Fluid Dynamics for Astrophysics
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Lecture – 15
Energy equation in a conservative form

Hello, so we are back and today we will do a mixture of topics before we get on to dimensionless numbers and similarity and everything; I thought I would go over a few aspects of some of some of the things we have done before. In particular, we will look at we very briefly introduced the Energy equation.

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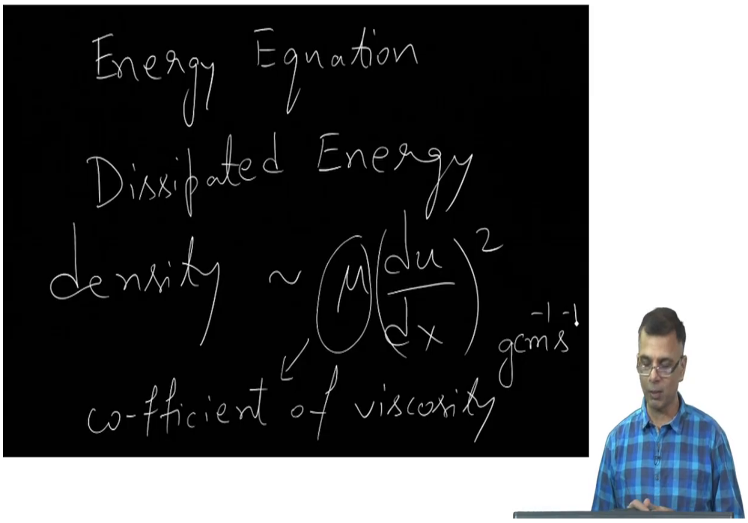

You remember, we talked about the energy equation when we finished the last time. And our main focus there was to look at viscosity and figure out that the dissipated energy; energy density went as mu; say this kind of thing right, where we really have d; d u i, d x j plus d u j;

d x i that kind of thing right. But this was the main focus and this was the coefficient of viscosity; the dimensions of which are grams like so ok.

So, sometimes they call this is the coefficient of dynamic viscosity and you will see, but keeping the dimensions in mind is very useful. So, this was the main focus of what we did last time and what we will do today is maybe write down. We also remarked that we already had seen the energy equation in a slightly different guys; in the form of a Bernoulli constant where you know you remember this right.

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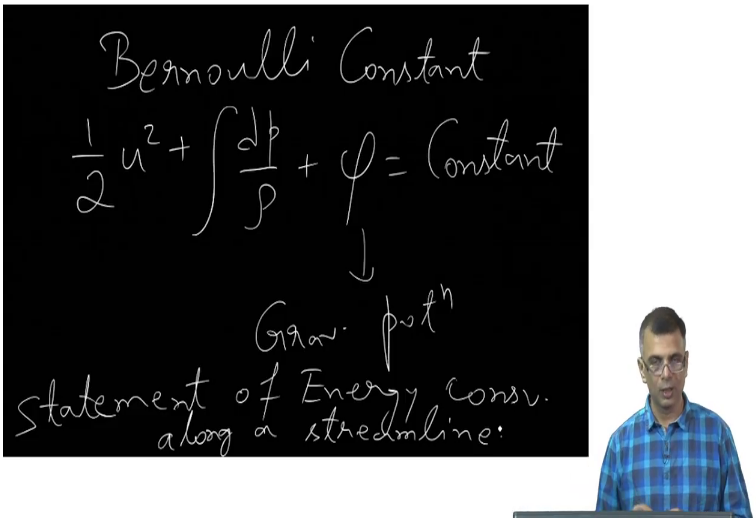

So, the Bernoulli constant which is something like 1 half u squared plus constant. This is true; however, only for a streamline and its true for irrotational inviscid flow right and this is not the velocity potential; this is the gravitational potential, please keep this in mind right. This is also a statement of a statement of energy conservation along a streamline; what we will do today; so, we along a streamline; along a streamline. So, we said these two things I just wanted to recap.

