Fluid Dynamics for Astrophysics
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Lecture – 22
High Re flows: Turbulent drag law, Vortex shedding and drag crisis

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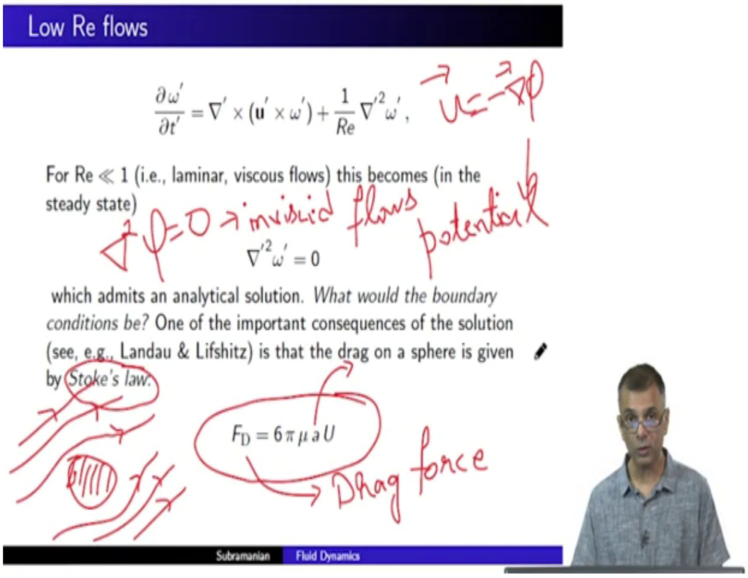

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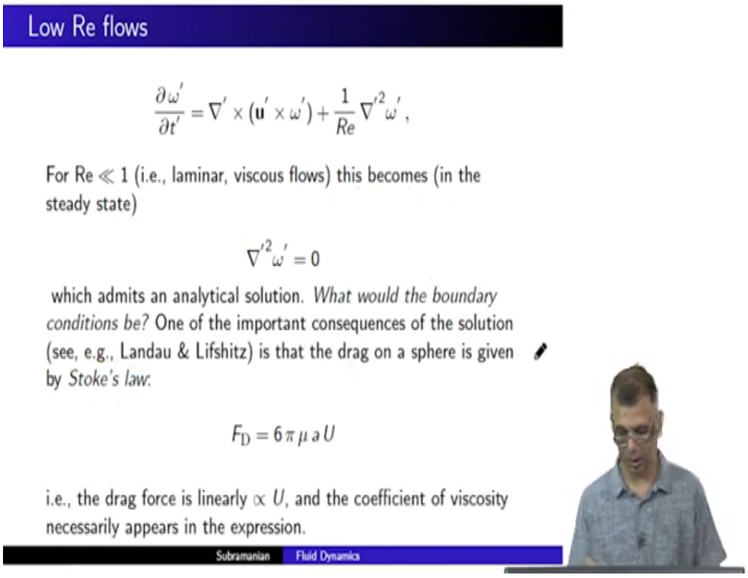

So, taking off from where we left off this. We were talking about the drag force on a sphere a smooth sphere when it is moving through a viscous fluid. And so, this is what we said the drag force is linearly proportional to the velocity right. And the coefficient of viscosity necessarily appears in the expression. I am simply stating this.

We have not derived this. I encourage you to look up Landau and Lifshitz or Kundu or any other you know popular fluid dynamics book for how this derivation is done, but it is a very important result.

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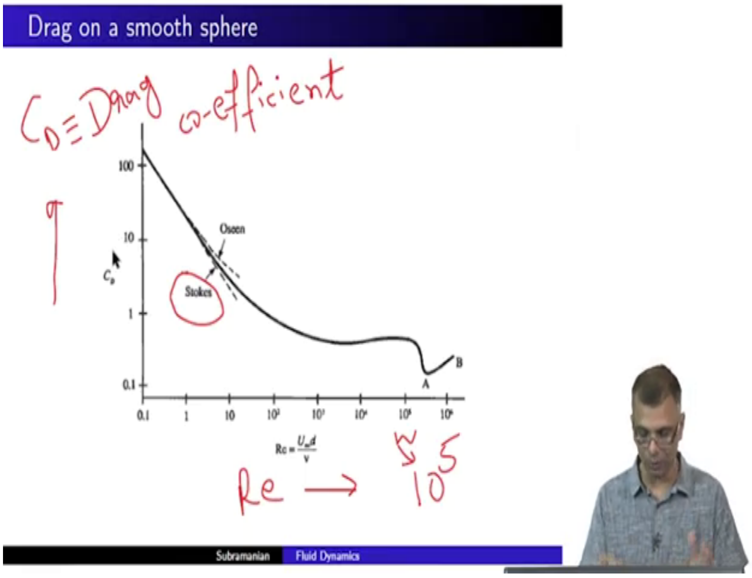

And this particular result is graphically represented in this form; we will return to this in a minute. For the time being, I will simply tell you what this means in one particular way and so what is on the x-axis is the Reynolds number; that is not how this is written mind you.

On an x-axis for instance, you would have the velocity, but that is not how this is plotted this is usually this is the usual way of plotting it. So, the Reynolds number is on the x-axis and on the y-axis is there is this quantity called C sub D which is called the drag coefficient.

And we have not even defined it at the moment and we will define it in a minute and when we start talking about the high Reynolds number law although. So, as you can see the Reynolds number goes from something like 1. So, it is 1 here, it is 10 here, it is 100 here and so on so forth.

