would see. And so, this represents this is representative of the internal energy. Of course, internal energy per gram; internal energy per gram right. Because there is also kinetic energy per gram, therefore, whatever energy this is also has to be per gram.

So, this would be the kinetic energy per gram plus the internal energy per gram in the downstream region. This would be the kinetic energy per gram plus the internal energy per gram in the upstream region. So, the internal energy has to do with heat has to do with thermodynamics and so, that is called the enthalpy.

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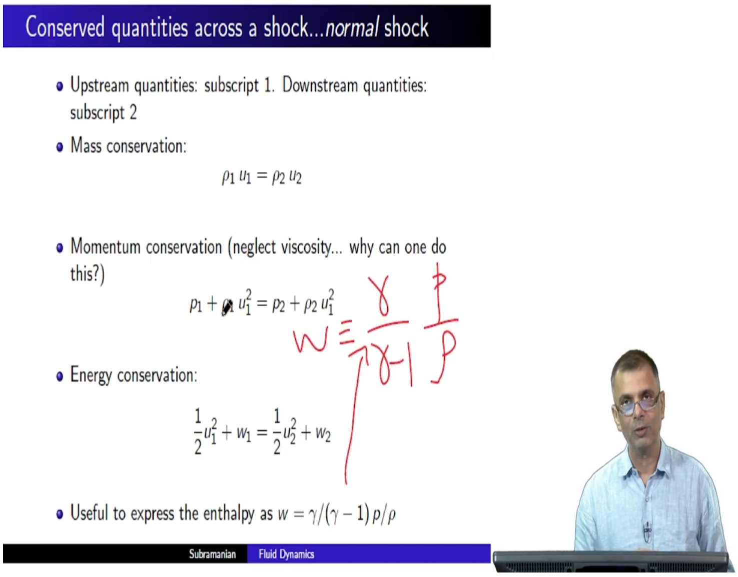

And the enthalpy can be related to the usual since this is a new quantity and enthalpy can be related to the usual pressure and density like this. What this is really this might be a slightly confusing way of writing it. It is essentially gamma over gamma minus 1 p over rho, this is enthalpy, ok. So, now, we have three equations and how many unknowns do we have?

We have the density 1, velocity 2, and pressure 3 right, but so, these are 3 you know variables and you have two copies of these variables right; one upstream copy, one downstream copy. So, we have 3 equations and 6 unknowns. Obviously, we cannot solve for them, but we can solve for the ratios.

With this it is possible to solve for u 1 over u 2, rho 1 over rho 2, p 1 over p 2. And what are these? These are the jump conditions that we are after right. So, using these three equations, it is possible to solve for the jumps and velocity, density, pressure and this is exactly what we are after.