Fluid Dynamics for Astrophysics

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Lecture - 25
Subsonic and supersonic flows

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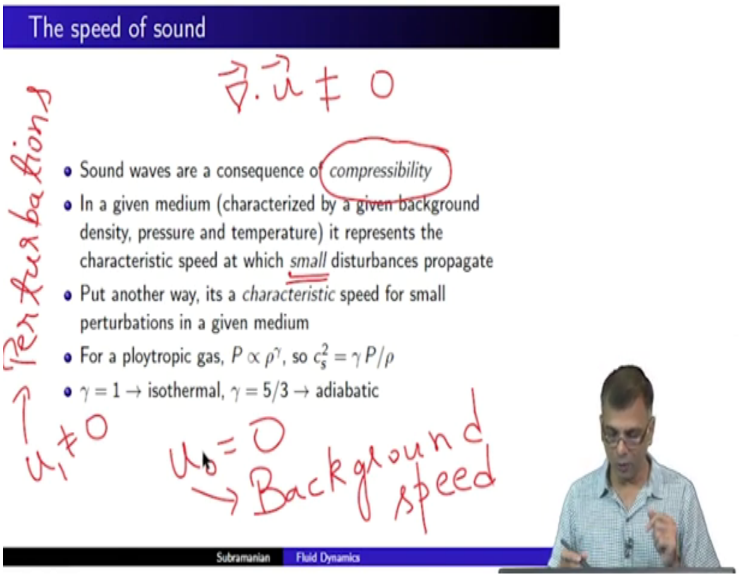

Hi. So, let us carry on with our discussion about the speed of sound and in this session, we will do a very fast recap of what we have done so far. And then, go on to do a brief recap of the Mach number which we have already seen.

But since the Mach number has to do with the speed of sound, we will start paying a little more attention to it. And we will pay some attention to the characteristics of subsonic and supersonic flows in other words, flows that are slower than the speed of sound and flows that are faster than the speed of sound. We will also discuss the concept of characteristics and so on so forth.

So, let us get right ahead and so, as we have discussed, sound waves are a consequence of compressibility right. So, in other words, the all important divergence of u being equal to 0 is no longer valid right.

However, one important thing to be kept in mind is that in a given medium characterized by given background density, pressure and temperature, it represents the characteristic speed at which small disturbances propagate and this is a very important adjectives small disturbances right. So, it is compressibility alright, but compressibility with regard to small disturbances ok.

Put another way, it is a characteristic speed at which small disturbances propagate in a given medium. Now, you have to if you recall the derivation of sound speed that we did you know during the previous session, we had assumed u naught to be equal to 0 and u naught was the background speed if you remember right.

However, the speed of perturbations which was what was this is the perturbation speed right, perturbations this was not equal to 0 obviously, not right.

So, you know I am talking, or I clap or something that causes density and pressure perturbations and u 1 is the speed of these small perturbations and that is not equal to 0 and it is the small perturbations which always propagate at the speed of sound, this is to be kept in mind. It has nothing to do with the background speed ok.

Although the derivation assume u naught equal to 0, this need not be the case, we can put u naught not equal to 0, we can generalize and get you know an equivalent derivation for u naught equal to 0.

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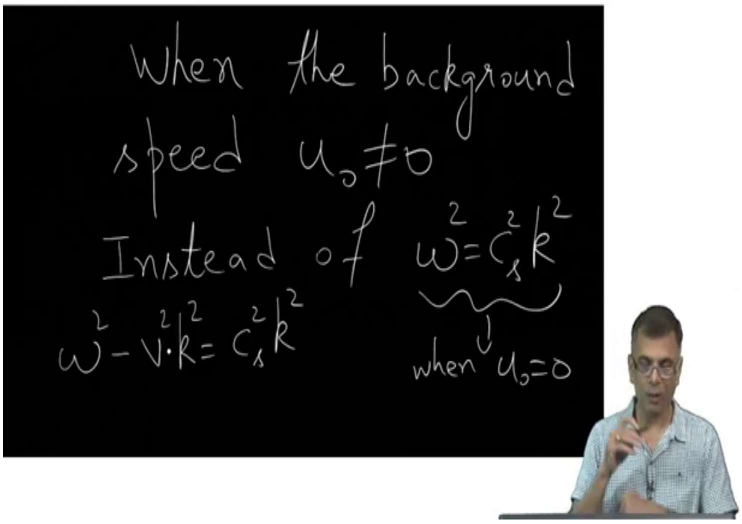

And so, what will happen when u naught is not equal to 0? What will happen is that when the background speed u naught is taken to be not equal to 0, you can go through the entire derivation and what will happen is that instead of the dispersion relation omega square equals c sub s squared k squared which happened when u naught is equal to 0.

We will have omega squared minus some sort of v squared k squared equals c s squared that is what will happen. In other words, there will be some kind of a doppler shift, this is very much like a doppler shift ok. So, when u naught is not equal to 0, but it is some finite value v.

