Fluid Dynamics for Astrophysics
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Lecture - 26
Propagation of sonic information, shock tube problem and piston problem

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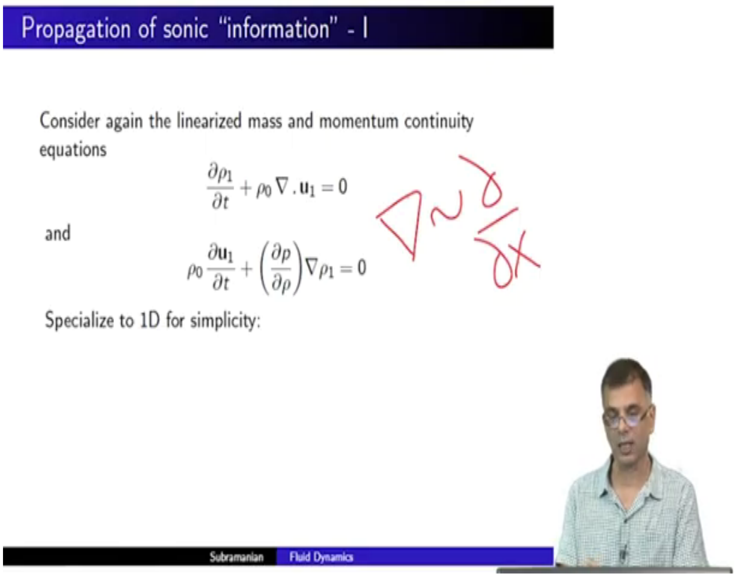

So, yeah, so we are back. And let us reconsider this problem of how information is propagated sound related information I must say. We saw this curious situation for supersonic flow.

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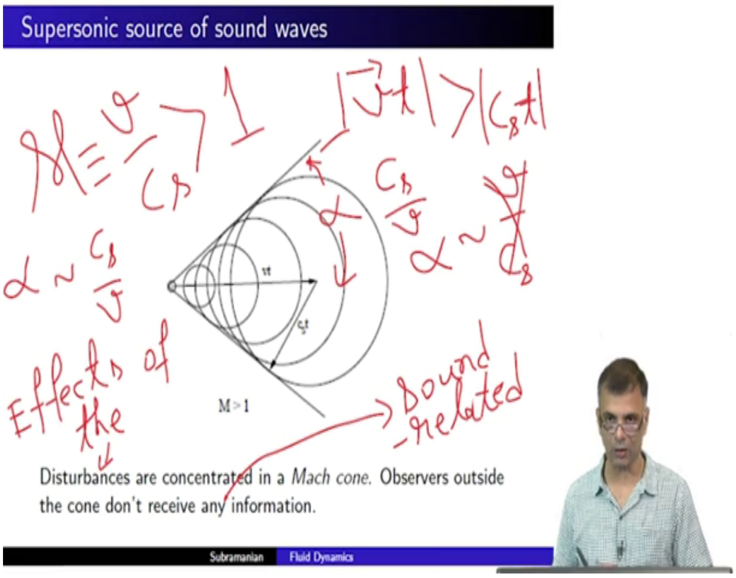

You remember this cartoon, where we found that sonic information in the case where the speed of propagation is larger than the speed of sound propagates in a rather peculiar way. It is only observers within some this thing called a Mach cone the width of which is roughly the half width of the cone is something like C s over V ok. So, it is only observers within this Mach cone who can who receive sonic information and observers outside the Mach cone do not receive this information.

So, let us consider this a little more formally. Now, let us consider again the linearized mass and momentum continuity equation. If you remember linearization essentially means considering only the disturbances only the u 1s and the p 1s, and the rho 1s, and not the u naught if it exists or the rho naught or the p naught, number-1.

Number-2, linearization essentially means that products of small quantities products of anything with subscript 1, suppose you have a u 1 times grad u 1, you neglect that. Anytime you see subscript 1 being repeated, more than once you neglect that term because the products are small quantities results in a quantity that is even smaller, so that is what linearization means.

And the linearized mass and momentum continuity equation are repeated here that is the this is the linearized mass continuity equation, and this linearized momentum continuity equation. And the dp d rho is of course, c sub s squared. So, this is what we have already seen.

And now we specialize to 1 dimension for simplicity. So, in 1 dimension, you know any these nablas are essentially d over dx strictly speaking ok, there is no d y and there is no dz. We are only considering one dimension. So, anytime you see a nabla you just say. So, for instance this nabla rho 1 would simply be d rho 1 dx that is it ok.

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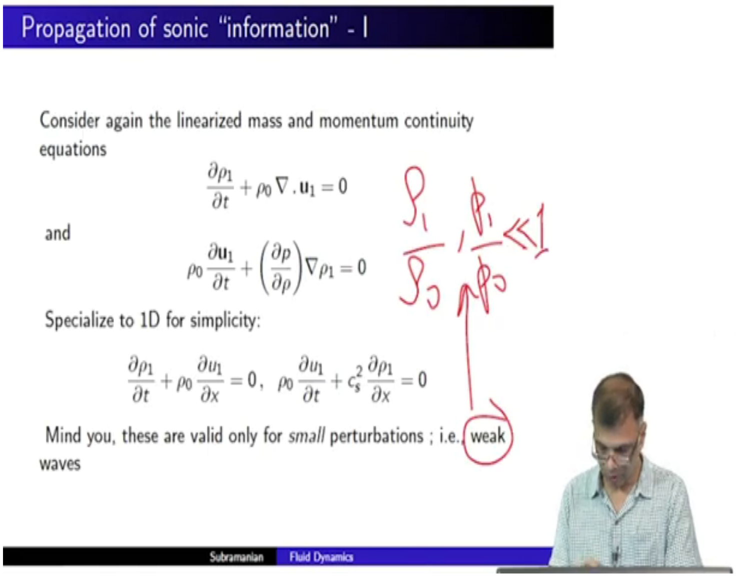

So, these two equations start looking like this. In other words, this is this and this is this that is all right. So, this is c sub s square and we said that nabla rho 1 is simply d rho 1 dx and that is it right. So, this is the linearized mass continuity equation and this is the linearized momentum continuity equation in 1D. Mind you these are valid only for small perturbations very important.

