Fluid Dynamics for Astrophysics
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Lecture - 23
Lift on a body, introduction to compressible flows

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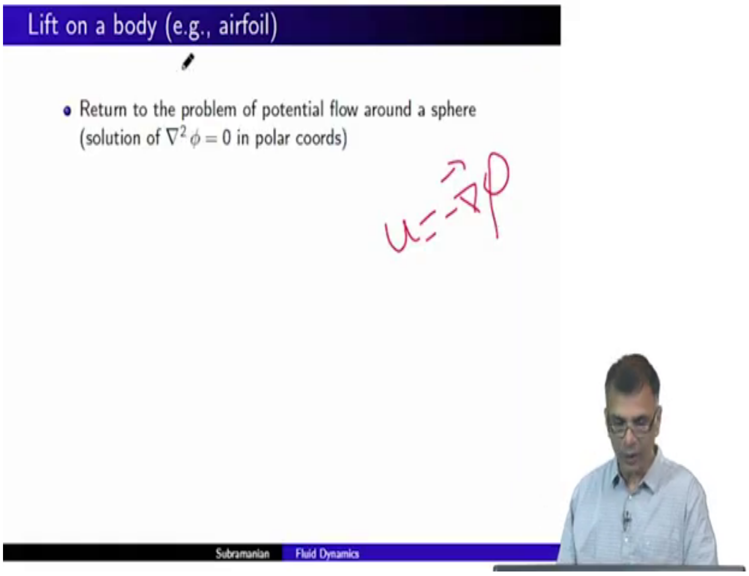

Hi. So, we are back. And we are now almost ready to start talking about compressible flows which is the next major topic we need to tackle. But before that, I thought before going on to compressible flows I had promised I would say something about lift on a spinning body. For that matter, lift on an airfoil any kind of body that has a circulation around it.

And in particular you remember I discussed top spin on a tennis ball. So, I thought I would take a little bit of time to discuss this. It is not to do with compressible flows not yet; it has to do with the income incompressible and inviscid flows really. The only difference being we can add circulation viscosity can be added.

So, this is not we are not yet discussing compressible flows, we will be doing this small thing and then going on to start discussing compressibility. It is just that there is one of the one of my favorite topics trying to demonstrate lift. So, I thought I would take a little bit of time to expound on this very briefly right.

So, before even talking about we are now specializing remember to incompressible flows not yet compressible right.

And let us now return before we add circulation let us return to the problem of potential flow around a sphere. Remember essentially we had said that problem of potential flow is essentially a solution of del square phi equals 0 the Laplacian, where phi where essentially u the velocity would be equal to the gradient of phi. In other words, this phi is really the velocity potential right.

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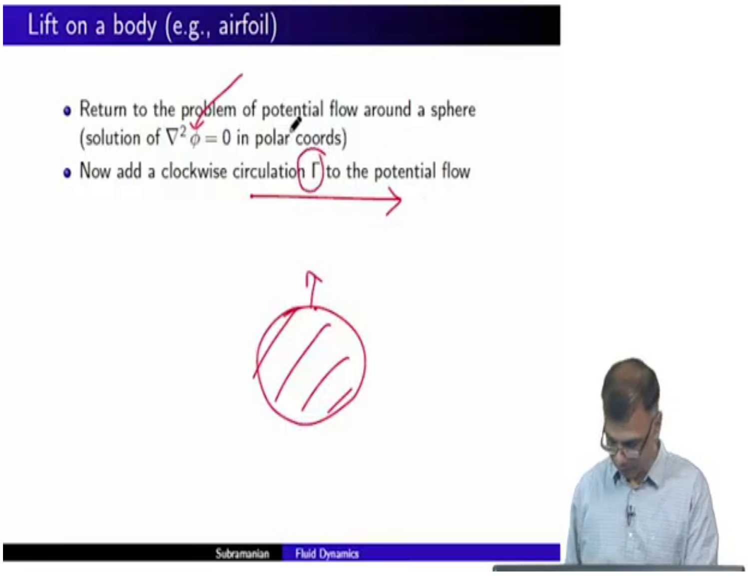

And in polar coordinates, you recall that we had written down the general solution to this equation. We had written down the general solution to this equation, and applying the appropriate boundary conditions. There were two boundary conditions; one was that the normal velocity, one was that. So, what we are doing here is we are considering a spherical body like.

So, and the two boundary conditions we had applied was that the normal velocity was 0, and the velocity at infinity was equal to the undisturbed velocity which is like this right. So, these were the two boundary conditions we had discussed. And so we had applied that to the general solution of del square phi equals 0 in polar coordinates and we had obtained the solution.

Now, the only difference where we are doing now the only difference in the treatment now is that we are adding a circulation a clockwise circulation this to that solution right.