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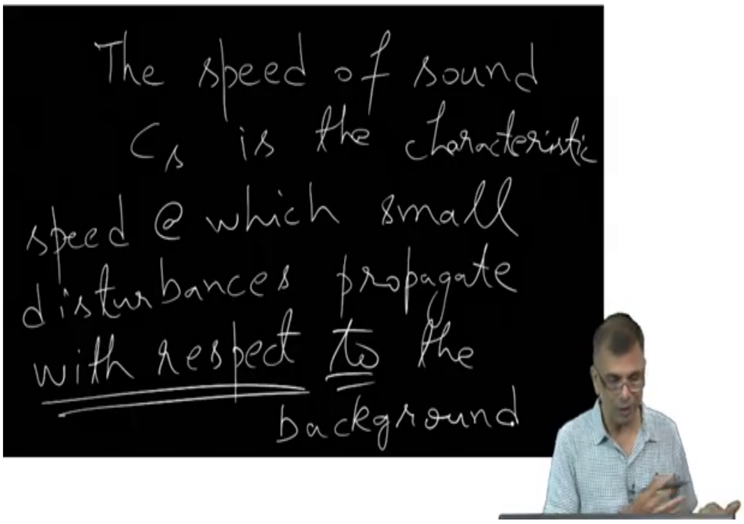

But the thing to keep in mind is that the speed of sound s is the characteristic speed all that is fine speed at which small disturbances propagate and this is very important with respect to the background and this is a very important statement ok.

So, you see the small disturbances, they are propagating at the speed of sound and that speed is always sees a mess with respect to the background, but what if the background itself is moving? In other words, what if the background speed u naught is not 0?

Well, one simple thing to do is just climb into the frame of the background flow and then, you will find that the characteristic speed at which these small disturbances propagate is simply c sub s; because you are in the frame of the background guess and therefore, now u naught is equal to 0 as far as your concern.

But what about the fellow who is standing in the lab frame? What about the Eulerian frame? Then what will happen is the person in the Eulerian frame will observe that the pressure, small pressure and density disturbances which characterize the sound waves are not propagating at the sound of speed, they are propagating at the speed of sound sorry I keep saying sound of speed, speed of sound, they are propagating at the speed of sound plus or minus the background flow.

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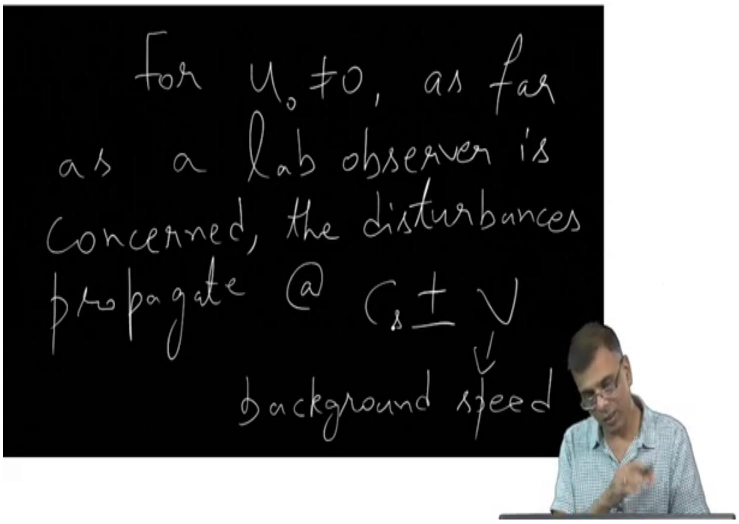

In other words, for u naught not equal to 0, as far as a lab observer is concerned, the small disturbances propagate at c s plus or minus v where v is the background speed and this is very important ok. It is only soc sub s I mean the speed at which these small disturbances propagate is indeed c sub s with respect to the background. If the speed of the background is equal to 0, well then, you know no problem I mean its c sub s as far as the observer in the frame of the fluid and the lab observer concern is the same thing.

But however, if you know the bulk flow is also moving at some v, then as far as the lab observer is concerned, the small disturbances are not simply propagating at the speed of sound, they can be propagating the speed of sound plus the speed of the background or minus depending upon which direction you know the bulk flow is moving and this leads us to the other important point.

As you can see, you know in which case the speed at which small disturbances propagate can indeed exceed the speed of sound, this is possible and so, this is something to be kept in mind very firmly.