Because in other words if there are waves and there are and then these waves propagate at c sub s at a velocity c sub s. These waves are weak right. Weak in the sense that; in the sense that rho 1 over rho naught and p 1 or p naught are much much less than 1. This is what we mean by weak waves ok right.

So, we say this because we will after we were done discussing this we will have an opportunity to relax this weak assumption. And do something that is very analogous to this ok. So, but for the time being, we are considering only weak waves right.

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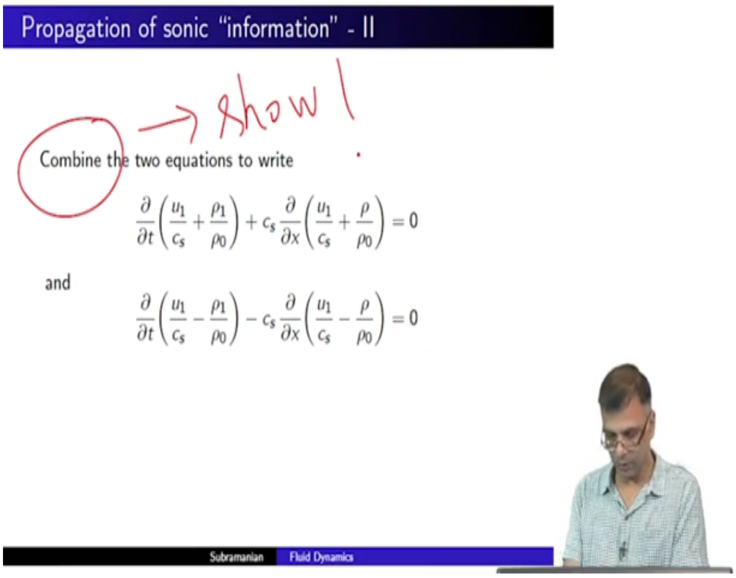

So, we combine these two equations to write this, this is the addition of both those equations you see there is a u over c s. And so I encourage you to see how adding these two equations gives you this and subtracting those two equations gives you this ok.

In other words, I urge you to show this ok. It is not very hard, it is a simple matter and so right. I urge you to show this. Now, what you can see from this is that this quantity and this quantity are the same for this equation. Similarly, this quantity and this quantity are the same for this equation.

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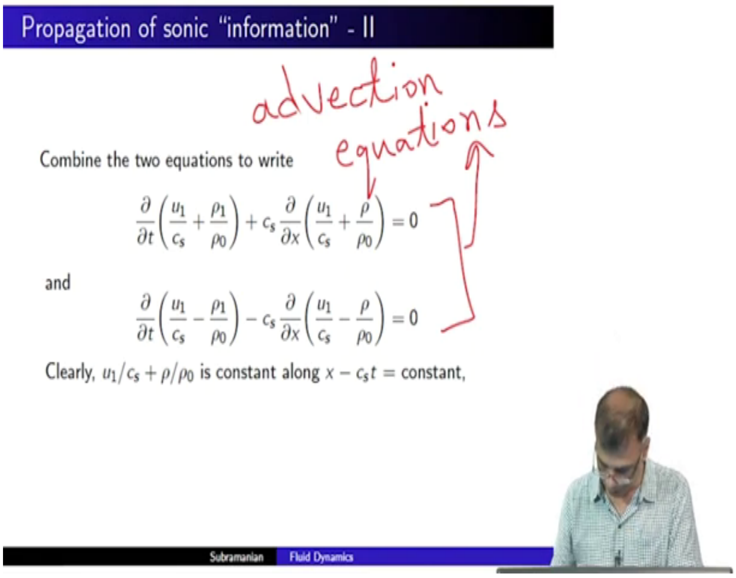

So, the quantity u 1 over c s plus rho 1 over rho naught which is the same here and here is constant along the line x minus c sub s t equals constant. This is an advection equation both of these are advection equations. The top equation represents a situation where this quantity u 1 over c s plus rho over rho naught is advected is carried along unchanged that is what advection means some quantity is simply advected along, it is carried along unchanged along the characteristic x minus c sub s t equals constant.

In other words, as long as I ensure that x minus c sub s t equals constant or as long as I am moving in the x t diagram as long as I am moving along a straight line with speed c sub s in the forward direction, this will always be constant that is what this equation is telling you ok, vice versa.

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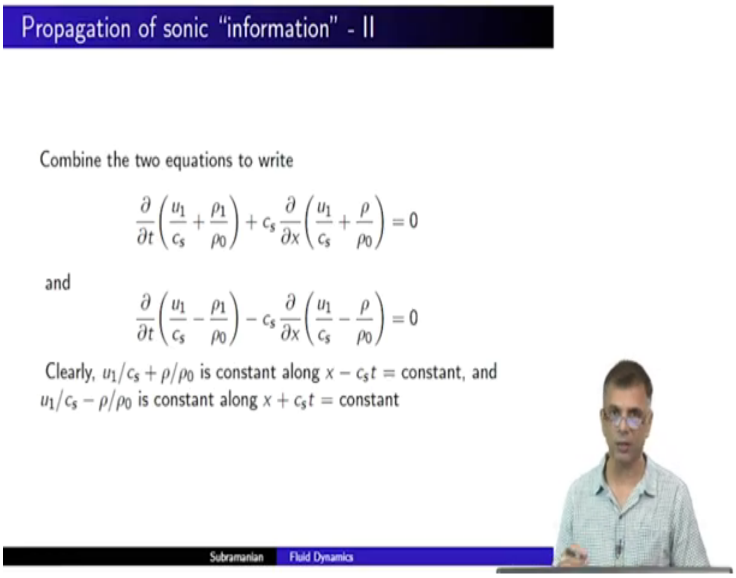

For this equation, this quantity is advected or is constant along this characteristic x plus c sub s; x plus c sub s t equals constant. This would represent propagation in the forward direction; this would represent propagation in the backward direction.

