Fluid Dynamics for Astrophysics
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Lecture – 18
Dimensionless numbers: Mach number, Reynolds number

Hi. So, we are back and we are kind of finished with a brief overview of the basics of Fluid Dynamics. If you recall we have laid down the basic equations that of mass momentum and energy conservation. We did not spend that much time on energy conservation, but nonetheless. And we spent you know a fair amount of time talking about the momentum equation various guises thereof.

And after all this you see now we are ready to sort of take the plunge and move forward and look at various applications of these equations having laid down the basics. One of the main things I thought, I would emphasize now is the concept of Dimensionless numbers; dimensionless numbers in fluid dynamics right.

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Essentially it boils down to you remember when we wrote down the Navier Stokes equation there were several terms. For instance we had the gradient of pressure term, we had the viscous term which look like so on so forth right.

So, the question is are each of these terms do we have to retain all these terms in every situation or are there are there situations where I can throw away one term in favor of another. For instance if we are talking about an inviscid situation, if we are talking about a situation where viscosity is not important clearly I can throw away this term this is not important right.

But, we have to while doing physics we have to qualify these things a little more. What do you mean by not important, not important with respect to what right? So, here clearly since both of these terms were appearing on the right hand side of the Navier Stokes equation we would have to say, well if I want to throw away the viscosity term. If I claim that viscosity is not important then this term had better be negligible in comparison to this term or some other term right.

So, we would have to compare terms and this is where the concept of dimensionless numbers comes in. Aids as a it serves as a very good aid to figuring out what term is important what term is not important and that kind of thing. There are also some other issues like dynamic similarity.

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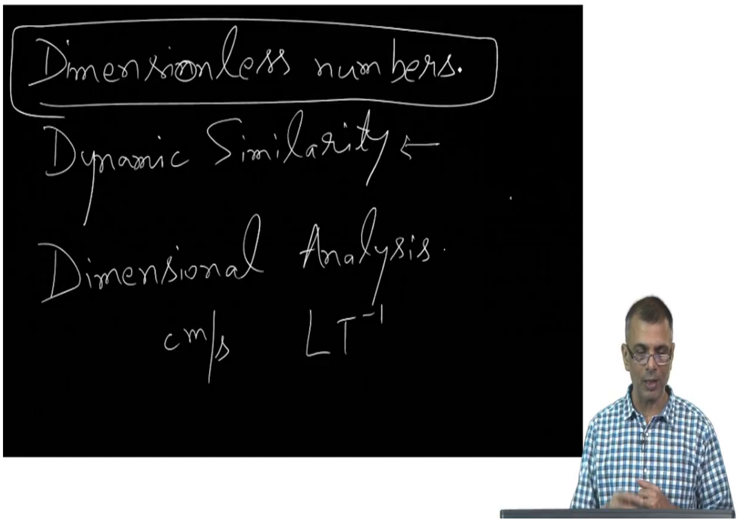

So, which I am simply saying this word right now, but I will not bother explaining it for the time being. But this is also something that we will concentrate on in this part of the course right.

And, the reason I am mentioning both of these in the same breath is because the concept of dynamic similarity is very closely related to that of dimensionless numbers ok. Now, underlying this whole thing about; so, we will start with this we will start with talking about dimension dimensionless numbers there are several, I will only start talking about a few right now.

And underlying this whole concept of dimensionless numbers is the familiar concept of dimensional analysis which you must be, you must be familiar with doubtless. Which is simply saying well; what are the dimensions of speed? Well the dimensions of speed are something like centimeter per second or L T raised to minus 1 so on so forth ok.

So, it is really you know closely related to the very familiar concept of dimensional analysis which I am sure many of you have been using for a long, long time. So, this is just to put you at ease this is really you know several of these dimensionless numbers simply arise from dimensional analysis.

So, without wasting much time let us get right to it. Consider the now familiar Euler equation right, well actually not quite the Euler equation let us say the Navier Stokes equation which includes viscosity also. Which you recall is nothing, but f equals m a right.