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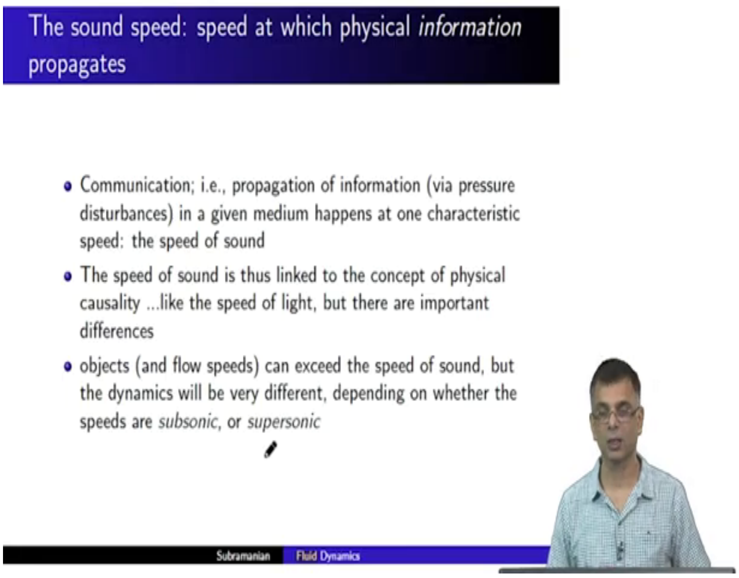

And so, as we emphasized the last time, the speed the sound speed is the speed at which physical information propagates. In other words, you can learn about me being here via the sound that I am emitting at the speed of sound right. You cannot tell that I am present here and I am talking before sound waves that I emit reach you right. So, it happens only at one characteristic speed of course.

And because you cannot tell that I am here and I am speaking, before the sound waves reach you, the speed of sound is thus linked to the concept of physical causality and in that respect, it is like the speed of sound light, the speed of sound is somewhat like the speed of light, but there are important differences.

Objects and flow speeds can exceed the speed of sound, there is nothing sacrosanct about the speed of sound, you can have flow speeds that are larger than the speed of sound ok. So, that is not true, that is not so for the speed of light of course, nothing can ever exceed the speed of light.

However, the dynamics will be very different depending upon the whether the speeds at these flow speeds are lower than the speed of sound in which case they are called subsonic or whether they are higher than the speed of sound in which case such flows are called supersonic right.

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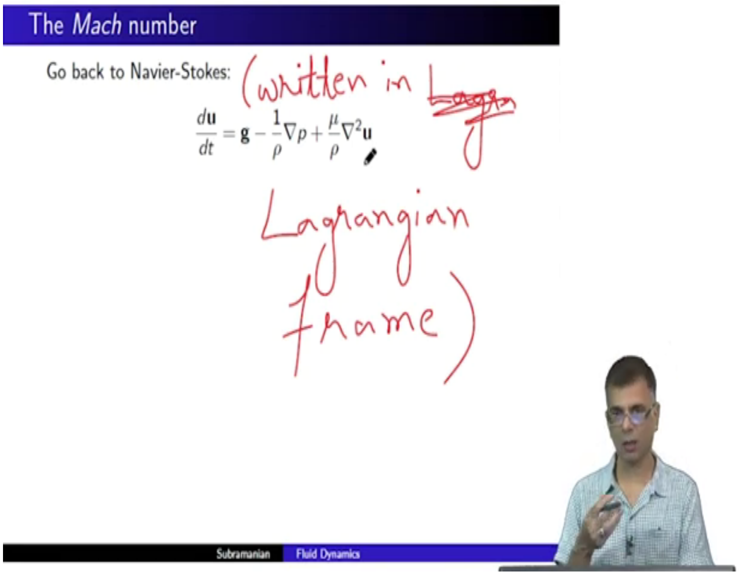

So, now let us go back to the Navier-Stokes equation and we have already done this, we have already encountered the Mach number, no harm and a little bit of repetition because now we are talking about the speed of sound and this is a very important topic. So, let us go back to the Navier-Stokes equation for a minute and so, this is what it was, this is of course, the Navier-Stokes equation written right. So, never mind this yeah.

So, this is the Navier-Stokes equation written in a Lagrangian frame and you can immediately tell by the fact that these d's are the straight ds and not the partial d's right and so, this is your ma and if you will and these are the forces, there is a force due to gravity, that is a force due to pressure gradients, then this is the force due to viscous terms right.

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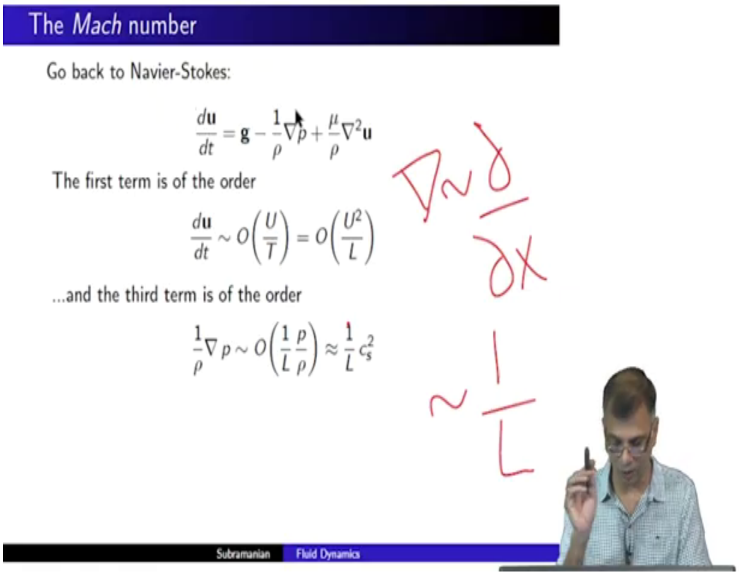

So, now the first term as we have already seen this, the first term is of order U over T where U is some kind of background speed and T is some kind of time scale, microscopic time scale and that that can also be written as U squared over L where L is some kind of representative macroscopic length scale right.