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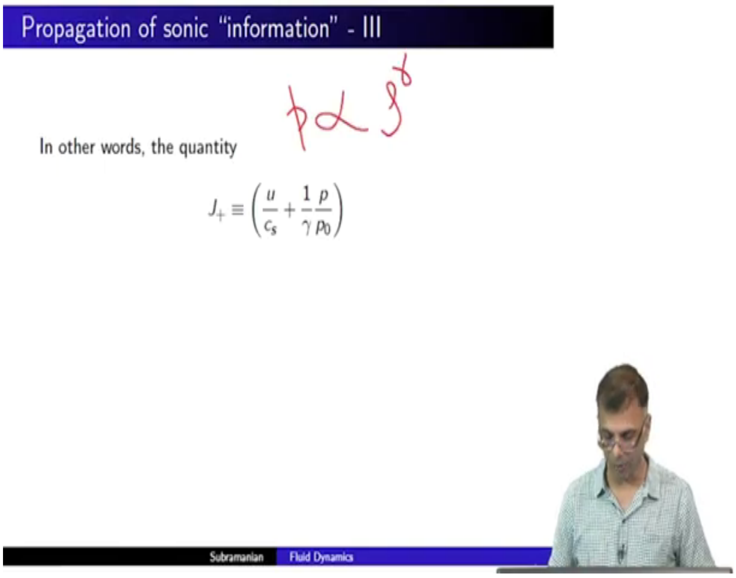

In other words, the quantity j when we now we are simply naming this ok. This quantity ok rho over rho naught we simply replace by p over p naught and divide by you know replace by p over p naught. And that leads to the appearance of this gamma where we have assumed that you know p is proportional to rho raised to gamma like that ok.

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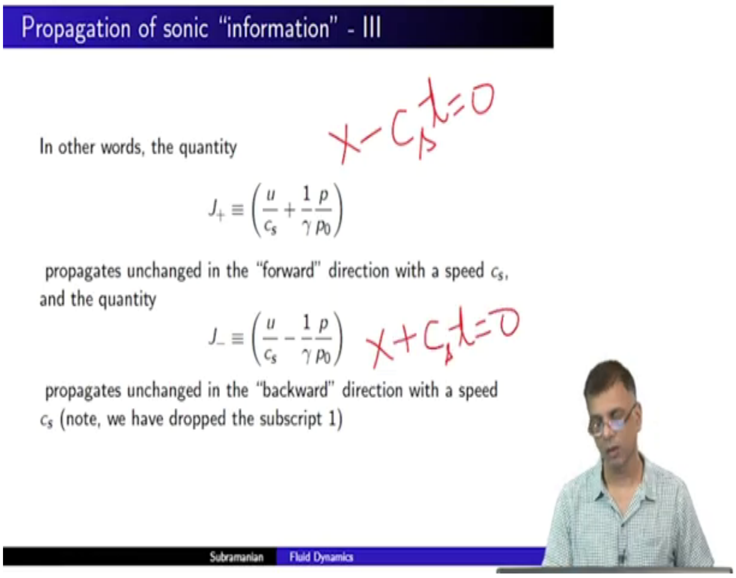

So, this quantity propagates unchanged in the forward direction with a speed c sub s, and this quantity propagates unchanged in the backward direction with the speed c sub s. And note we have dropped the subscripts there used to be a p 1 here or rho 1, there used to be u 1 here, we simply drop that ok just for simplicity.

But you should keep in mind that these are perturbations ok. These are not background ok, the backgrounds still have the subscripts 0 ok right. So, these are conserved quantities, quantities that are this quantity is conserved in the forward direction ok. Along the forward characteristics this quantity is conserved along the characteristic x minus and this quantity is conserved along the characteristic ok right.

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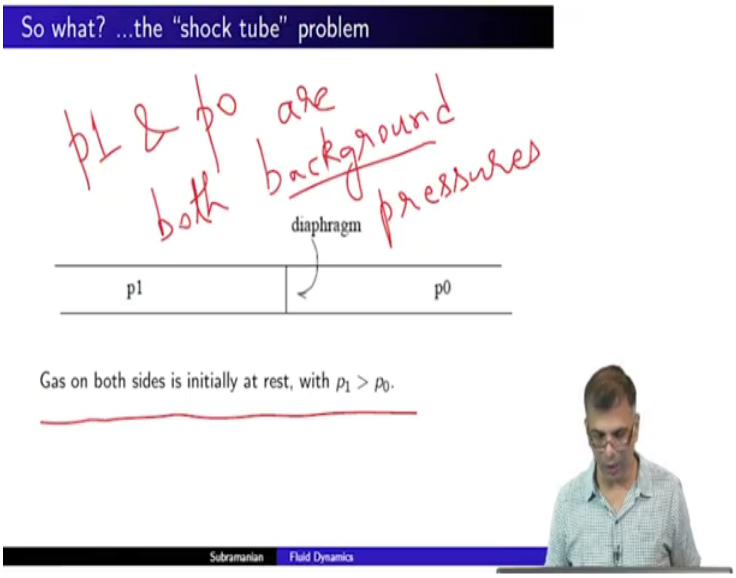

So, now ok, how do we apply this? So, let us consider something called the shock tube problem. If you for the time being we are not really discussing shocks not yet, we will come to shocks, but we are not yet discussing shocks. But this is a classical setup for a shock for studying shocks both from you know analytical theoretical point of view as well as in the lab.