So, we are really roughly speaking anything above a Reynolds number of about 20 or 30 or so, is considered to be high Reynolds number regime and by the time. So, there is a logarithmic axis and by the time you go beyond a Reynolds number, you probably cannot see this. This is something like 10 raised to 5.

So, by the time you go beyond a Reynolds number is 10 raised to 5 which is really really high, the character of this graph changes drastically. And so, what is being plotted here is what is called the dimensionless drag coefficient which we have not defined yet and we will define in a minute as a function of Reynolds number and the contents of the Stokes law. This law are essentially given by this dashed line here, this one this thing called Stokes this dashed line.

So, as you can see at least from this graph, for low Reynolds number the predictions of the stokes law are and so this solid line is. I do not want to say experiment results of an experiment, but this is an idealized depiction of experimental results ok. Experimental results are of course, much messier you will have lots of data points you will have scatter and so on so forth.

But from a large number of experiments this is a general behavior ok. So, for the time being for the purposes of this particular lecture, you can think of this solid line as being the you know and experimentally obtained line. And so, this is the experimentally observed the solid line represents the experimentally observed drag coefficient as a function of Reynolds number.

And the main thing from to note from this graph is that, for relatively low Reynolds number and the dashed line here this dashed line the lower dashed line represents the prediction of the stokes drag. And so, for low Reynolds number the experimental data confirms pretty well with the Stokes law ok. So, that is the main thing to take away from this graph although, we have not quite defined what the C sub D is and why the Stokes law looks like this when. So, we will come to that in a minute.

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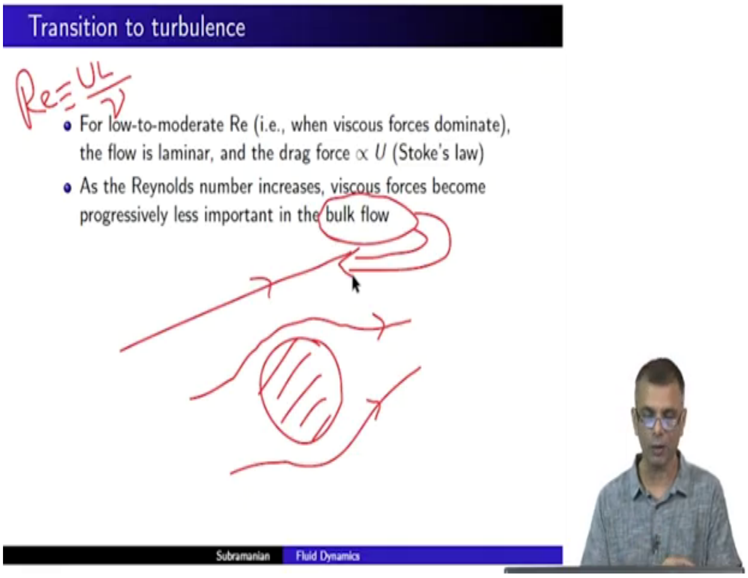

So, moving on. So, we will now, try to see what happens as you transition from low to high Reynolds number right. So, for something like low to moderate Reynolds numbers say any anything from one to about 20 or 30. In other words, when viscous forces are still important remember the Reynolds number is there is a ratio of the inertia term to the viscous term right.

So, if the Reynolds number is relatively low then, viscous forces dominate yeah the flow is laminar. Again, to quote an everyday experience you know the flow of honey is generally laminar if you think of the streamlines in a flow of honey in a motor oil or some such viscous fluid. They are generally laminar, they are not tangled and as we said the drag force is proportional to the velocity which is what Stoke's law tells you.

As the Reynolds number increases, viscous forces become progressively less important in the bulk flow. It is important to note this particular adjective in the bulk flow ok. We are not talking about the boundary layers here. The boundary layer complicated stuff and we will touch upon it, but we will not go into any great detail of this particular aspect right. So, but again Reynolds number increases, because the Reynolds number is defined as its worth writing this over and over again right.

As this increases this means, when the Reynolds number becomes larger, the importance of the denominator becomes progressively lesser right. How will it become larger? Well, in many ways maybe, you are increasing the system the scale length, but that is not typically how you know in everyday situations.

If you increase the velocity, the Reynolds number increases right and so, viscous effects become progressively less important in the bulk flow ok. We are not talking about the boundaries, we are not talking about a situation where you know for instance; if this is the sphere and this is where the flow gets perturbed right.

This would be kind of the boundary layer a very thin layer. That is not what we are talking about by bulk flow. We are really talking about the undisturbed flow right. So, this is what we are talking about by when we say bulk flow right.

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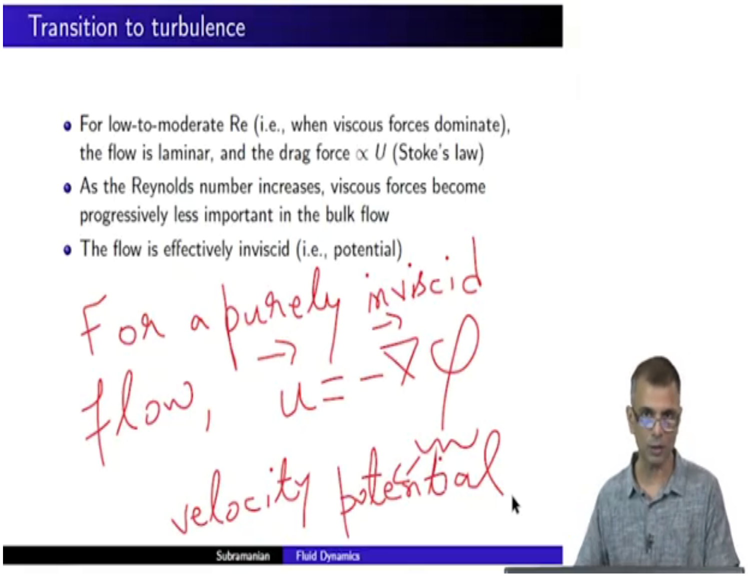

So, in the bulk, the flow is effectively inviscid. Same thing; we are saying the same thing over and over again in slightly different language. It is useful to do this so that you turn the concept around in your head and it you know gets embedded well. And as we said earlier, for a purely inviscid flow right for purely inviscid flow, the velocity can be given as a gradient of a scalar potential right. And this quantity is called the velocity potential right we have seen this.