And we are now concerned with the comparison Mach number is concerned with the comparison between the first term and the third term right.

So, this is the inertia term, and this is the pressure gradient term and so, the third term, this term is of order 1 over rho gradient p that the gradient is as good as 1 over L right because why is that and the gradient is something like d over dx right. So, it is something like 1 over L right, the x we replace by a length scale right.

So, we keep the pressure and everything as it is. So, the gradient gets replaced by 1 over L and so, you have p over rho and although, strictly speaking c sub s squared is really d p over d rho, we just roughly speaking, we simply write p over rho is c s squared right. So, the third term is of order 1 over L c s squared yeah.

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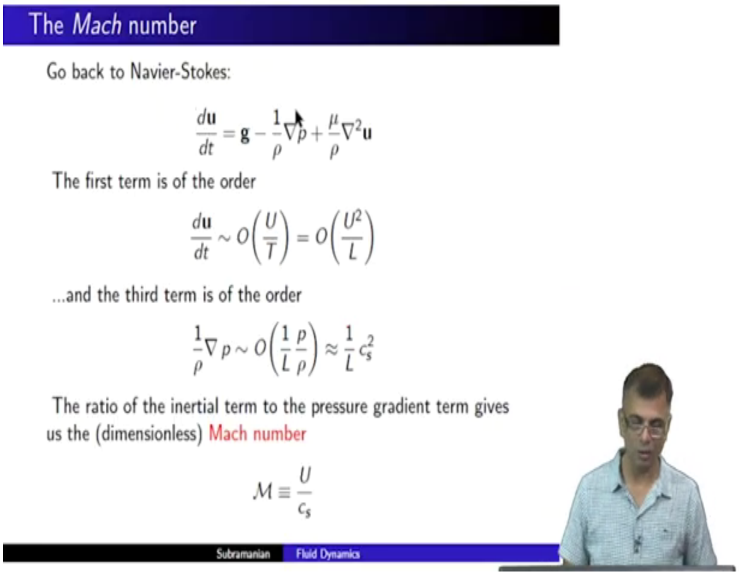

So, you want to compare this term with this term, you want to compare the inertial term with the pressure gradient term and the ratio of the inertial term to the pressure gradient term as we have seen before gives us the dimensionless Mach number which is well.

Actually the ratio of the inertial term to the pressure gradient term is the square of the Mach number; where the Mach number is defined as the ratio of the flow speed to the sound speed and this can be M can be less than 1 or greater than 1. If it is less than 1 such flows are called subsonic. If it is greater than 1 such flows are called supersonic and the characteristics; characterize these flows are very very different.

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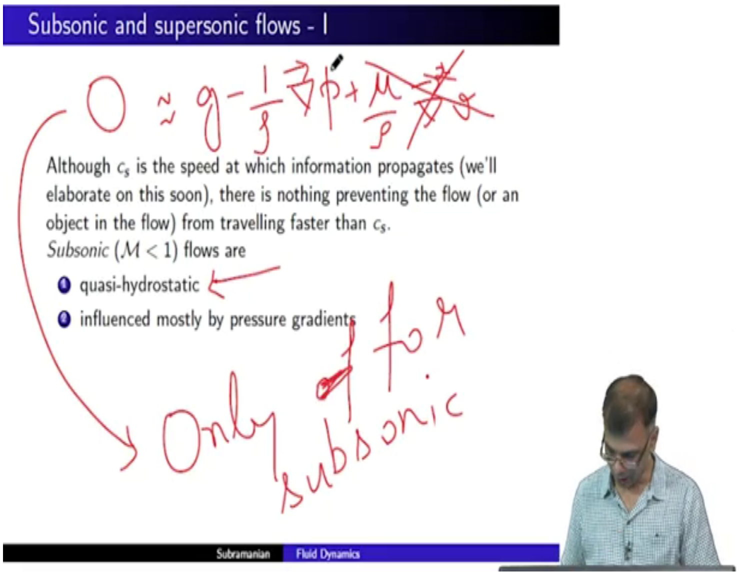

So, although the speed of sound is the speed at which information propagates, we will elaborate on this. There is nothing preventing the flow or an object in the flow from traveling faster than the speed of sound. So, the thing about subsonic flows, subsonic flows being once for which the Mach number is less than 1 right.