And, this setup comprises you know a tube with an airtight piston rather this would be a piston normally and you shove the piston forward and you will find that when the speed of the piston exceeds the local speed of sound there will be a shock formed in front of the piston so on so forth that is why it is called a shock tube. Just Google shock tube and you will find lots of sources.

For the time being, we are not considering shock formation we are and we are not considering a piston either. We are considering a much simpler situation where you have you know infinitely long tube like so. And you simply have a diaphragm an airtight diaphragm with two pressures on two sides of the diaphragm ok. You have p 1 on one side of the diaphragm, the pressure is p 1 on one side of the diaphragm and the pressure is p naught on the other side of the diaphragm ok.

Yeah so do not confuse this with perturbations and background in this case, these are simply background. Both of these are background pressures, p 1 and p naught are both background pressures.

The difference simply is that there is one background pressure on the left of the diaphragm and there is another background pressure on the right of the diaphragm ok. And gas on both sides is initially at rest with p 1 greater than p naught ok. The pressure on the left is larger than the pressure on the right. So, this is our setup ok.

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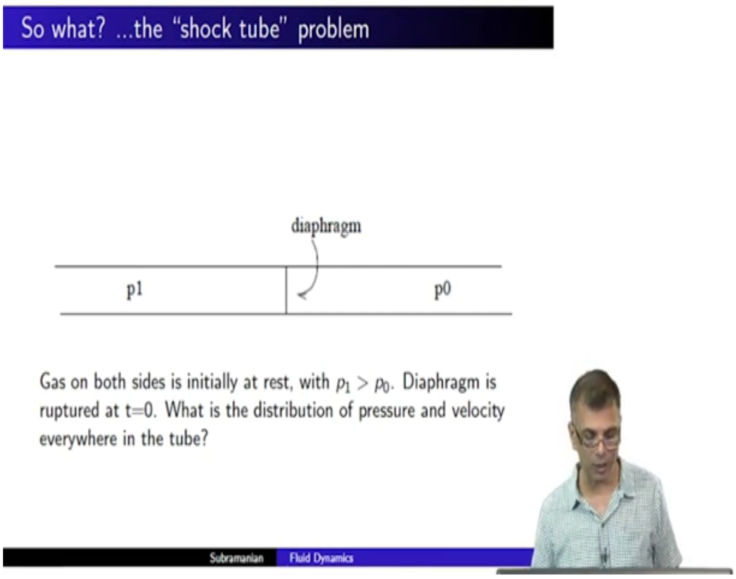

Now, what do we do? We rupture the diaphragm, we create a little hole here right at t equals 0, time t equals 0. And we see and what will happen we know you know our intuition tells us that they be mixing right. The gas will flow from the left side to the right side, because the pressure on the left is larger than the pressure on the right, right. So, gas will flow from left to the right.

And let us see what happens right. So, and we will find that this concept of these conserved quantities is a very useful one. In trying to figure out exactly what happens when the diaphragm is structured as time progresses how exactly you know the gas mixes ah. So, this is very useful. So, what is the distribution of pressure and velocity everywhere in the tube this is the question we want to ask.

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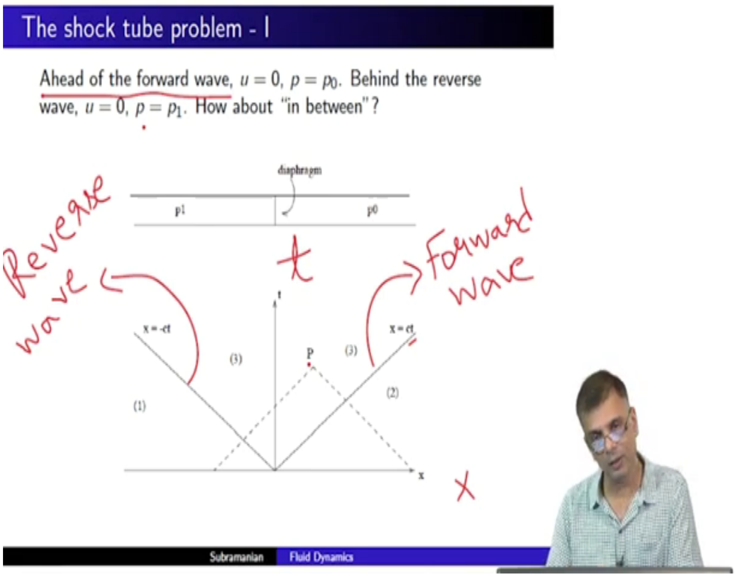

So, you see information propagates at the sound speed right. And there is no restriction on which direction you know the information propagates in. It can propagate to the right and also to the left. In other words, there is a forward wave as well as a reverse wave right. So, the thing is ahead of the forward wave, the information the diaphragm is ruptured has not yet reached ok.

So, the gas was initially at rest and it will remain at rest because the information is not yet reached. The gas does not know that the diaphragm has been ruptured. So, the background velocity is still equal to 0. And, the pressure is still equal to the background pressure whatever it was. And the same is true behind the reverse wave there is a wave propagating to the left which is the reverse wave.

And behind the reverse wave, in other words, for the points toward the left where sonic information has not yet reached is the same thing. The gas was initially at rest the undisturbed gas was at rest and it remains at rest. The undisturbed gas was at a pressure p equals p 1 and it remains like that, so that is fine.