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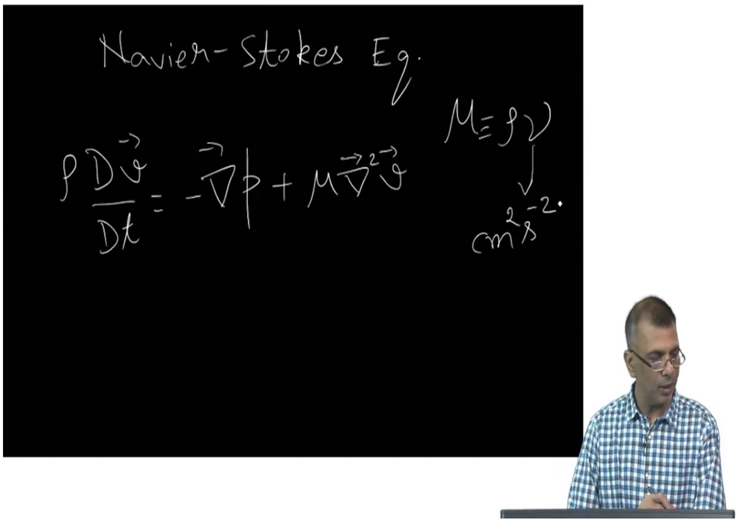

So, written in the Lagrangian framework it is a little more compact to write it that way. In other words if you recall the Lagrangian framework is one where you are sitting on top of the fluid parcel remember that right. So, in terms of the Lagrangian framework the Navier Stokes equation is simply rho, m a yeah is equal to and the f's are minus, if there is a gradient of pressure right plus mu. This is a vector Laplacian I want to; I want to beg your pardon, I did not write this very well sorry. Yeah, this is a vector Laplacian yeah.

Now, the mu is the coefficient of you know dynamic viscosity, many times we write mu as the density times this and this is also often confusingly both of them are often called the coefficient of dynamic viscosity.

And the distinction I want to make is that the dimensions of this are something like centimeters sorry; I seem to be making several writing mistakes today, like that. I am not sure that is right let me leave it as it is. It is the dimensions of it the nu has the dimensions of the coefficient of dynamic viscosity divided by mass density.

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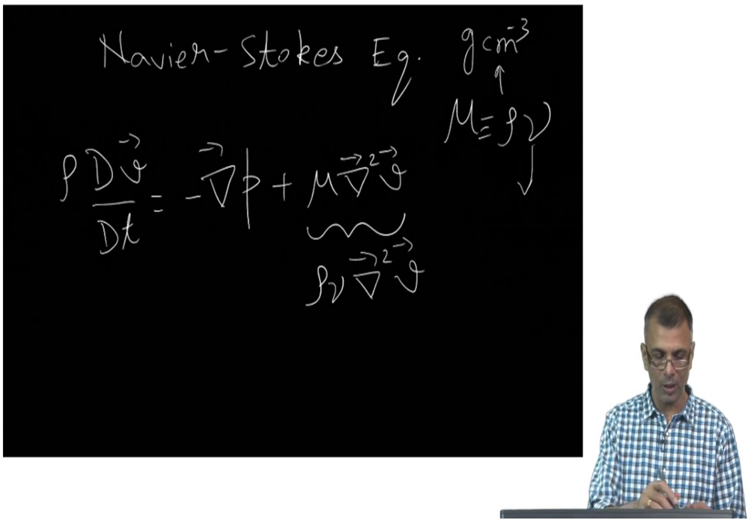

And this is simply the dimensions of this are simply grams per ok. So, and in terms of this if you write it this way then this can equivalently be written as right that is obvious. So, now, what are these different terms? Well this is m a this is the pressure gradient the fluid is flowing because it is a pressure gradient there is a high pressure here low pressure there fluid flows under the influence of the pressure gradient right.

And this is the viscous term which represents you know rubbing together of layers and so yeah so that is what it is. And it contributes to momentum transport of course, that is why it is in the momentum equation right.

And we have also seen that you can bundle both of these together both of these terms together into one pressure tensor we have discussed all that. But for the time being this is a more convenient way of representing the Navier Stokes equation because especially for what we want to discuss right now yeah.

So, now as we said earlier the question now is, how do I decide which term is in a given physical situation can I throw away one of these terms? For instance can I assume that the can is justifiable for me in a low viscosity situation to simply throw away this term.

So, that I am only left with these two. You realize of course, I am not including body forces like gravity so that is neglected for the time being just for simplicity right. So, if for instance I was to throw away this term you would have to compare it you would have to compare this term with this term or compare this term with this term and that is exactly what we are going to do right. So, before that let us write down what is called the order of magnitude of this of these terms.