So, the flow is effectively inviscid in other words, potential. You can see this by looking at the Euler equation which does not include the velocity. You can think of the Euler equation as the Navier-Stokes equation minus the viscous term. That is essentially what it is right.

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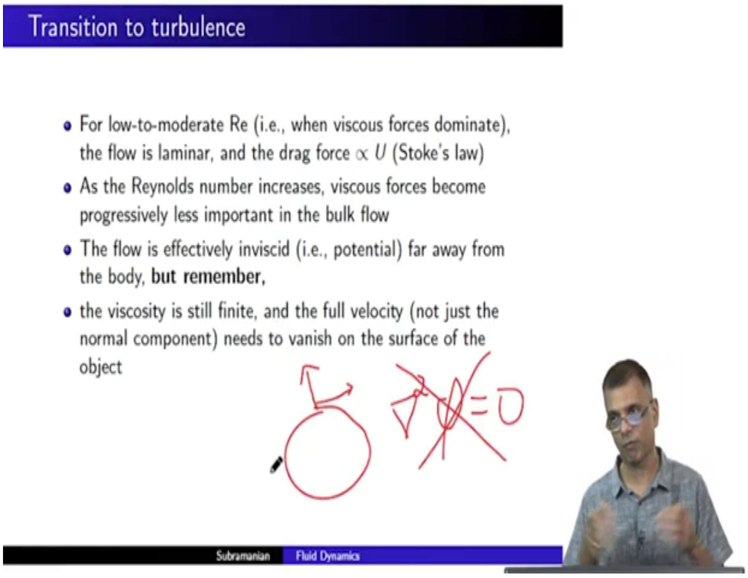

So, the flow is effectively potential far away from the body in the bulk flow not near, but remember and this is in bold phase very important statement. The viscosity is still finite, it is not technically zero ok. And the full velocity and not just the normal component needs to vanish on the surface of the object. What do I mean by this? Ok. You remember, for a potential flow in other words, for a strictly inviscid flow the tangential velocity can slip as much as possible right.

So, the boundary conditions for a perfectly inviscid flow; if you want to specify the boundary conditions at the object right. So, you say that the normal velocity. So, this would be the object right. So, the normal component of the velocity goes to 0 and maybe usually you simply stick to that and then, you say that the velocity at infinity is equal to the undisturbed velocity ok.

You do not specify any more components in any more boundary conditions. You specify only one boundary condition on the surface and other boundary condition is at infinity. You need two boundary conditions, because you know the solution you need to solve the Laplacian right.

So, this is a second order differential equation. You need two boundary conditions. So, one of the boundary conditions is typically specified at the boundary and that would be the normal component of the velocity. You demand that the normal component of velocity equal to 0.

And the other boundary condition is typically specified at infinity and that is a common sense boundary condition. It simply says that the velocity at infinity is what it would have been in the absence of the object, because you know very far away from the object the object does not exert much influence on the flow right.

However, you remember. So, this is technically valid only for a perfectly inviscid situation and the combination of these two boundary conditions, when applied to the general solution yields a solution where the flow is allowed to slip infinitely. The tangential component of the flow is allowed to slip infinitely on the surface of the sphere and this predicts no drag at all and this is the D'Alembert's paradox right.

But as we know no flow is perfectly inviscid ok, it might well be true that the Reynolds number is large and therefore, viscous terms might be relatively not so important in the bulk flow, but remember the viscosity is still finite, it is not technically 0. And therefore, the full velocity both and the boundary conditions for viscous flow or that both the normal component as well as the tangential component vanish on the surface of the object and so here now right now, we are not talking about this situation.

So, in other words, what if both the normal and as well as the tangential component or the velocity vanish on the surface of the object this is essentially saying that the flow sticks to the object which is what one would expect for in a viscous situation.

This is why you find that you know dust sticks to the blade of the blades of your ceiling fan right and that is, because the velocity of the air right at the boundary right where the you know the air comes very close to the blade, it is technically 0. That is why the dust particle actually sticks in well.

There are some other things. I mean the dust particles are oily that increases the velocity. So, and that increases the viscosity. So, it is not as if. So, there are some other complications there also roughly speaking this is what it is. For in a viscous situation, both the components of velocity have to be 0 on the surface of the object. You do not bother about the boundary conditions at infinity right. So, this is what I meant by this statement right.