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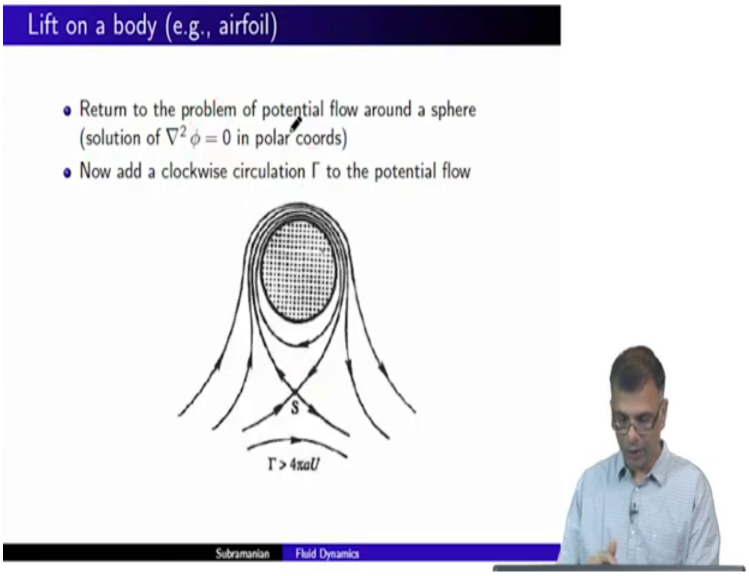

So, we are add, by hand we are forcibly adding a circulation to the flow you have seen this diagram before. So, we are adding a circulation gamma capital gamma to the potential flow.

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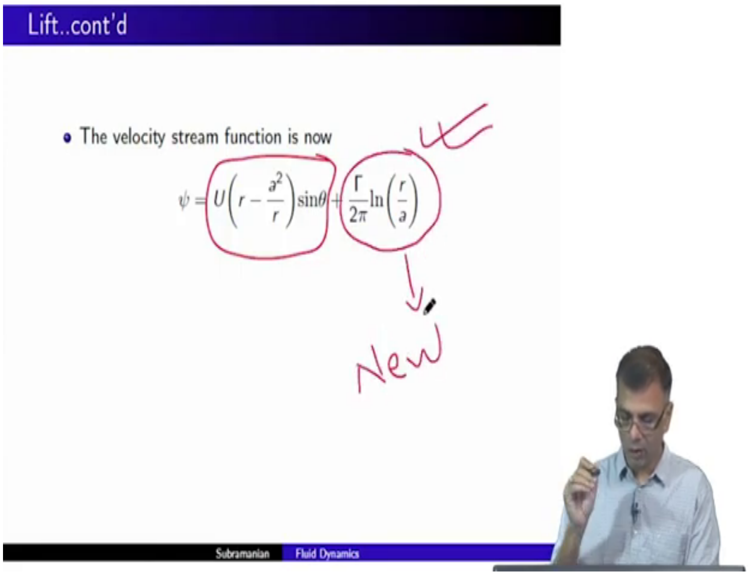

And that makes the velocity stream function, not yet the velocity the velocity stream function to be this. Earlier we had just this much ok. And now with the addition of the circulation, we have this piece. So, this is what is new now. Due to the fact that we have added clockwise, this kind of a clockwise circulation to the already existing potential flow right, so that is what we have done. So, there is the new piece we have added right.

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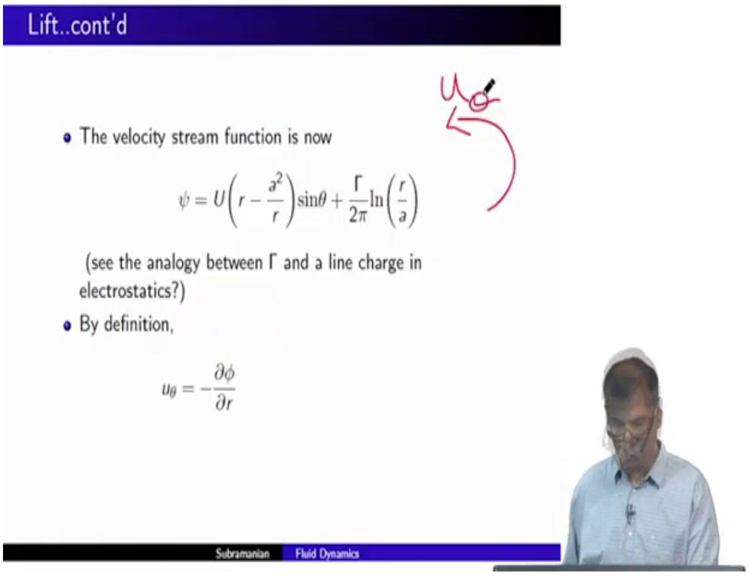

And notice see the analogy between the gamma this term and a line charge in electrostatics. Remember an infinite line charge the potential due to an infinite line charge looks exactly like this. There is a logarithmic dependence on r right. So, the analogy is exact right. So, now this is the velocity stream function.

And by the definition of the stream function, you remember how the velocity is derived from the stream function right. So, you take the curl and so by the definition of the stream function well I mean there is a relation between the stream function and the potential. And so by definition the velocity the theta directed velocity this kind of velocity yeah is equal to this yeah.