Subsonic flows are quasi-hydrostatic, we will see why in a minute yeah. We have already seen it actually, subsonic flows are quasi-hydrostatic, they are influenced mostly by pressure gradients and the pressure gradients are influenced by boundary conditions right. So, we will go back to this why is this so?

So, now, you see let us go back to this again, you see the Mach number is essentially the ratio of the inertial term to the pressure gradient term right. So, if the Mach number is less than 1, it means that the pressure gradient term is much more important than the inertial term right. So, as a result, what will happen is the inertial term is essentially equal to 0 right.

So, that is why so the equation essentially becomes 0 is approximately equal to g minus 1 over rho of p plus mu over rho v this is what the Navier-Stokes equation starts looking like right. So, this is the vector Laplacian. Now, this is not that important for the time being, we can just drop this.

Now, if you just take these two terms and set it equal to 0, what does this look like? This is a hydrostatic situation right, there is no appearance of velocity now and this is valid for subsonic flows, this is true only for subsonic flows. You can put a 0 here because the inertial term is not so important, that is a whole definition of subsonic flows.

So, you look at this in this, it is essentially a hydrostatic equation right. So, there is no velocity and so, that is why, we say that subsonic flows are quasi-hydrostatic and they are influenced mostly by pressure gradients that is our point. Apart from the from the gravity term, they are mostly influenced by pressure gradients. Pressure gradients play a large part in subsonic flows ok.

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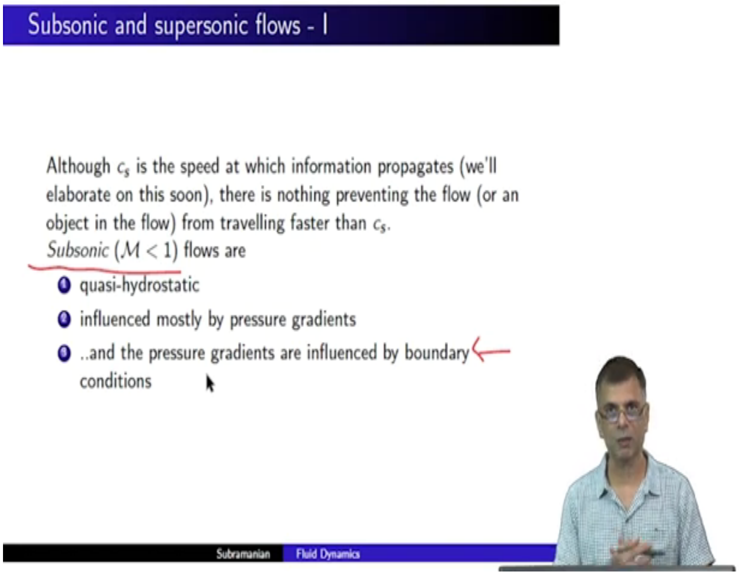

And the pressure gradients are influenced by boundary conditions. What do you mean by pressure gradients? Well, the pressure is larger here and smaller here. In other words, you have to specify how much is the pressure here, how much is the pressure here, you have to specify the boundary conditions.

So, that is what this statement means and all of these are true for subsonic flows where the inertial term is negligible or essentially can be replaced by 0. So, all of these three statements are very very true for the lower the Mach number the true are the statements are.

If the Mach number is much much less than 1 say 10 raised to minus 3 or something, these three statements are very true if the Mach number is something like half or so, these statements are not as true, they are still true, but not as they are not as accurate as what they would have been if the Mach number was 10 raised to minus 3 for instance ok.

Why is that? Because you know the lower the Mach number, the better the approximation of setting the inertial term equal to 0 that is all, nothing else right. So, this is what you know subsonic flows are all about, subsonic flows. All these three statements are true for subsonic flows and the more subsonic the flow, the truer these three statements are right.

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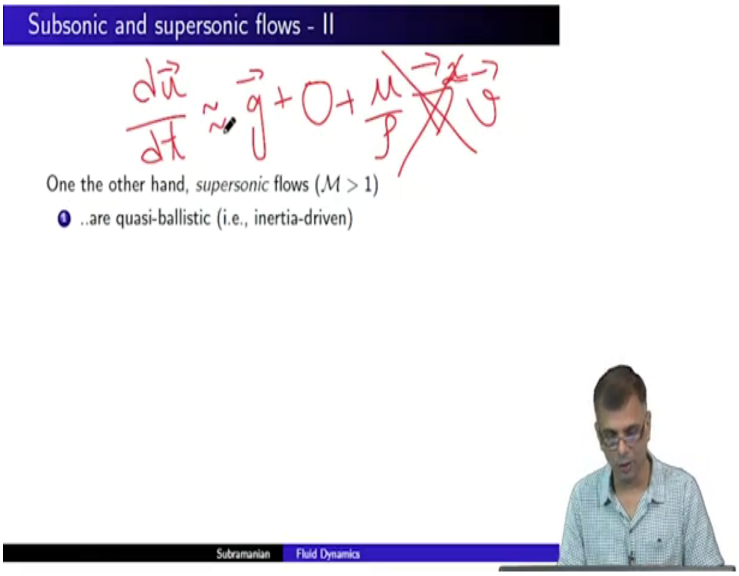

On the other hand, supersonic flows for which Mach number is greater than 1 or essentially quasi-ballistic. In other words, they are inertia-driven, again a no brainer right.