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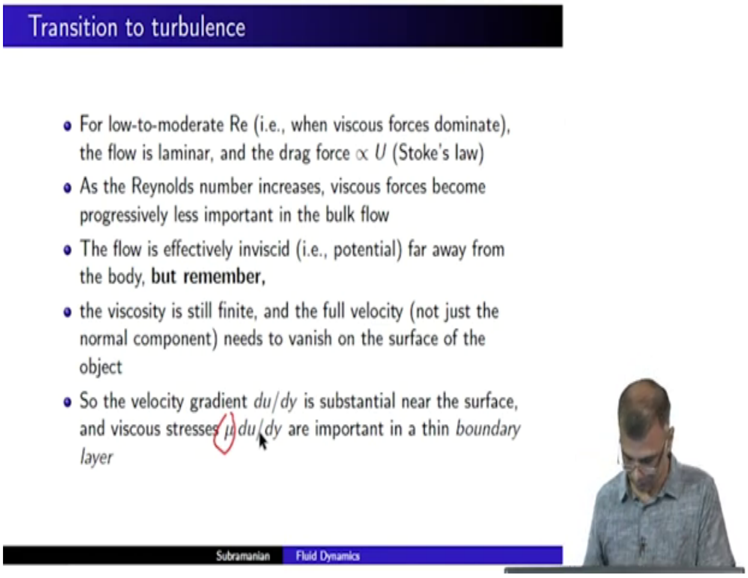

And which means that if the velocity has to come to a crashing halt right at the surface, the velocity gradient very close to the surface is quite large; the du dy is quite large near the surface. And therefore, even if mu is small as would be the case for a high Reynolds number situation.

The viscosity is small, that is why the Reynolds number is high in the bulk flow, but this combination this is a viscous stress right mu times du dy, because du dy is large, the combination mu times du dy becomes important. And where is it important? It is important in a thin boundary layer; not in the bulk flow, but in a very thin boundary layer near the object ok.

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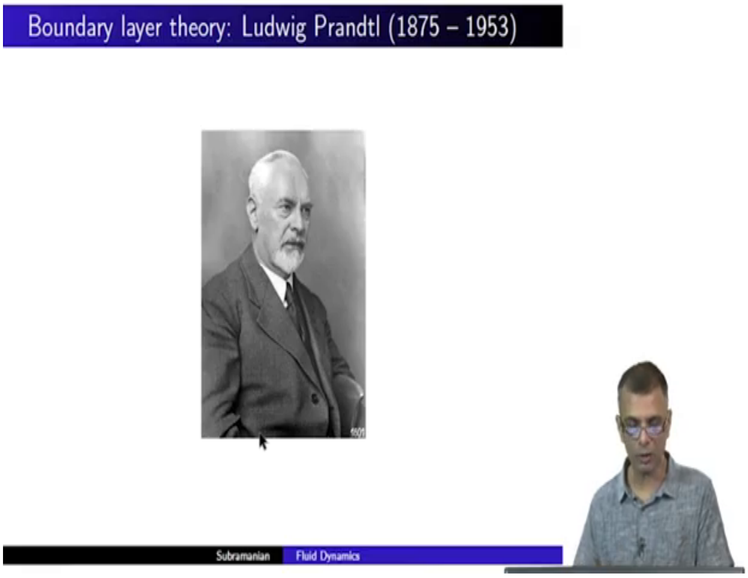

And boundary layer theory was investigated extensively by Ludwig Prandtl. Prandtl also, I mean I think it is a student's of Prandtl who put together a book on fluid dynamics. A very nice easy to read book I would highly recommend this; presents facts in a very we saying intuitively appealing way. A little advanced if I may say, but still the facts are presented in such an intuitive way that it is well worth reading. So, with that historical aside, we will proceed right.

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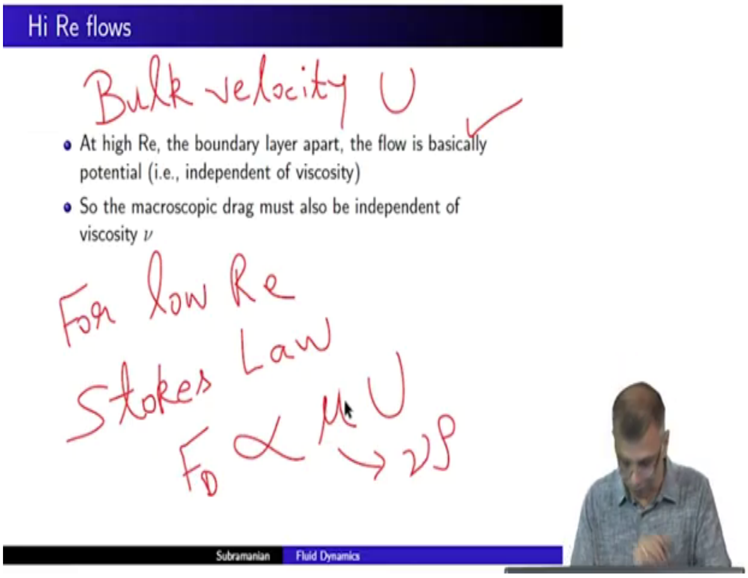

So, when we talk about high Reynolds number flows; in other words, say high velocity flows for instance. The boundary layer apart and this is very important. Except for the boundary layer except for the very thin boundary layer around the solid object, the bulk flow is basically potential ok.

It is basically inviscid. So, viscosity is not important in the bulk flow, it is important only in the thin boundary layer around a solid object yeah. So, the flow is basically potential; in other words, independent of viscosity this is what saying the same thing.

And so, if that is the case this drag force that we talked about a little while ago remember the drag in a low Reynolds number situation, we said that the Stokes law which gives the drag force predicts that the drag force is linearly proportional to the velocity right and the velocity there was the bulk velocity. And what is the bulk velocity? The bulk velocity is essentially the velocity that is far away from the boundary from the solid object essentially the undisturbed velocity right.

And so, here we are again talking about the drag force, but for a high Reynolds number situation ok. And now, if you accept this statement that in the bulk the viscosity is not important the flow is essentially potential. If that is the case, the macroscopic drag which is essentially the drag you know felt in the bulk if you will, there is no way it can depend upon the viscosity right.