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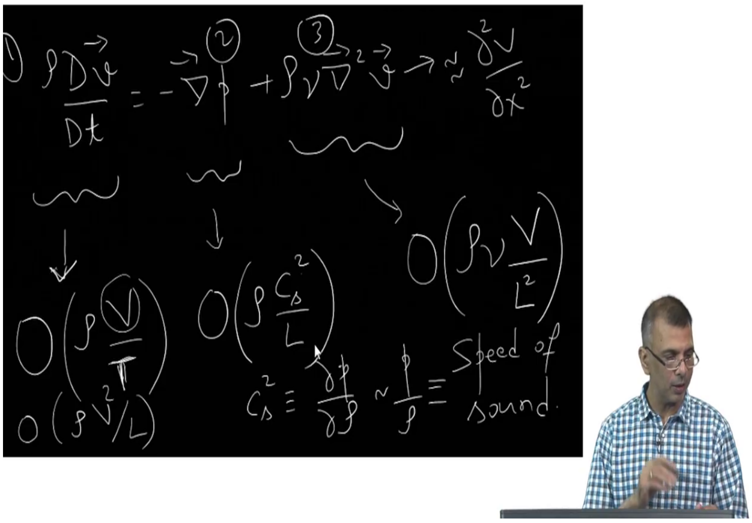

Let me again repeat this let me repeat the Navier Stokes equation for a minute, it is important to have this on the same slide as what we are writing things down. So, I thought right. So, now what is I claim that this term is of order rho V over T. Now, this is essentially dimensional analysis. Whatever the dimensions of you know rho are grams per centimeter cube that I keep that as it is.

Now, this D v D t is something like some kind of velocity divided by some kind of time. Now, I realize it is a little vague yeah, what do you mean some kind of velocity? What do you mean some kind of time a representative velocity?

When you are talking about a breeze flowing through this room you are certainly not talking about a you know some something flowing at several kilometers per second. No, you are talking about velocities that are much smaller than that, so of the order of so many centimeter per second that kind of thing.

So, that is what I would call a characteristic velocity for a given situation. Similarly, one can arrive at a characteristic time scale right. How fast do you think do you expect things to change in this room? Yeah do you expect things to change over the time scale of seconds or do you expect things to change over the time scale of hours.

If you ask these questions you will very quickly be able to come up with a reasonable time scale ok. So, this is what we call a characteristic time scale ok. So, although it is although this concept is a little vague it still make sense right. It still makes sense to talk about a characteristic velocity and a characteristic time scale. And so using this these concepts we can say that this term is of the order of rho times V over T this term right.

Let me now write down this term I will leave this for a for later you know we will talk about this in a minute. So, this is of order I leave these two terms as it is ok. Now, remember this is the second derivative right it is like d square v d x square or d square v d z squared so on so forth.

So, it is essentially velocity divided by some length scale squared right this is something like that right. So, it is some kind of characteristic velocity divided by some kind of characteristic length scale squared that is the order of magnitude of this term right.

I might simplify this by saying that V over T is something like of the order of rho, I can say this 2 right this is the same as this. In that the T, I am writing as a V over L ok. Just because I want only V's and L's all throughout the equation I do not unnecessarily want to introduce a T if I can help it in this particular case I can right.

So, now we come down to this term now what is this you know roughly of the order of. I claim that this is of the order of rho where I am introducing a new term called C s squared right which is a speed of sound. We will discuss the speed of sound this is something that is central in fluid dynamics we will discuss this in some detail a little later.

But as of now I am sure you have come across this definition the speed of sound is ok. And a little dp d rho right that is a sphere. And so carelessly we can write sort of write this as p over rho ok. So, instead of p I write rho times C s squared that is all I have done here ok.

So, I have written this rho times C s squared gives roughly p yeah and this is a gradient so it is like a d over dx kind of term. So, that I simply write as d over d x is 1 over L, where L is some characteristic length for the dimensions of this room for instance, characteristic length would be something like a meter certainly not a millimeter and certainly not a mega meter its of the order of a meter. So, that is what I mean by characteristic length so I plug that in.

So, please remember this when we go to the next slide these are the orders or magnitudes of these three terms. Now, we are in a situation to say well, if I claim that this term is smaller than this term or this term is smaller than that term then all I need to do is essentially divide this by this for instance to compare the magnitude of this term to this term. And this gives rise to the concept of dimensionless numbers.

Complete this, this is the speed of sound, we will discuss this in great detail as we go along, but for now I have just introduced it this way right. Let us now so this would be you know term 1, term 2 and term 3.

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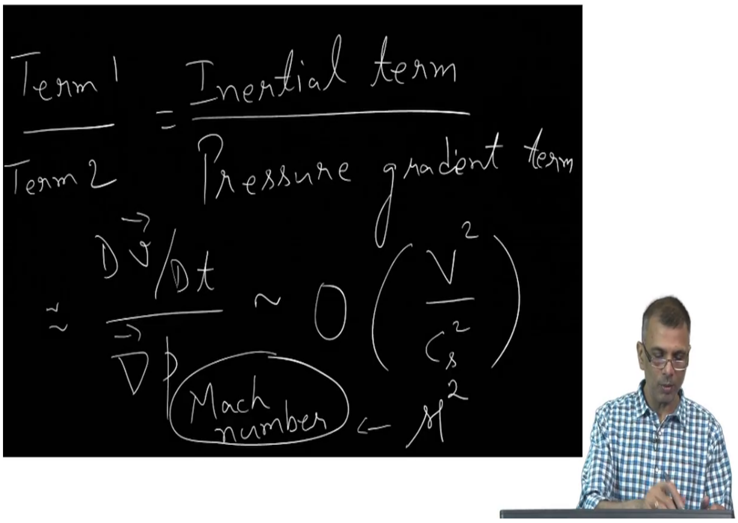