So, what happens is the third term is not that important in this case, you simply write du dt g, the second term is almost equal to 0, the pressure gradient term and then you of course, have the viscous term which for the time being we will ignore right. So, here, you used to have the pressure gradient term in the full Navier-Stokes equation.

But now, for supersonic flows by definition, the inertial term is larger than the pressure gradient term. So, in comparison to the inertia term, I am able to set this third term equal to roughly equal to 0 and then, what does this look like? du dt is equal to g right. So, this is as good as a bullet.

The whole point, the whole distinguishing feature for fluid flows is the appearance of this pressure that is the reason the flow of a fluid requires a different treatment from the dynamics trying to study the dynamics of a fluid, you need to take the fluid pressure into account, it is in that respect that it is different from the dynamics of a stone or a bullet or something which just a something that is ballistic.

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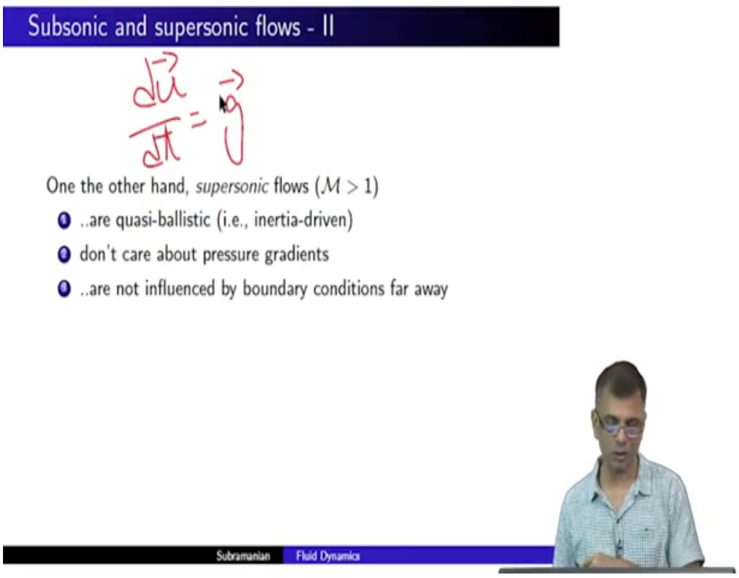

In this particular case for supersonic flows, the flows actually even though they are fluid, they actually behave more like a; more like a stone, they are quasi ballistic, they are inertial driven, they do not care about pressure gradients right.

The pressure gradient term is equal to 0 approximately equal to 0 and they are not influenced by boundary conditions far away. You see, we simply wrote that is all there was right. They do not care about the boundary conditions you know far away especially, if you are sitting on the partial of fluid, the boundary conditions do not really matter that much.

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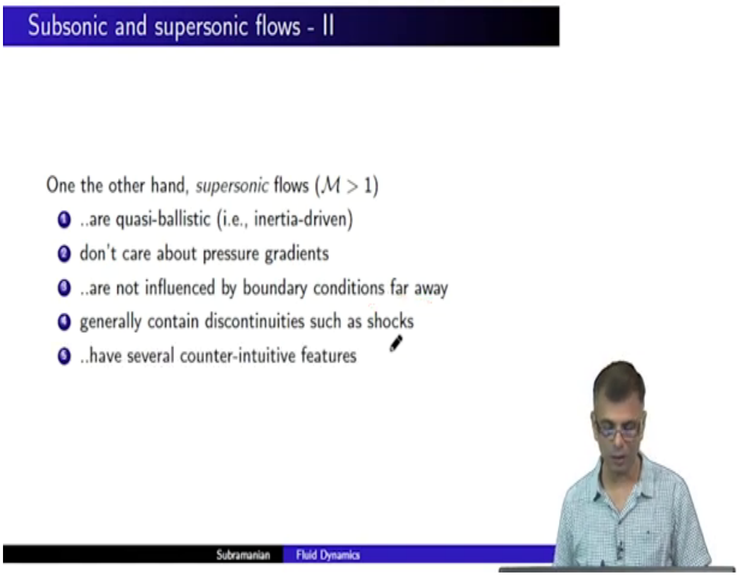

And they generally contain discontinuities such as shocks and this is a very important thing. We will talk about this in a minute. There are some counterintuitive features that arise in in supersonic flows and the situation gets especially complicated when one tries to transition between subsonic and supersonic flows, if a flow is transitioning between the Mach number less than 1 to Mach number greater than 1.

You have to pass through what is called the sonic point where the velocity of the flow becomes equal to the speed of sound and so, one has to be really careful in you know we will come to all these, for the time being, just to remark that supersonic flows generally contain discontinuities such as shocks and they have several counter-intuitive features as we remarked yeah.