It cannot depend upon nu or mu as the case might be; mu is simply a nu is simply mu over rho. So, it cannot depend on the viscosity, because in the bulk flow the viscosities are important right. So, the statement we are making is that for high Reynolds number flows the macroscopic drag on a solid object must be independent of the viscosity ok. This is exactly diametrically opposite to the Stokes law where the viscosity appeared if you remember, there was a mu ok.

So, the drag force was proportional to low Reynolds number, the Stokes law said that the drag force was proportional to mu and to U right. So, the viscosity appeared and this is just nu times rho right. The viscosity appeared, but whereas, here what we are saying is for high Reynolds number flow, because the bulk flow is effectively inviscid; viscosity is not important. This cannot be the law. In other words, the drag force cannot contain the viscosity.

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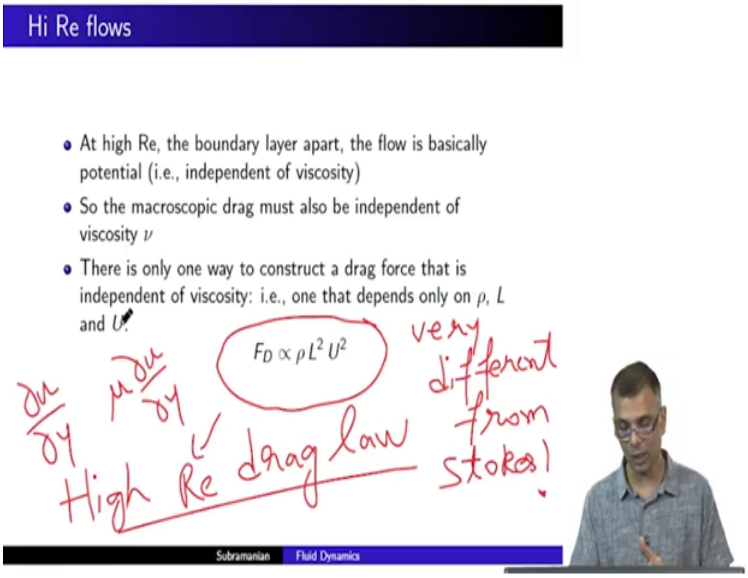

If that is the case, purely from dimensional analysis; there is only one way and you can try this out to construct a drag force that is independent of viscosity ok. To construct something with the dimensions of force, using yeah something that depends only on the density of the fluid the dimensions of the object say some L yeah.

If you are considering a sphere, the L would be say, the radius of the sphere or the diameter of the sphere it does not really matter just a factor of two. And the bulk velocity, because the bulk undisturbed velocity. So, you have to construct a drag force only from these quantities. The viscosity cannot appear, because you know the bulk flow is essentially inviscid.

That is the case. Then, there is only one way you can construct something with the dimensions of force; the drag force in this particular case from these three quantities and that is this way. So, this is the high Reynolds number drag law and I would say it is very different from Stokes law. Why is it very different? Well, first of all, there is no appearance of the viscosity that is one thing. Secondly, if you remember, the Stokes law depended linearly on the velocity.

The power here was 1 whereas, here the power is 2, very different dependence ok. And it is also been experimentally. So, here you have the appearance of an L squared which is kind of like an area and it is been experimentally determined that in the high Reynolds number regime, the drag force experienced by an object is proportional to the cross-sectional area that it affords to the flow and the cross-sectional area is something like L squared ok.

Now, here is a situation where the bulk flow is essentially inviscid and yet we are saying that there is a drag. As one would expect in a real life situation, there will be a drag even for the most low viscosity of I mean even for the most inviscid looking of flows, there will still be a drag on an object that you are trying to drag through this very inviscid fluid ok.

Specially, if you are trying to drag it at very high Reynolds number, there will be a drag. Yeah, the flow might well be potential in the bulk ok; but still there will be a drag and that is, because of boundary layer effects right. At the boundary layer even though the viscosity is unimportant in the bulk right at the boundary layer, because the viscous term the viscous stress appears in the combination mu times du dy right.

And, because U which is the velocity has to come to halt right at both components of velocity have to you know become 0 right at the boundary. So, the gradient you know of the velocity is very large. The velocity has to vanish from the bulk value all the way to 0 within a very small distance and therefore, you know du dy simply, because this y is very small the y is of the order of the boundary layer thickness.

Simply, because it is very small this gradient du dy is large and therefore, the quantity mu times du dy is appreciable, even though mu itself is small in the bulk flow ok. And therefore, there will be a drag and that is due to boundary layer effects ok, but as far as the bulk flow is concerned well, you have to construct a drag force that does not depend on viscosity.

And only way to construct something on the dimensions of force from something that depends only on the density of the fluid, the dimensions of the object and the bulk velocity is this and this is the high Reynolds number drag law and it is very different from the Stokes law. And this proportionality constant is what is called the C sub D. We will come to that in a minute and so, this is the turbulent drag law.