So, ahead of the forward wave and behind the reverse wave, you know the conditions are undisturbed, whatever it was prior to the rupturing of the diaphragm. What is interesting is in between how does the gas mix? So, this would represent the reverse wave in x t, you see this is x, and this is t. So, this would represent the reverse wave because x plus c t is equal to 0 and this would represent the forward wave.

So, and this would represent the reverse wave ok. So, ahead of the forward wave and behind the reverse wave, so in other words, in this region or in this region, the conditions are undisturbed. Ahead of the forward wave the conditions are this and behind the reverse wave the conditions are this. The question is what is in between in here?

For any arbitrary point p say ok, for any arbitrary point p, what are the conditions? The thing to remember now is what you do is you link this point p with a line with a forward characteristic that is parallel to this.

This is parallel to this and this is parallel to this ok. And the intersection of these two lines will give you the physical conditions at any arbitrary point p here, here, here, here. Any arbitrary point p that is within this v shape thing here and let us see how that is done right.

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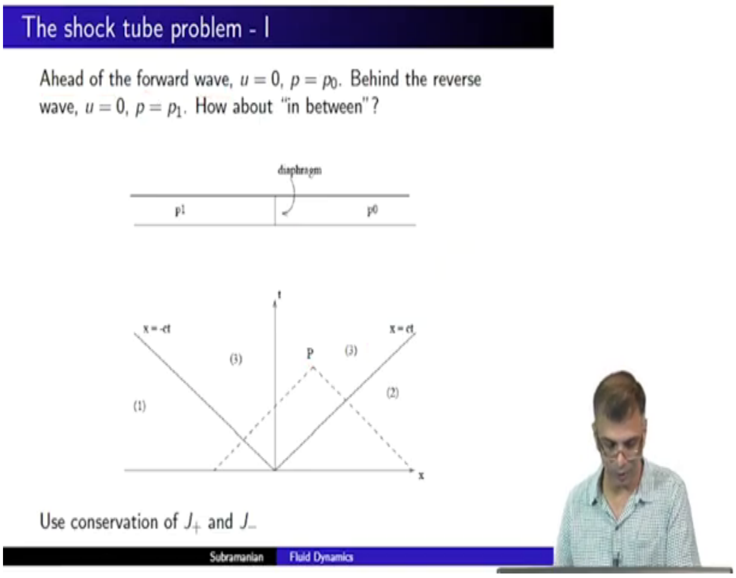

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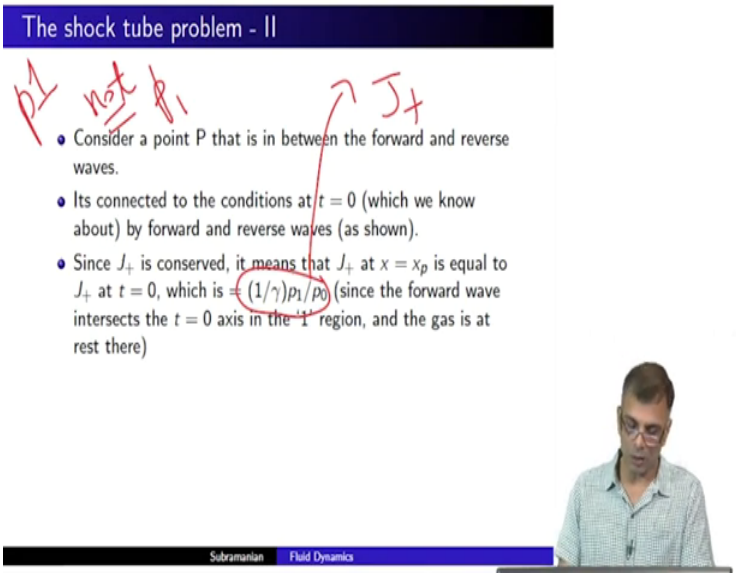

Using conservation of j plus and j minus what we do consider a point P that is in between the forward and reverse waves like we said, this point P. It is connected to the conditions at t equals 0 which we know about by forward and reverse waves.

In other words, what is the use of this dotted line and this dotted line? The point is you see on this axis t is increasing. So, right here t is equal to 0 right. So, this connects this dotted line connects the conditions here to the forward wave the propagation of the forward wave at t and this is t equals 0 here as well as here.

And we know what the conditions were at t equals 0. Before the rupturing of the diaphragm, we know what the conditions were. We know that u is equal to 0 on the left and the right and the pressure is equal to p 1 on the left and pressure is equal to p naught on the right. We know this right.

So, point p is connected to the conditions at t equals 0 by forward and reverse waves as shown and since J plus is conserved which means it means that J plus at x equals x p is equal to J plus at t equals t naught ok. And j plus is equal to 1 over gamma p 1 over p naught, this is J plus.

Why? You go back to the definition of J plus you see J plus, and u is equal to 0, therefore, all that remains is this 1 over gamma p over p naught that is why J plus at t equals 0, u was equal to 0, that is what and that is therefore all that remains is 1 over gamma p 1 over p naught ok. This is a little bit of a you know confusion science.

I said that I was dropping the subscripts 1, but here I have retained the subscripts 1. But what that means, with this p 1, it is really not p sub 1, this is really p 1 not p sub 1 ok, this one. This really should be p 1, p yeah. So, and the gas is at rest there as we said.