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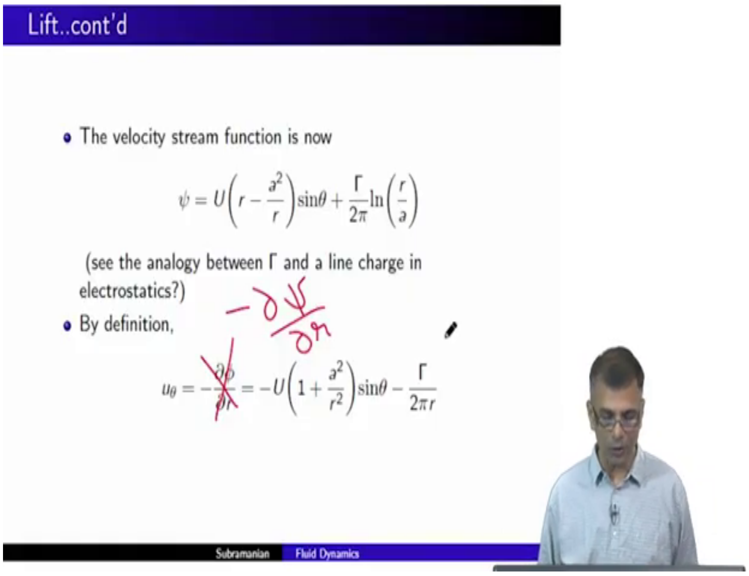

And that becomes I beg your pardon this really should not be, this is as a mistake here; this should be right. So, that is so. So, you take the differentiation of this and you can see that so it becomes this right. So, now, the theta directed velocity becomes this right. So, remember there is this funny term sigma gamma over 2 pi r.

So, there is a funny behavior when r tends to 0. But remember as we were as we had emphasized earlier we are only talking about the exterior solution the solution exterior to the sphere, and so r equals 0 is of no consequence for us. So, we do not need to worry about the pathological behavior of this piece of this piece when r tends to 0, that represents the interior solution and that is another solution. And we do not have to worry about that what it is.

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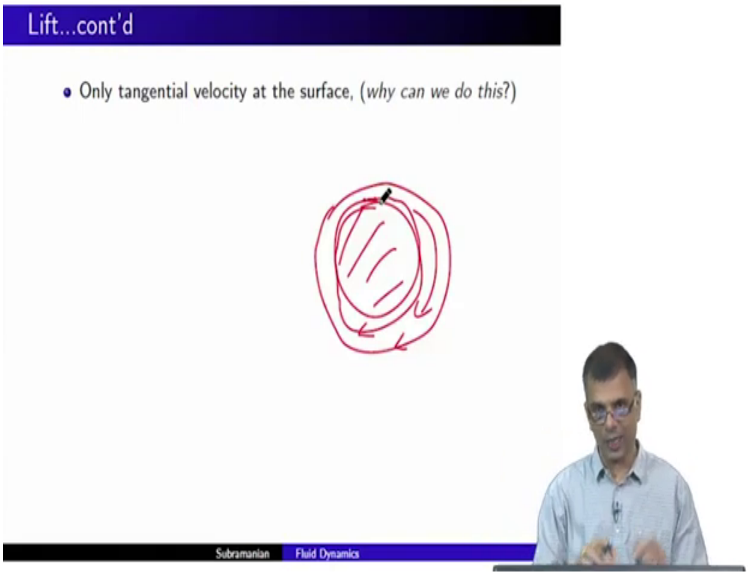

And now we allow only for tangential velocity at the surface. Remember, we are still talking about potential flow; the only difference between being that we have artificially added a circulation to it. We are still talking about potential flow, and therefore, we do not allow for normal velocity we only allow for tangential velocity ok that is why we can do this.

But it is important to also recognize that we are indeed using the solution for potential flow, but we are really trying to simulate a non-potential or of an a viscous flow with this solution ok.

So, what is happening is the ball is actually spinning in the clockwise direction. And in a real situation, in a real world situation, there is finite viscosity. And as a result of the spinning, that the fluid layers that are closest to the ball, the fluid layers that are really. So, this would be the ball and it is actually spinning like this.

So, what is happening is the fluid layers that are closest to the ball are actually being dragged along with it. My drawing is not very good, like this. So, there is a little bit of dragging, but this dragging is effective only in a very thin boundary layer like this, farther out no dragging; farther out, the flow is effectively potential. Very close by there is a bit of a non potentiality ok.

But what we do is we retain the potential flow solution ok. And we simulate the effect of this dragging by adding a circulation like this by adding a circulation like this that is what we are doing.