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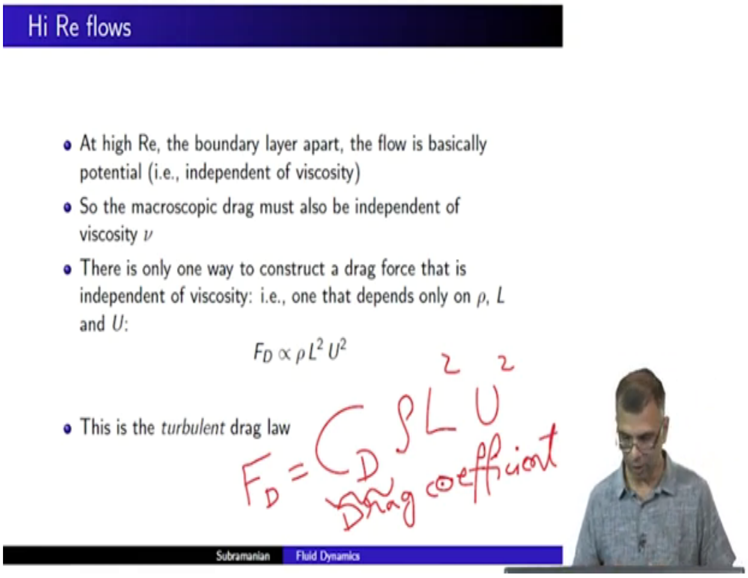

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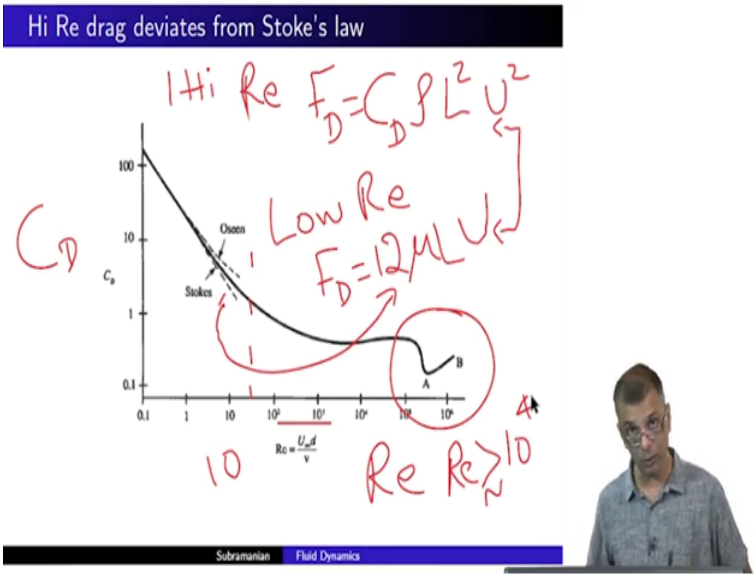

So, I did not write this down on the slide. So, this is called C sub D times rho L squared U squared. And this C sub D is the drag co-efficient and this was the quantity that was being plotted on the y-axis and. We will come to this. So, let us write this down here. The high Reynolds number high sorry.

Now, remember I am not writing a proportionality here anymore, I am writing the equality rho L square U square just this right. And Low R e; however, we had the Stokes law and F of D was equal to some 12 times mu and we had the U here and the L here, something like this ok. I hope I am not making a mistake please please verify this right.

So, now what we are plotting here is the Reynolds number on the x the very same graph that we saw before. We are plotting the Reynolds number on the x-axis against this dimension and if you see the combination of rho L squared and U squared has the dimensions of force. So, per force therefore, and necessarily this C sub D is has to be a dimensionless constant and that is what is being plotted on the y-axis. But you might wonder for low Reynolds number I mean I thought there was no C sub D right.

There is no real C sub D here. So, what is up? What we do is, we forcibly we essentially dig out a dimensionless drag coefficient for the low Reynolds number law by simply equating this to that by simply equating this F sub D to that F sub D. And you can extract the C sub D and that turns out for this particular law that kind of C sub D turns out to have this dependence against Reynolds number ok.

So, that is what is being plotted here on the y-axis it is C sub D and on the x-axis, it is Reynolds number. So, for low Reynolds number if you simply equate this to this, that is not saying that the low Reynolds number drag law is the same as a high Reynolds number right no no no no no.

I am simply trying to extract an equivalent C sub D for low Reynolds number flows and that turns out to be this dashed line, this dashed line here ok. And the solid line is an idealized version of experimental results and it seemed to agree very well with the Stokes law right.

Now, what happens at relatively high Reynolds number? Say, from about this is about 10 right here and this is something like 100 this is 1000 10000 10 raised to 5 and a million ok. So, what happens above say about say 20 or so something around here what happens here?

You see the Stokes law was is a straight line this dashed line if I extrapolate it right. You start seeing deviations of this solid line from the dashed line. I am not going to talks too much about this or scenes law we will at the moment it is not terribly important for this particular discussion.

So, but the main thing is this the solid line starts to deviate from this dashed line and the deviation starts (Refer Time: 26:15) about Reynolds numbers at little larger than 10 something like 20 or so right. So, and this particular behavior is given by the high Reynolds number law. If you were to, if you were to plot C sub D as a function of Reynolds number and for that I know the Reynolds number contains a nu right.

So, if you were to so you cannot do it exclusively from this formula of course, but if you were to introduce a Reynolds number into this formula by hand, making sure that the formula still remains the same obviously. Then you know you would be able to figure out how the high Reynolds number law behaves with the how the dimensionless drag coefficient for the higher Reynolds number law behaves with Reynolds number right.

And this is the experimentally observed curve. And clearly the experimentally observed curve deviates significantly from Stokes law from the low Reynolds number law beyond Reynolds numbers of about 20 or so ok. So, as you can see the drag kind of flattens out it does not and at very high Reynolds number, there is a certain plummeting of the drag which we will talk about in a minute and yeah. So, we will let us go ahead yeah.