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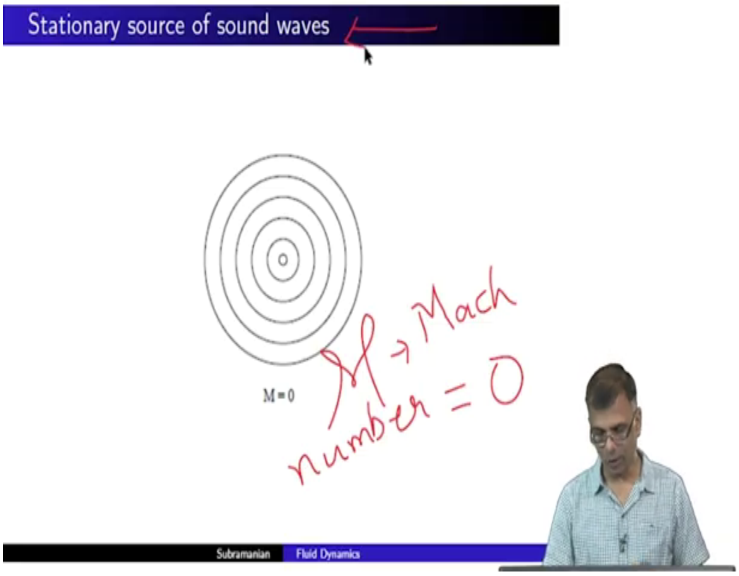

So, now, let us go on to a bit of a graphical illustration. As we said a little while ago, the speed of sound is the characteristic speed at which small disturbances propagate with respect to the background fluid ok.

If the background fluid is moving or if the source of sound is moving same thing right as; I mean it does not matter either the source is moving through the fluid or the source is stationary and the fluid is moving, it is exactly the same mathematics right let us now adopt you know a scenario where the source is moving, it is either stationary or moving ok.

So, now, our first illustration is pertains to a stationary source of sound waves. In other words, the Mach number, the this really should be a script M is technically equal to 0 right because u over c s and u is equal to 0 therefore, the Mach number is technically equal to 0 and what these circles indicate are iso-contours of pressure or density for definiteness let us consider them to be contours of pressure.

So, the disturbances and so, so the source is located at the center and the pressure disturbances are propagating isotopically outwards right and these are the contours of iso-pressure right. So, the pressure is constant around any one of these contours and of course, it is decreasing as you go out yeah. So, this represents a stationary source of sound waves yeah.

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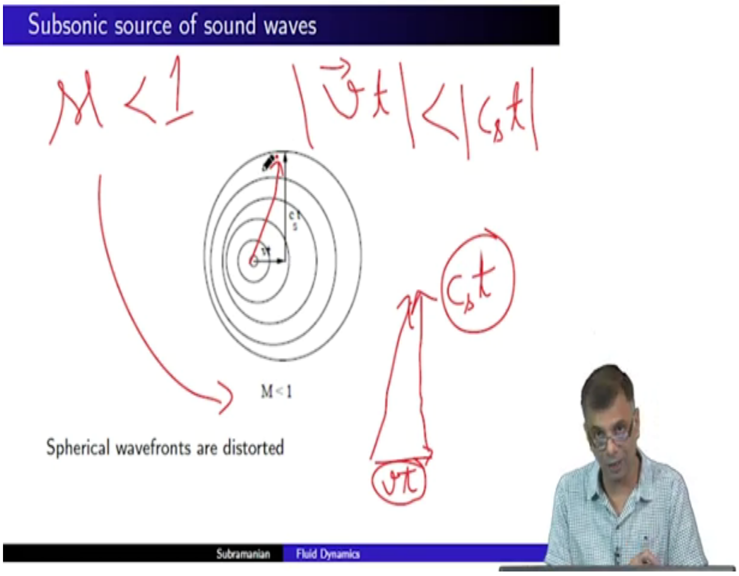

Now, let us set the source in motion yeah and in particular, this is a situation where the Mach number is less than 1 here yeah. So, what happens is so, the source is moving to the right yeah. Now, all of this is with regard to the lab observer. The observer is not sitting on the source of sound, the observer is outside and he or she is observing how the iso-pressure contours behave now.

Now, that the source is moving yeah. So, the source is still there, and the source is moving with a velocity v and so, the distance the source moves is something like v t right and the distance which the pressure disturbances move is something like c sub s t. Therefore, and the source is right here right. So, that would be the resultant vector, this is essentially, this is supposed to be a straight-line. So, that would be the resultant vector right.

So, the point I am trying to make here is that first of all there are two things. First of all, unlike the situation for you know for a for a stationary source of sound, the contours are now squished, the contours are now distorted, they are squished to the left that is point number 1. But the other more important point is that, but they are still not intersecting, the contours are still nested, they are still within each other right.