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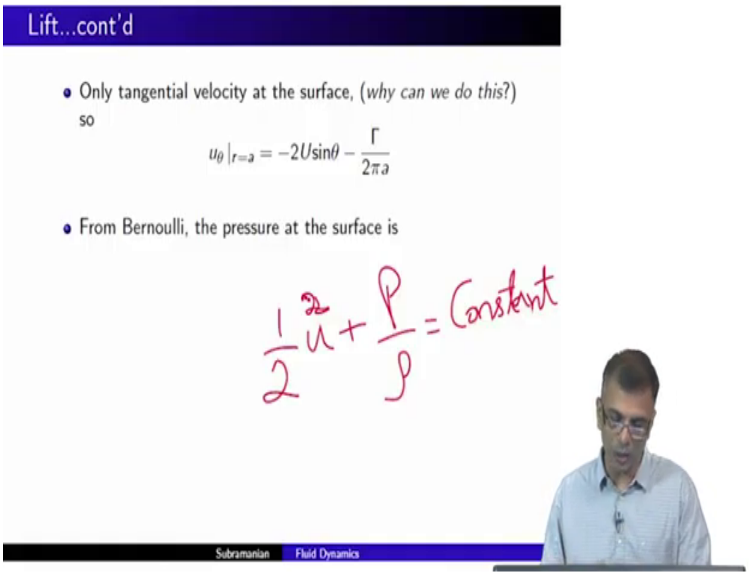

But and since we are retaining the potential flow solution while actually considering a solution, a situation where the viscosity is finite definitely so near the boundary condition.

But nonetheless as far as the mathematics is concerned, it is still that of potential flow that is why we can have tangential velocity the surface. Unlike it would be in a real world solution with viscous flow where even tangential velocity is not allowed. Both the components of velocity vanish for viscous flow at the surface.

But here what we are doing is we are looking at the mathematics of a potential flow in which tangential velocity is allowed. And so the tangential velocity at the surface you simply put r equals a, you simply put r equals a in this solution and you get this ok, so that is what you get.

And you remember Bernoulli’s theorem right. So, Bernoulli essentially tells you one-half u squared plus p over rho equals constant right. This is what Bernoulli is telling you right. Since, the ball is relatively small we do not have to worry about gravitational potential right. So, from this, so this is constant.

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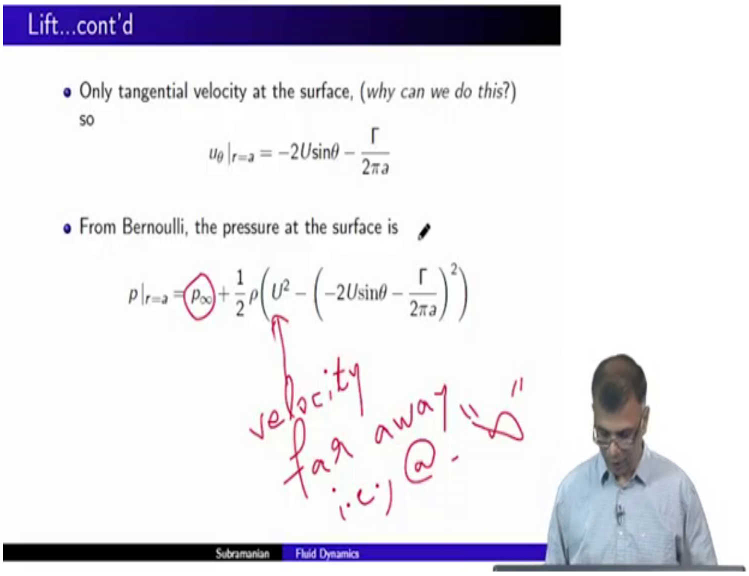

So, what this is saying meaning, so from that we get the pressure at r equals a, we can relate the pressure at the surface of the ball to the pressure at infinity, this is the pressure at infinity like this right. So, the velocity at infinity is this. This is the velocity far away. In other words, at infinity right, so that is the velocity far away undisturbed by the presence of the ball here.

And this is the velocity very close to the surface, so that is how it works out. And this is simply a consequence of applying Bernoulli's constant right.

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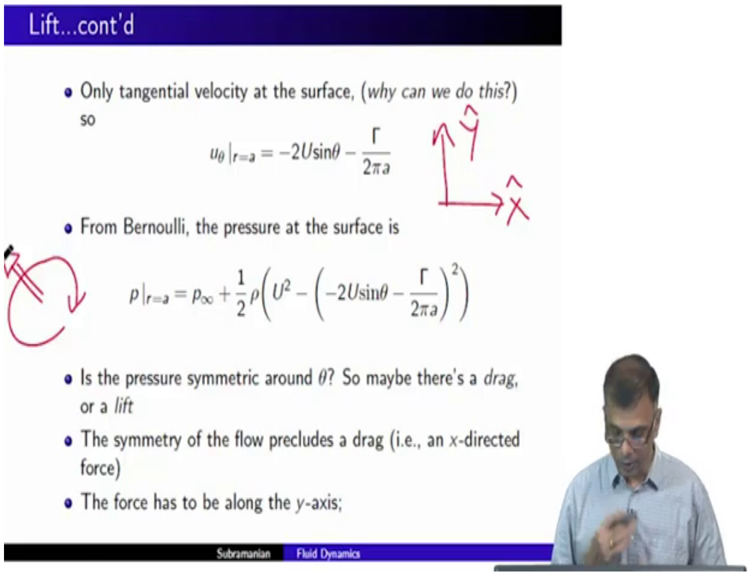

And so, the first thing to notice from this expression is near the surface of the ball. So, this is just a constant right, as is this U is just a constant, and there is also just a constant, a is the radius of the ball right. So, all of these are constants, now how does the pressure behave with theta right? So, the way theta works is this is theta equals 0 and theta increases like that right. So, theta increases in the counterclockwise direction.