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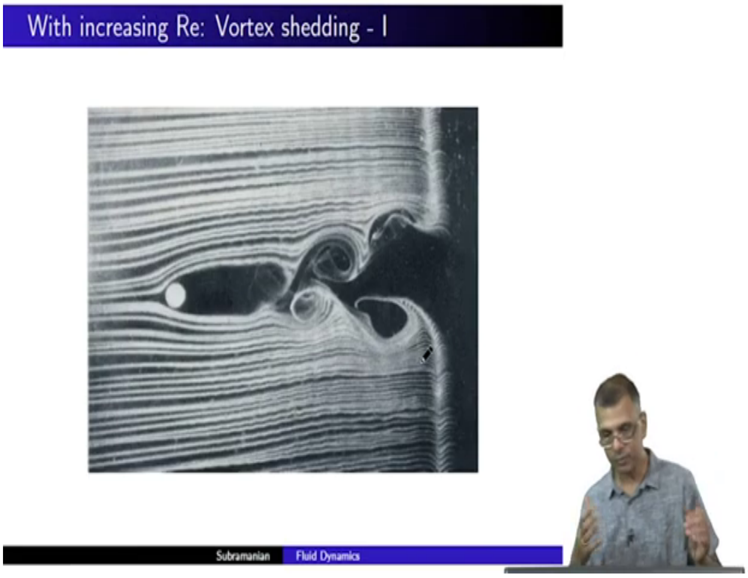

So, physically having talked about this graph for a minute, let us now, look at some pretty pictures encourage you to look up say Google vortex shedding and you will find plenty of very you know very appealing movies and this would be a snapshot from one such movie. With increasing Reynolds numbers what happens is. So, the flow here would be going from you know left to right and what happens is there is a formation of vortices behind in the wake behind the flow ok.

And this is this phenomenon is called vortex shedding and it is believed that it is this phenomenon of shedding vortices that is central to the drag. And so, it is a formation of these vortices that leads to the dissipation of energy and the dissipation of energy is essentially due to a frictional kind of effect which is what gives rise to the drag. And so, as the Reynolds number of the fluid is increased beyond a certain critical Reynolds number it is observed that such vortices are shed.

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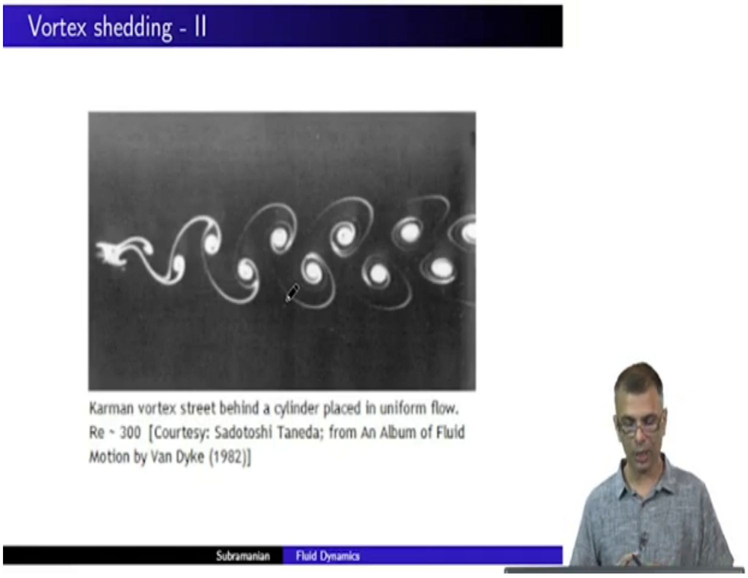

And in particular some sometimes what is observed is the vortices are start appearing alternately on both sides and beyond the point, there is a certain symmetricity in the appearances vortices and these are really very beautiful images. And fluid dynamics has all kinds of you know computational as well as experimental depictions of fluid flow are really truly very beautiful. I encourage you to Google some of these vortex shedding in particular and you will. So, I encourage you to explore this subject yourself.

For the time being, I simply want to emphasize that is this phenomenon of vortex shedding behind the solid cylinder behind the object behind this object is this phenomenon of vortex shedding that is central to the phenomenon of drag at high Reynolds numbers? Right.

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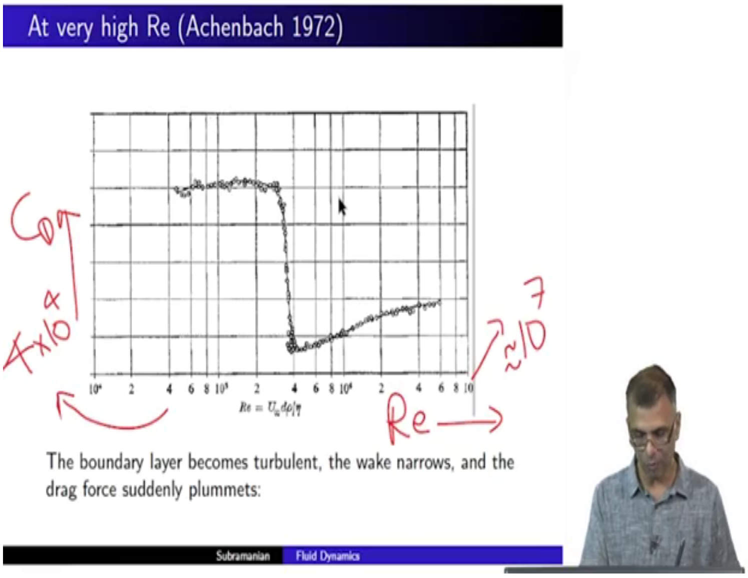

Now, going on to very very high Reynolds numbers right. So, going on to somewhere around here going on to this region ok. These are Reynolds numbers in excess of 10 raised to 4 or 5 or something like that yeah and so, what we are going to do is zoom in and here I will show you actual experimental results. So, again this is the Reynolds number being plotted on the x-axis and this is the dimensionless drag coefficient being plotted on the y-axis.