But unlike let us keep the unlike you know adjective to the next slide. The main point to be noted is that the source is still in causal contact with all the points on the farthest contour. This is the distance which you know so, this would be the resultant of v t and c sub s t.

So, what this is showing is that the source is in causal contact is in communication via these small pressure waves is in communication with all the points here even on the farthest contour. This is large here and this would be smaller here of course ok, it would be smaller here so on so forth, but it is still in communication with all the points and this is true as long as this v t is smaller than c sub s t.

In other words, the magnitude of v t is smaller than the magnitude of c sub s t. In other words, v is smaller than c sub s. In other words, the Mach number is less than 1 right this is true as long as this is you know satisfied and you know it is obvious what we are building up to?

We are building up to a situation where this need not be true if the Mach number exceeds 1 what will happen is this v t, the length of the v t vector can be larger than the length of the c sub s t vector.

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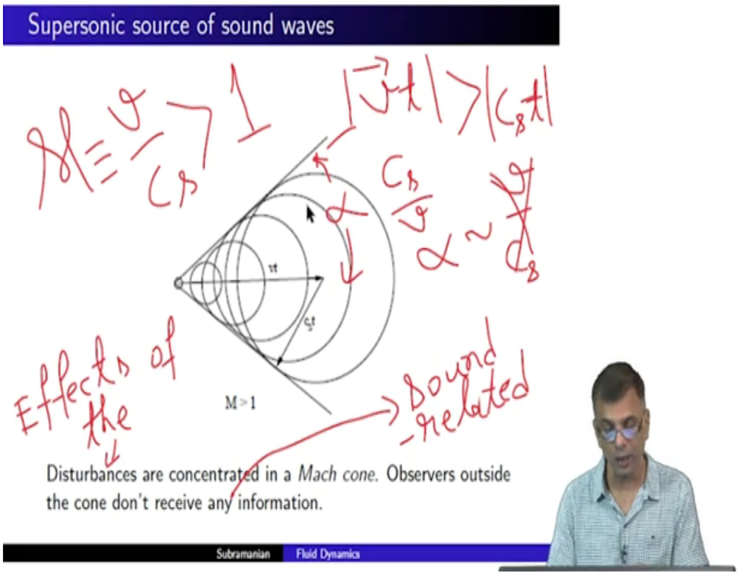

And what happens then? This is what happens. You see in this case; the v t is in this case is larger. The length of this vector is larger than the length of this vector, the source is still here and is still moving to the right except the Mach number which is defined as v over c sub s this is now greater than 1. As a consequence, v t is larger than c sub s t and therefore, what happens here is that the source is not able to be in causal contact with say this point or that point ok.

The source can be so, if you draw the resultant, this is v t and this is c sub s t and this would be the resultant. The resultant outlines a tangent like so. So, the disturbances what the effects of the disturbances, it is not so much the disturbances what we can say is that the effects of the disturbances are concentrated in a Mach cone and our observers outside of this cone do not receive any information. More properly we should say sound related information or sonic information ok. So, this is what defines a Mach cone.

So, and the angle of this cone, the angle, this half angle let us call it alpha, alpha is approximately equal to v over c sub s ok. I beg your pardon it should be c sub s over v sorry, it should be always smaller than 1 and you can verify this for yourself by dropping a perpendicular. So, the half angle is something like c sub s over v ok.

In other words, it is approximately something like that the inverse of the Mach number. The larger the Mach number, this is the more concentrated the cone and this is behind the phenomenon of the sonic boom that you might be you know familiar with when a supersonic aircraft flies fast, you hear a sonic boom and so, it is only observers within this cone who feel so, the aircraft is flying fast and it is only observers outside of this cone cannot say anything about the aircraft, they might be able to see it ok.

But that is that I mean seeing the aircraft is where the speed of light, but as far as hearing the aircraft goes, they would not hear the aircraft ok.

And so, so this is essentially, this illustrates the basic differences between subsonic and supersonic sources of sound waves. The other peculiar thing one can notice from this is that these are iso-contours right, these are contours of pressure. So, all along this, this, this contour, the pressure is constant say p 1, all along this contour the pressure is also constant say it is p 2 right.

Now, for supersonic sound waves, you see that these two contours are allowed to intersect, there is an intersection of these two contours here, the intersection of these two other contours here and so on so forth. So, add this intersection point, what is the pressure? Is it p 1 or is it p 2? Well, that is a good question right. I do not know that is the answer. The contours are intersecting so, I cannot tell whether it is p 1 or p 2 is indeterminate and so, this illustrates one of the other problems that can arise in in supersonic flows.

In other words, this is indicative of a mathematical problem. It is indicative of the fact that there can be discontinuities in the physical quantities such as pressure, velocity, density so on so forth ok.

It is indicative of the fact, it is not yet demonstrator, but it is indicative of the fact that there can be discontinuities in these physical variables and so, so we will come to that and these discontinuities are often shocks, but not always. There can also be things called contact discontinuities and we will come to that in a minute. So, that is it for the time being.

Thank you.