So, like if you look at it that way, we can ask the question is the pressure symmetric around theta? That is the key question to ask right. So, it is not, it is not. Especially, if you only take 0 to pi, the pressure is not symmetric around theta owing to the sin theta term.

So, now, maybe there is a drag, there is a lift. If there is no, in other words, if the pressure on one side is different from the pressure on the other side, there is a force right. So, remember, the force is essentially at the gradient of a pressure right.

So, if the pressure is different, then if there is a gradient in pressure, there is a force. And what is that force? Essentially it is a drag or a lift; it depends. A drag would be something we normally refer to a drag as an horizontal kind of force and we normally refer to a vertical kind of force as a lift right.

Drag or a push we do not distinguish. It is the same thing. And a lift upward lift or a downward lift, we do not bother, it is the same thing ok. One would just be the negative of the other. So, because of the presence of sin theta term, we know that there is a break in the symmetry and maybe there is a drag or a lift ok.

Now, the symmetry of the flow precludes a drag, because you see there you see this it is symmetric like so, it precludes a drag. There is no way because the flow is symmetric in this direction, there is no way there can be a force either in this direction or in this direction yeah. So, there has to be either a lift like so or either an upward lift or a downward lift ok, so that is what it is now.

In other words, there is no x-directed force. The force has to be along the y-axis, where this is x, and this is y. So, the force has to be along the y-axis. Why is there a force again? Because there is a gradient in pressure, the pressure is different below the ball and above the ball. So, naturally a grad p gives rise to a force, and this is the lift. Now, turns out that we can actually compute the lift ok.

So, what are we really doing? We are saying that here is a ball that is spinning that is spinning like so, that is spinning like so. And as a result, it experience as a lift like this, that is what we are really saying ok.

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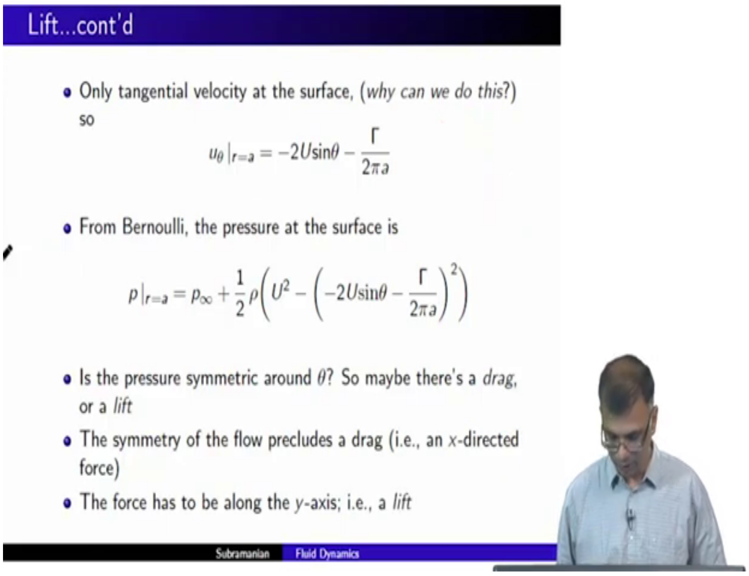

And, and we are now trying to compute the magnitude of this lift ok.

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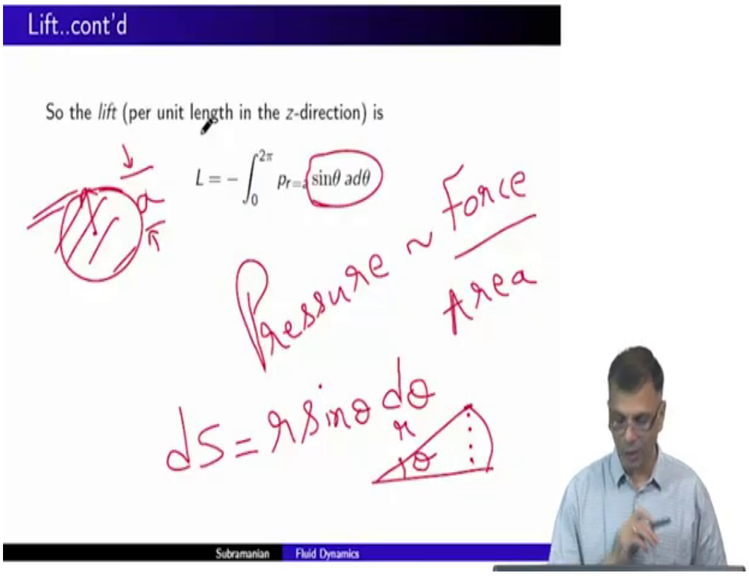

So, what are the dimensions of pressure? Pressure, we all we have is a pressure now. And you take the gradient of that pressure and you get the force right. But now the way the other way to look at it is that the dimensions of pressure as we know force per unit area, yeah we know this.