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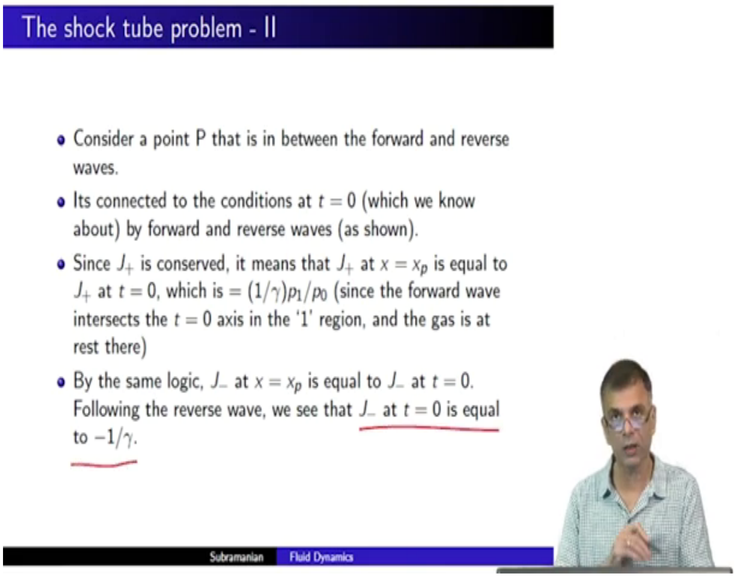

By the same logic J minus at x equals x p at the point P is equal to J minus at t equals 0, and J minus at t equals 0 is simply equal to minus 1 over gamma. Why is that? You see here J minus is equal to you know this is obviously 0, because u is equal to 0. And so is it is whatever condition whatever pressure there is divided by p naught, but you see p is simply equal to p naught along J minus you see.

Out here when you go along this characteristic we know that on the right is simply equal to p naught. So, p naught over p naught is simply 1, that is why we get J minus is simply equal to minus 1 over gamma that is why we say J minus at t naught t equals 0 is simply equal to 1 over gamma ok right.

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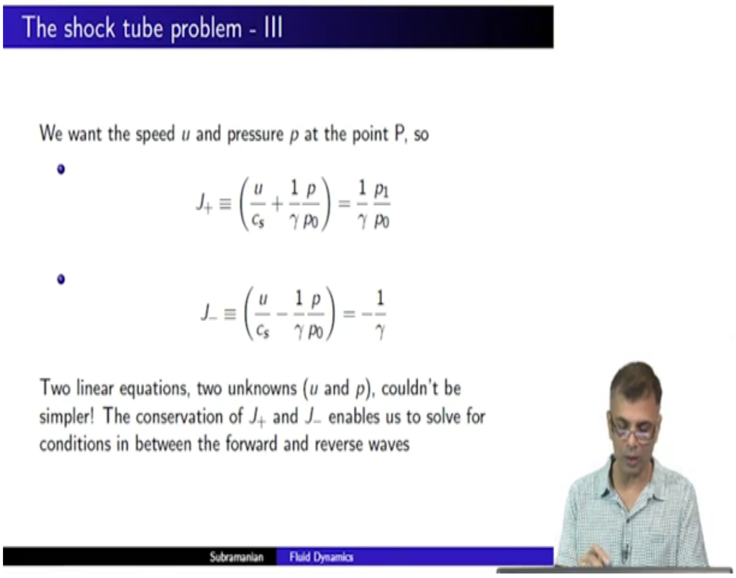

So, what is the objective of this entire exercise? We want the speed and the pressure. And therefore, once we know the pressure via the sound speed, we can relate it to the density also right. So, we want the speed and the pressure at point P at the point x sub p right.

So, therefore, J plus is equal to 1 over gamma p 1 over p naught and again I apologize for this. This is really; this is really p 1 not p sub 1 ok sorry. And J minus is this 0 and p over p naught is just it is essentially p naught over p naught, so it is simply minus 1 over gamma right.

So, there are two linear equations. So, I want p right. So, at x equals p, I do not know u, I do not know p; I do not know u, I do not know p. So, there are two unknowns, there are there is u and the p. And I have two equations right. So, there are two unknowns u and p, and there are two, so and I just solve them. So, this enables us to solve for the conditions in between the forward and reverse waves.

At any given point which is characterized by a velocity u and a pressure p, I can figure out what u is, and what the velocity is, and what the pressure is, by solving these two linear equations. The right hand sides are completely known; p 1 is given to us; p naught is given to us; gamma is assumed to be say five-thirds from a monoatomic gas right.

So, essentially what this is saying is that the conservation of J plus and J minus enables us to solve for the conditions in between the forward and reverse waves. In other words, anywhere here anywhere in this v, so that is what this enables us to do ok.

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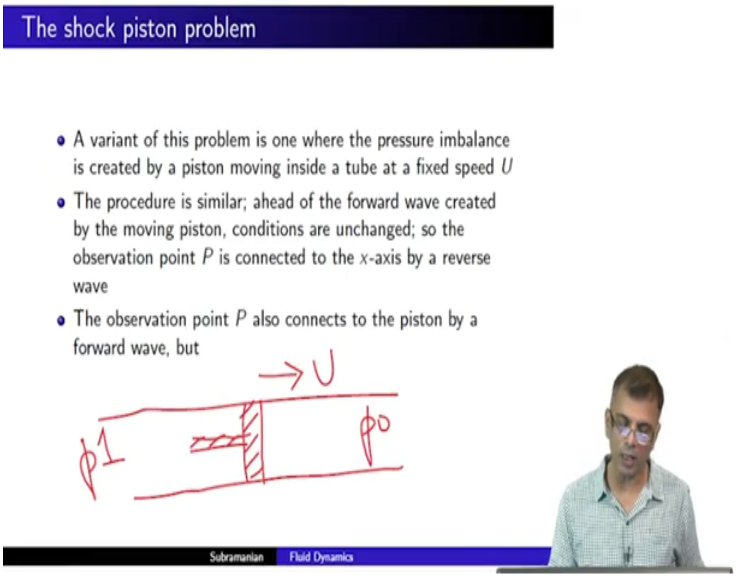

Now, a variant of this problem is one where the pressure imbalance is created by a piston. So, instead of a diaphragm, what you have is a piston. And the piston is moving inside the tube at a fixed speed U ok. So, what you have is a piston. So, you have the same shock tube right. And as before you have p 1 and p naught, the only difference being the piston is moving at a velocity U, this piston is moving at a velocity U. And what happens? Same question, we are asking right.