And so, this is a logarithmic scale and so this is something like 10 raised to well 4 times 10 raised to 4. This would be something like 4 times 10 raised to 4 and this would be around 10 raised to 7. So, this would be truly fantastically high Reynolds numbers and these dots that you see are actual experimental results ok. So, what is this graph telling us? Beyond about something like 4 times 10 raised to 5, there is a sudden plummeting of the drag.

At very high Reynolds numbers and this is taken from Achenbach, the drag suddenly plummets and this is also a logarithmic scale. So, this is really a substantial reduction in the drag and this is called the turbulent drag reduction and then, there is a gradual increase in the drag beyond this critical Reynolds number, why this is happen?

This is really a rather complicated phenomenon this is probably, because people think that even at sufficiently high Reynolds numbers at about the kinds of Reynolds numbers where these kinds of things happen, the boundary layer which is you know this very thin layer around here this would be the boundary layer.

The boundary layer is still laminar ok. There is no turbulence in other words as you can see even from this very rough photograph very old photograph. You know the stream lines are relatively laminar, there is no evidence for streamlines becoming tangled ok; whereas, for to super high Reynolds numbers the boundary layer becomes turbulent and the way here, this becomes narrow and in a complicated way, this is roughly why the drag force plummets and this is what is called the drag crisis ok.

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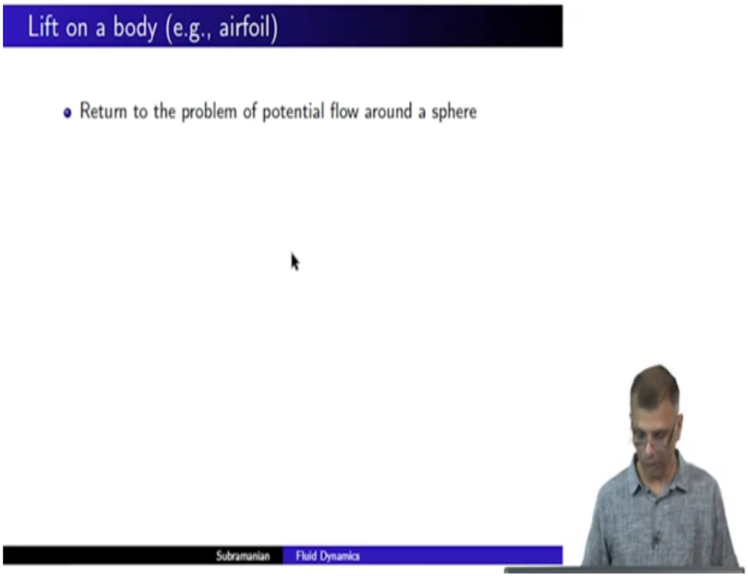

And this is important because. So, we will return to the other aspects in a minute. So, this is important because you know for instance; for supersonic planes for supersonic flights which used to be very popular I think in the 80's, until it became commercially and unviable, because you know they consumed too much fuel.

And so, it suddenly fell out of fashion, but supersonic flights you know you go from relatively low Reynolds numbers where the Stokes law is valid to high Reynolds numbers, where the turbulent drag law is valid this kind of law, this kind of law yeah is valid.

But if you keep increasing the velocity even further between the Stokes law and in this law, the behavior of the drag is relatively I mean you know there is there are no drastic changes, especially if you understand it well; I mean you can tune your thrust that you are providing the aircraft you know in accordance with this. But at really super high Reynolds numbers what happens is there is a sudden reduction in drag.

So, when you are at really high speeds, the drag suddenly plummets and this is something that you should be very aware of ok. The drag certainly plummets and so, your if you continue applying the same thrust, the results can be pretty unpleasant. So, you have to be very cognizant of this fact of this sudden drop in the drag and your engineering has to be has to take this into account right and so, this is called the drag crisis. Why am I talking about this in an Astrophysics oriented course?

That is, because and this pertains to some of my research and that is, because as I said astrophysical fluids are often most of the time astrophysical flows are in a very high Reynolds number situation in particular. For instance, to take a concrete example, coronal mass ejections from the sun yeah when they are travelling through the solar wind the Reynolds numbers for this situation are supposed to be very very high in excess of a million ok.

Next well in excess of about a million right and so, you are we are essentially in the super high Reynolds number regime. So, we have to realize that the drag law will probably be given by something like this. Not by something like this which would be this kind law and certainly, not by the Stokes law which is the lower Reynolds number law ok.

So, we are in particular for the motion of coronal mass ejections through the solar wind, the relevant drag law should be something that is described by this ok. We might well choose to describe it with this kind of an equation that is ok right.

But the drag coefficient that we extract should reflect the fact that we are in this regime ok. So, hence this bit of emphasis on the super high Reynolds number regime ok. So, having talked this far about Reynolds numbers and so on so forth, we will next transition to calculating the lift of an on an airfoil. I alluded to this some time ago. We simply said that we would you know appeal to something called the Kutta-Joukowski theorem.

And this also has something to do with the Magnus effect that I also talked about. The reason why tennis ball you imported you impart top-spin to a tennis ball and it dips ok. Or a cricket ball you know when a spinner gives a vicious tweak to the cricket ball, there is a dip and that fools the batsman and so.

We sort of discussed that in qualitative terms. We will come to that we will discuss that in a little more detail and the connection with the Reynolds number will become the connection with the rest of this discussion will become apparent, when we talk about it in some more detail. So, that is it for now and.

Thank you.