And in spherical polar coordinates, what is a differential area? A differential area is something like r sin theta d theta that just comes from this right. It is really this the length of this L times r, but the L can be approximated as if this is r right; L can be approximated as r sin theta, but this is r sin theta.

So, it is really r sin theta d theta ok. And, that is what we have written down here. But the a is really the r, we really concerned with the pressure at the surface of the ball this is a the radius is a.

And we are concerned with the pressure right here yeah, so that is why instead of the r, we have an a. So, this is the area element yeah and then. So, this is p d s essentially yeah. And in particular, we have we are writing down the p at r equals a and we integrate this from 0 to 2 pi. And using and that is what we call the lift pressure, so that is the force essentially pressure times area that is a force, that is a lift force right.

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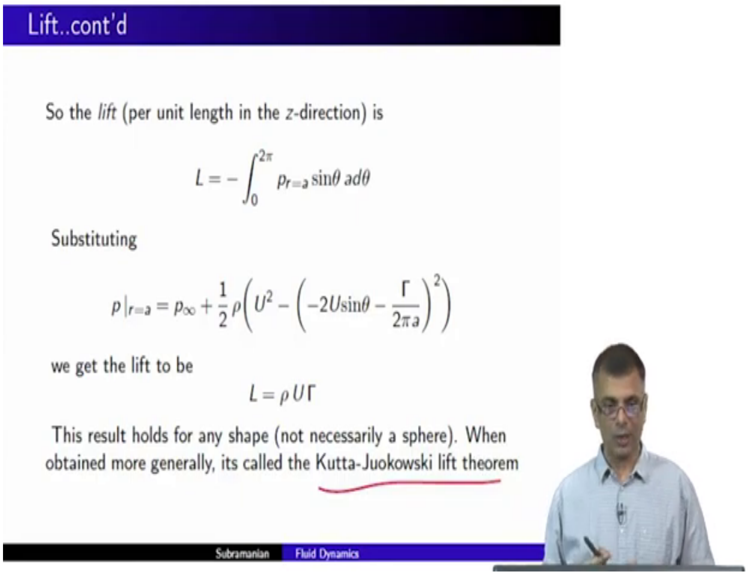

So, substituting this value of p r equals a, I am simply repeating this from the previous slide substituting for this in here, we get this magical formula. This entire the all these complications magically vanish away when we do the integration and you get this very nice formula for the lift ok. It depends upon the velocity at infinity.

Remarkably enough, it actually depends upon the velocity in infinity yeah it depends upon the density of the fluid that you would sort of guess right. I mean if the fluid is less dense or more dense I mean that the lift would be proportional to that this you can guess from intuitive considerations as well, and not surprisingly it depends upon the circulation.

In other words, you spin the ball viciously, you spin the ball a lot, and you experience more top spin yeah. You spin the ball a lot and the tennis ball experiences larger force downwards as opposed to if you spin the ball you know not so much the downward force is not that much that is what this term is saying yeah. But interestingly, it also depends upon the horizontal velocity the velocity at infinity or the undisturbed velocity ok.

And so this lift as it turns up, although we have you know derived this particular result only for a you know spherically symmetric situation. It turns out that this result it actually holds for any shape not necessarily a sphere. And this particular thing I am not going to show you I am simply stating it. And this is when obtained more generally in other words when this result is obtained for a general shape not necessarily a sphere this is called the Kutta-Juokowski lift theorem.

So, this kind of finishes or small discussion about lift. And so, from now onward we are going to do is we are going to we are going to talk start talking about compressibility right. So, we are done with talking about incompressibility. And from now on, we are going to start talking about compressibility.

We have just finished discussing lift on a spinning ball, and that pretty much completes discussion of potential flows incompressible flows and then we have also done a fair bit about viscosity.

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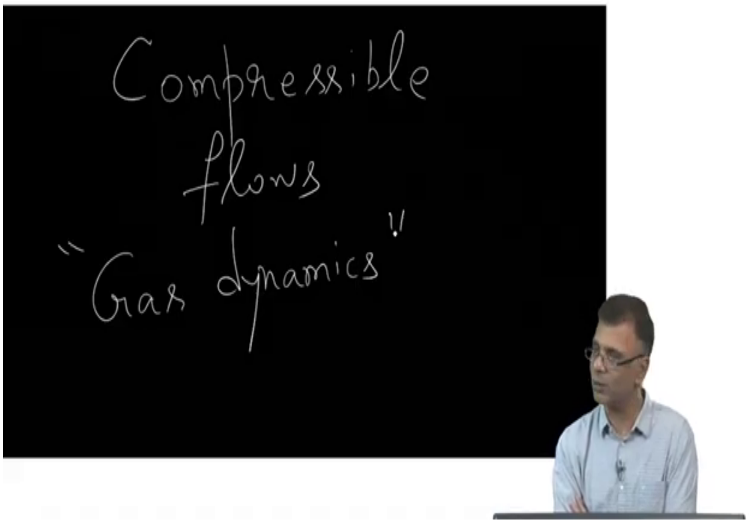

Now, we turn our attention to compressible flows. This is a bit of a major shift paradigm shift from what we have been talking earlier. And really this is something that astrophysicists mostly concentrate on ok, when you look fluid dynamics books that generally talk about gas dynamics. These are the ones which discuss compressible flows roughly speaking ok.