The observation point, so again you have forward and reverse waves, so the observation point P also connects to the piston with a forward wave, the piston can only I mean since its moving forward it can connect to any observation point here only via a forward wave.

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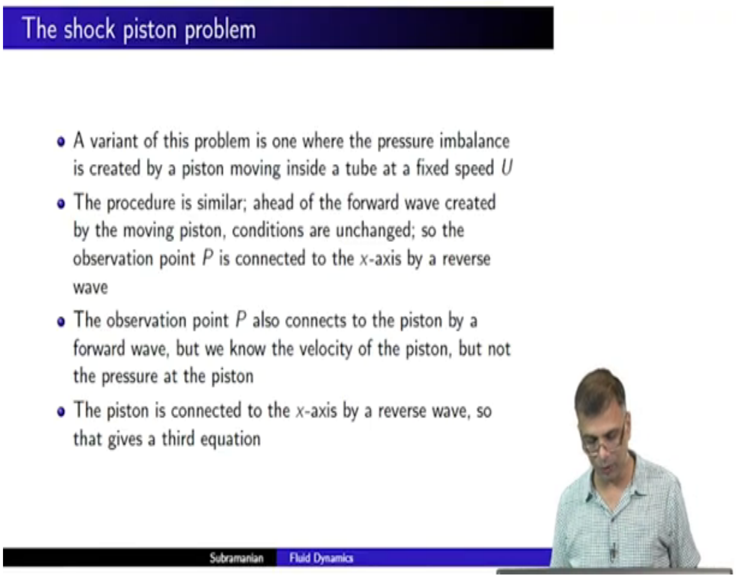

But we know the velocity of the piston. So, the velocity of the piston is known, but not the pressure. The velocity of the gas right at the piston is known to be used simply because the gas is sticking to the piston, is not it? So, the gas is sticking to the piston. So, the velocity of the gas at the piston is known, but the pressure is not known ok, so that is one thing.

So, as we said the gas is being pushed by the piston right. So, right at the surface of the piston, since I know that the piston is moving at a velocity you know U, the velocity of a gas is also U right at the piston, but I do not know the pressure of the gas at the piston, this I do not know right.

So, I do not know the pressure at the piston. I know the velocity. And the velocity of the piston is given by U, but the pressure is not known that is ok. But the piston is connected to the x-axis by a reverse wave ok the observation point is connected to the piston by a forward wave, but the piston is connected to the x-axis by a reverse wave.

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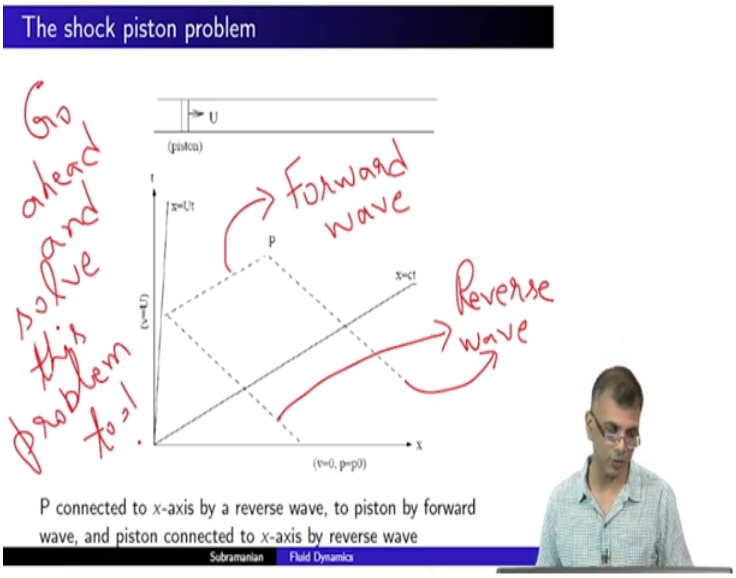

So, what happens is you have a given observation point like so, and the observation point is connected to the piston via right. And the piston is connected to the x-axis via a reverse wave. And the point P of course, as before the point P is also connected to the x-axis via a reverse wave right. So, P is connected to the x-axis by a reverse wave to the piston, P is connected to the x-axis by a reverse wave to the piston via a forward wave. And the piston itself is connected to the x-axis via a reverse wave right.

So, what gives now? The thing is additional unknown if you see in comparison to the previous problem is that we do not know the pressure right at the piston. We know the velocity, but we do not know the pressure right at the piston. So, we have one additional unknown, but we have one additional.

So, we have three equations 1, 2, and 3. So, we have one additional unknown in comparison to the previous problem, but we have one additional equation, so no problem. So, it can be solved just like the previous problem. And I encourage you to go ahead and solve this problem too right. So, so this illustrates the utility of considering characteristics ok.

The important difference being that we have in this treatment we are we have restricted ourselves to smart perturbations, and therefore, weak waves in what we will treat going ahead we will relax this assumption. But before that and when we relax this assumption that the perturbations will be large and so that is the essential difference and even there we will find invariants ok.