So, now the ratio of Term 1 to Term 2 ok in other words the term 1 you see is the what is what one would call the inertial term ok. This is what represents inertia m a by Newton's second law that is the inertial term. So, I would say the ratio of the inertial term to the pressure gradient term right.

The inertial term to this pressure gradient term that would be some rho V squared over L divided by rho C s squared over L right. So, in other words this is something like the D v D t term over the gradient of pressure term ok. This is of the order and this simply follows from this and this ok.

Because you know the L's cancel out and so you and the rho's cancel out also so all you are left with is V squared over C s squared ok. So, this one, and this; this very important quantity is called is often denoted by the letter M and this is something that is called the Mach number. This is the first dimensionless number that we will encounter the Mach number.

As such you see the Mach number is simply the ratio of the flow speed more accurately not really the ratio of the flow speed itself the ratio of some characteristic speed in the system ok. Divide well the square this is the actually the Mach number squared, I beg your pardon this is let me just erase this a little bit yeah.

So, this is the square of the Mach number this whole thing ok. So, the ratio of the flow speed to the sound speed is called the mach number, but where did they come from, why did I simply you know pop it on you? It came from this fact is the ratio of the first term in the Navier Stokes equation to the second term in the Navier Stokes equation or the ratio of the inertial term to the pressure gradient term.

So, clearly if the Mach number is large; that means, the inertial term is more important than the pressure gradient term and vice versa. The Mach number is low then the pressure gradient term is more important than the inertial term.

So, this dimensionless number that this is the first dimensionless number we are encountering and this gives you a good idea of which term which of at least these two terms are relatively more important than the other. So, this our first encounter with you know and this is how it comes about right.

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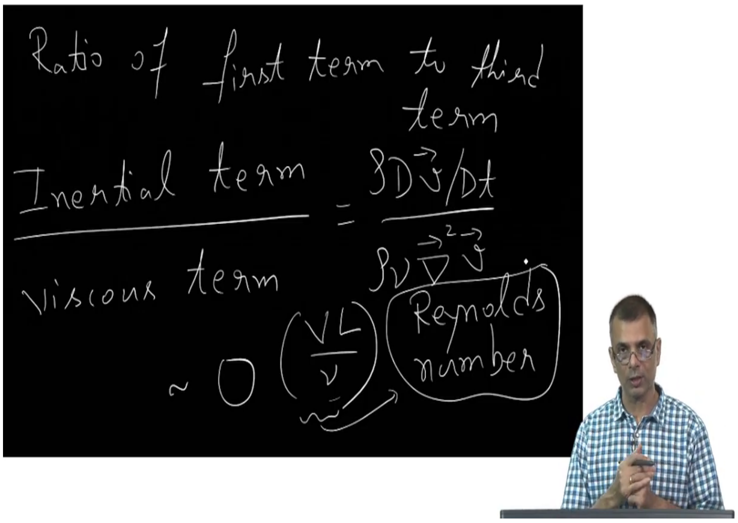

So, another example, the ratio of the first term to third term again first term to third term right inertial term to viscous term. In other words over viscous term; which is essentially the rho D v D t over rho nu right.

The ratio of these two terms is notice this is of order rho v squared over L and this is of order rho nu V over L squared; obviously, the rho's will cancel each other right. So, and this so therefore, this turns out to be of order V L over nu. This is essentially some characteristic velocity in the fluid times some characteristic length scale in the fluid on the fluid flow to be precise. And this is the coefficient of dynamic viscosity.

And this term is another dimensionless number you can verify that this is dimensionless indeed, you can verify that there are no dimensions the dimensions are 0 ok. And this term is something called the Reynolds number, another very important number in fluid dynamics the Reynolds number ok.

This gives you; the Reynolds number gives you the ratio of the inertial term to the viscous term. The Reynolds number is large the viscous term is relatively speaking negligible if the Reynolds number is low then the viscous term is quite important. So, this is how these things come about.

So, we will stop here for the time being and we will take this philosophy of these terms a little further. What I will do is I will instead of writing things down, instead of invoking these terms in a systematic fashion as we have done now. I will simply list out a few other dimensionless numbers that are commonly used in fluid dynamics and then we will go on so.

Thank you.