And whereas most engineering applications, well, aerodynamics sometimes you do start venturing into in into compressibility. But most other engineering applications flows of flows past nozzles and so on and so forth. They do not bother about compressibility, it is not so important to them ok.

Incompressible flows are good enough, whereas you know in astrophysics compressibility effects are very very important. Now, before starting to talk about astrophysics, let us you know can you think of an everyday situation where compressible effects are very, very important?

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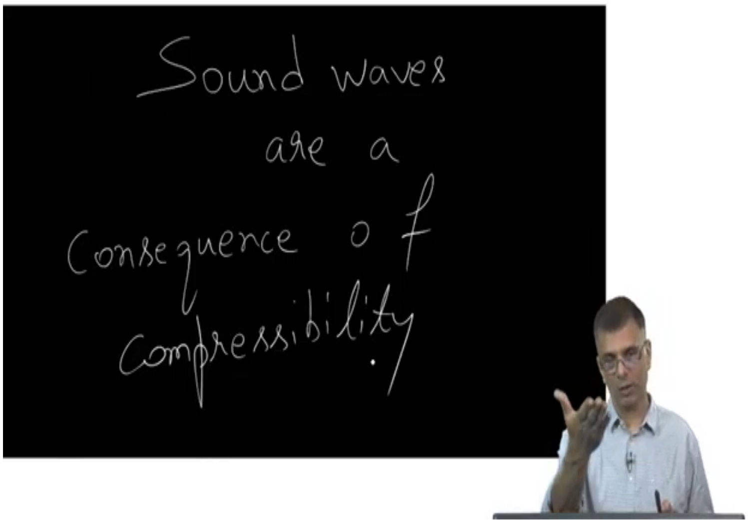

Well, let me tell you the speed of sound or rather you know sound waves which is how what I am talking reaches you eventually right. The waves the sound waves are a consequence of compressibility, no compressibility, no sound waves right. Why? Because sound waves are all about density, compressions and rarefactions is not it? You know, I am talking and I am what I am really doing is I am setting little density compressions and rarefactions in motion.

And these things are if you were sitting in a classroom, they would directly reach your ears and you would your eardrums would vibrate in response to these density compressions and refractions. And then it would get conveyed through a series of truly wonderful transformations, and reach a brain, and your brain would figure it out as sound.

In this case, what is happening is the density compressions and rarefactions that I am generating are reaching my microphone, and being converted into electrical signals, and being recorded. And then the electrical signals are when you are listening to it on your laptop or your phone or so on so forth. Those electrical signals are being converted back into audio vibrations.

Again density compressions and rarefactions and those are reaching your ear and you are hearing them. But the point is the point I want to make is here is an everyday example ok of compressibility. So, compressibility is central to lot us of things you know in life like sound waves right. And so one of the main things we will do as we go along is derive the speed of sound essentially and you know this very well from maybe the 11th grade onwards.

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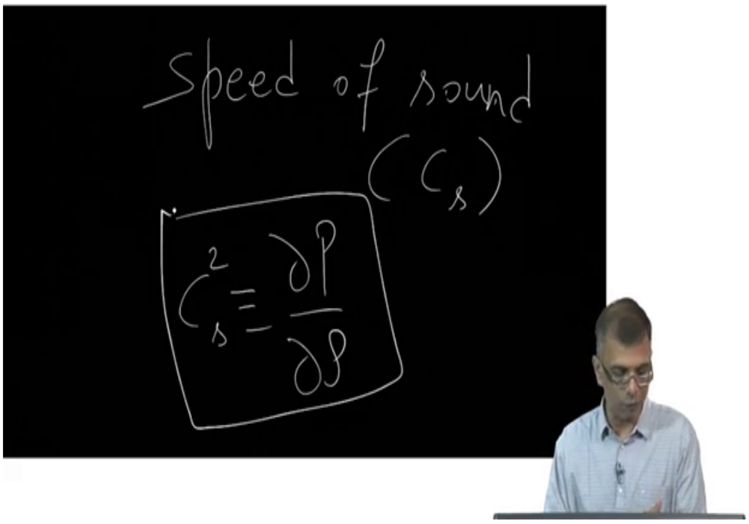

You know that the speed of sound generally denoted by C sub s is given by where P is the pressure and rho is the density right. So, we will derive this. We will see where this comes from ok, so that is our basic in our first goal. We will figure out how is it that the speed of sound which is essentially the speed at which small disturbances travel is always this ok.