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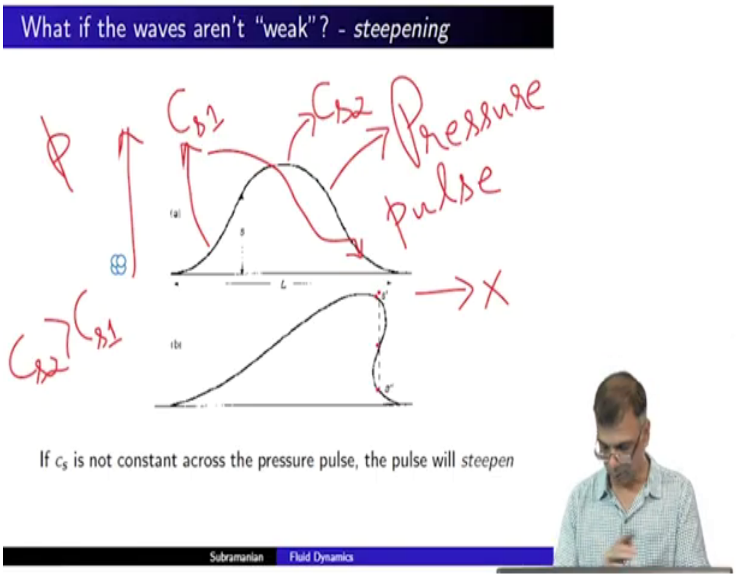

But let us see what happens qualitatively let us first try to understand what happens when the disturbances are not are not small. In other words, what if what happens when the waves are not weak ok. We will, encounter this phenomenon called steepening ok. And let us try to understand this qualitatively first.

So, let us say that this represents the profile of a pressure pulse ok. So, this entire thing represents a pressure pulse right. So, the pressure is large here, and it is small here yeah. So, this is a pressure pulse that is propagating in space. So, this is this, this x-axis this is essentially x-axis and the y-axis is just p pressure ok.

So, the pressure is larger here as compared to here that is the main point. And so because the pressure is larger here as compared to here, the speed of sound say C sub s let us call this C sub s 2 and let us call the speed of sound here as C sub s 1.

Clearly because if we say if we consider that the density at both these points is the same, clearly C sub s 2 is greater than C sub s 1. The speed of sound here is larger than c speed of sound here that is obvious, no problem there.

In other words, the top of the pulse is moving faster than the bottom of the pulse. And it is if it is C sub s 1 here, it is just it is something like that here too a symmetric point you know. So, I should also draw here ok. So, the sound speed at the top of the pulse is larger than the sound speed at the trailing edge of the pulse as well as a leading edge of the pulse ok.

In other words, one can envisage a situation where the top of the pulse the information, the sonic information from the top of the pulse can overtake that from the leading edge of the pulse you see. So, the top of the pulse kind of tries to overtake the top of the pulse which is trailing this edge can often one can envisage a situation where it overtakes the leading edge, and the pressure pulse starts looking like this ok.

Now, this is a problem. Why? If you draw a dotted line like so, you see it intersects the pressure pulse at two points, yeah 1 and 2, and in fact, 3 ok. Now, this is not allowed. What is the pressure here? Is it this, or this, or that? You have to give me a definite answer. You cannot say maybe this, may be that, maybe. No, you cannot say that. You, it has to be one answer.

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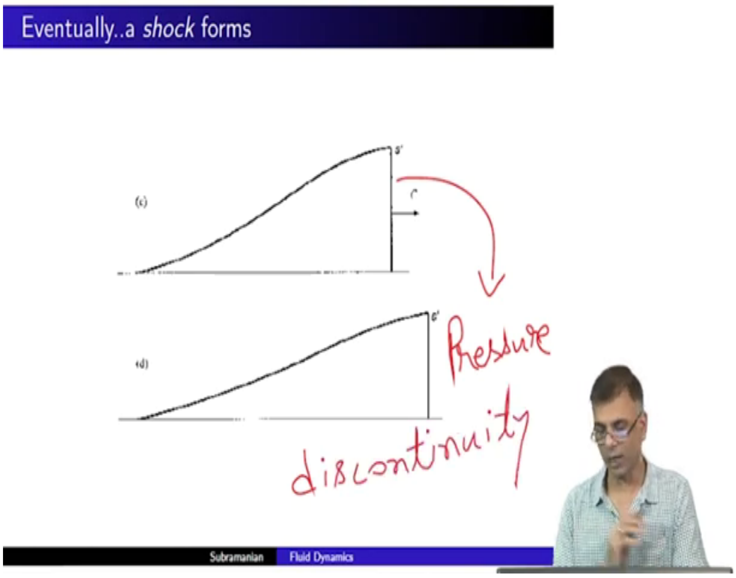

In other words, yeah, so eventually what happens is this kind of a situation is not allowed ok. And eventually what happens is this kind of a situation leads to a steepening ok. And this marks a pressure discontinuity that is what happens.

So, these kinds of you know things are not allowed ok. And instead what happens is this steepens and forms a discontinuity like so. And this is what a shock is essentially. It represents a discontinuity in physical quantities such as pressure, density, for that matter velocity also.

And this can only happen when the amplitude of the pressure pulse is so large that the sound speed is appreciably larger at the top of the pulse as compared to the leading at the edge of the pulse or for that matter trailing edge of the pulse. If the pressure pulse was small the difference between C s 2 and C s 1 would not be that much these kinds of effects start you know becoming noticeable only when the pressure pulse is. So, large your C s 2 is appreciably larger than C s 1 ok.

And if when that is the case the waves are no longer weak, and we start seeing these kinds of phenomenon ah which essentially which lead to discontinuities its essentially a mathematical problem ok. And it is not simply the point is we would not be talking about this if it was simply mathematics.

The point is pressure waves shock waves or indeed observed in everyday life both in the lab and you guessed it. And they are very, very important in astrophysics which is why we are starting to talk about shock waves at all.

And this happens when we will see that it happens a and when the speed of an object exceeds the speed of sound b, when the pressure disturbances when the disturbances of pressure or density or whatever are so large that the linearization assumption that we made all along is no longer valid. So, we will stop here. And we will take up a detailed study of shockwaves when we meet next.

Thank you.