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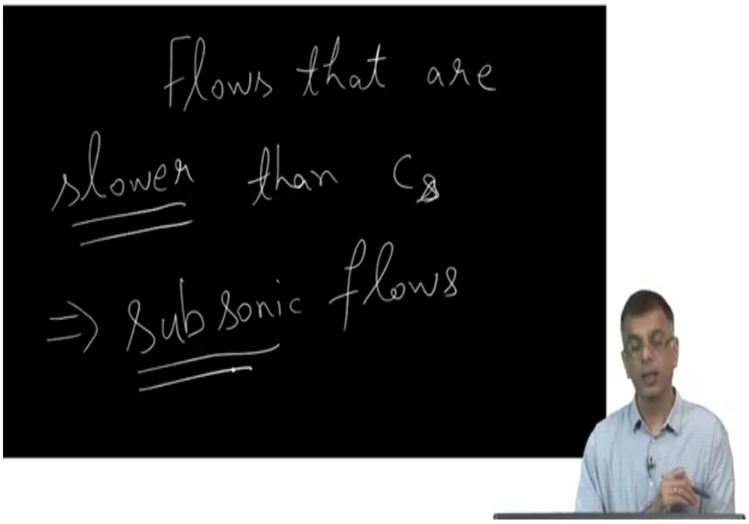

Afterwards what we will do is, we will say well how about speeds, how about flows where the flow is slower than the speed of sound flows that are slower than the speed of sound right. And so these are called sub sonic flows ok.

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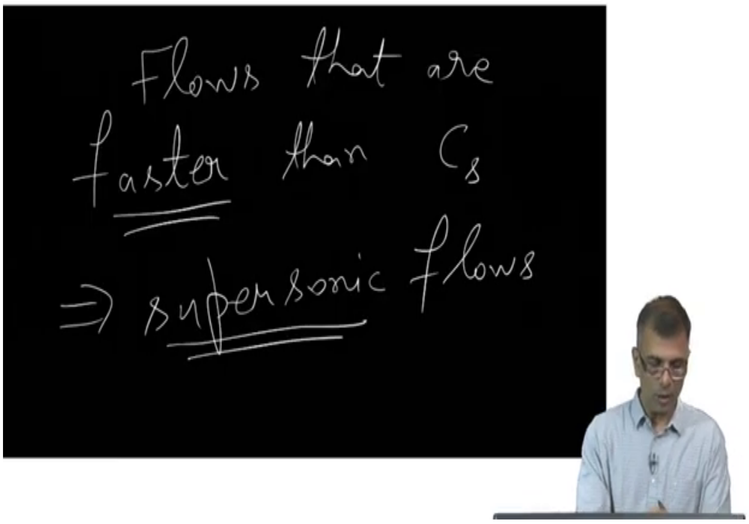

And flows that are faster than the speed of sound. These are called supersonic flows. So, these are the two kinds of flows that we will concentrate on. And we will also we will see that supersonic and subsonic flows are very different in character ok.

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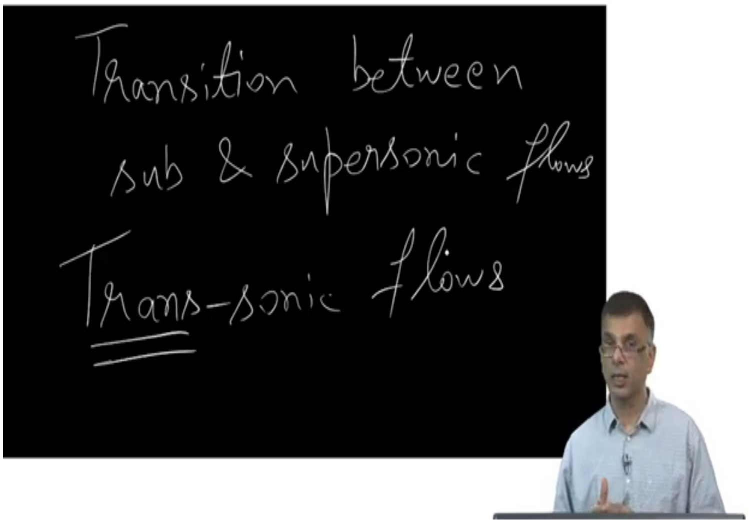

And they are very different in character. And we will also investigate this peculiar phenomenon where a flow can transition from subsonic to supersonic, and we call the transition between sub and supersonic flows. This is very important ok, because like I said the character of these two kinds of flows are very different.

So, when a flow transitions from sub to supersonic these are called trans-sonic flows. In other words, these are flows that transition from a subsonic flow to a supersonic flow because the character of these two kinds of flows are very very different you have to pay special attention to flows that are trans sonic. And you guessed it right.

Why are we mentioning this? Because transonic flows are very very important in astrophysics. Examples of transonic flows are the solar wind also certain kinds of accretion on to compact objects such as black holes and neutron stars. All these are examples of flows that can be trans sonic ok.

And transonic flows need special attention because the character of the flow changes substantially while going from sub to supersonic. So, we will discuss all this when we take up the subject in some more detail.

Thank you.