What we will do today is try to write down an energy equation in the conservative form; just like we have write written down you know the other equations in conservative form, mass conservation, momentum conservation and so on so forth.

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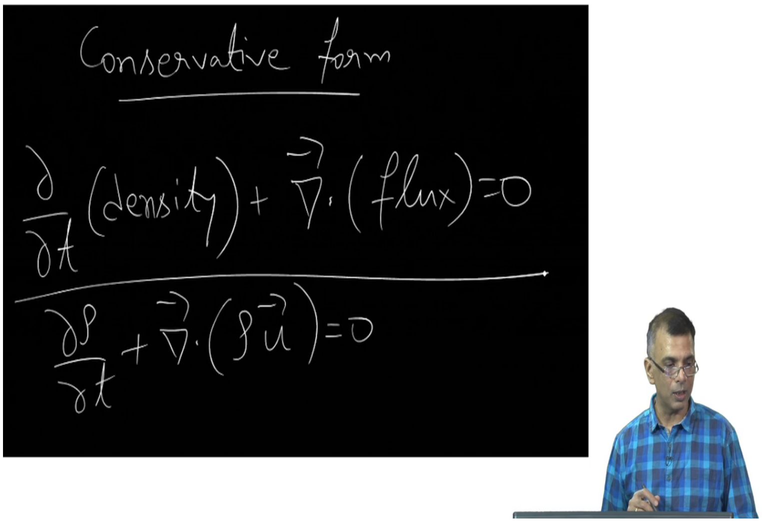

So, before that just to recap the conservative form of writing down any conservation equation is partial time derivative which means that the moment you see these kinds of partial time derivatives. You are you know in the lab frame right; of the density of whatever quantity you are talking about if you are talking about mass conservation its mass density right plus divergence of the flux of this quantity is equal to 0 right. So, suppose this was mass density; you would have and the flux of mass is right.

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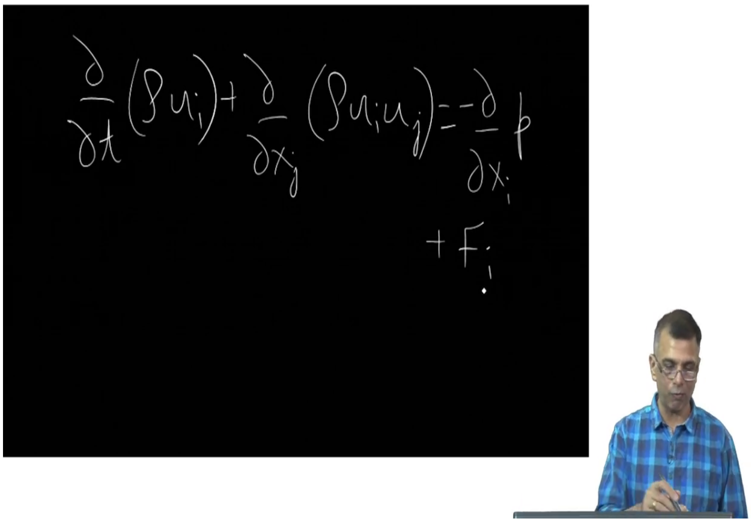

And if this was momentum density, you would have d t like that; this would be momentum density and the flux of momentum density. Now, this is divergence essentially what we are writing here is divergence rho. We sometimes wrote this as an outer product, this is the same thing is equal to if we neglect; if we neglect a node right. So, this would be the gradient of the pressure and this would be any external body forces.

So, this would be the conservative form of the momentum continuity equation ok, this is the conservative form and same thing. So, this would be 0 as such and you would be able to do that; you would be able to write down the right hand side is 0; as long as you are able to pull this term in here.

And this could be written as the gradient of a potential; in which case you could write it down exactly in the same form as this, exactly in this form to be precise; exactly in this form ok.

Now, in very similar fashion; let us try to write down the energy equation in conservative form.

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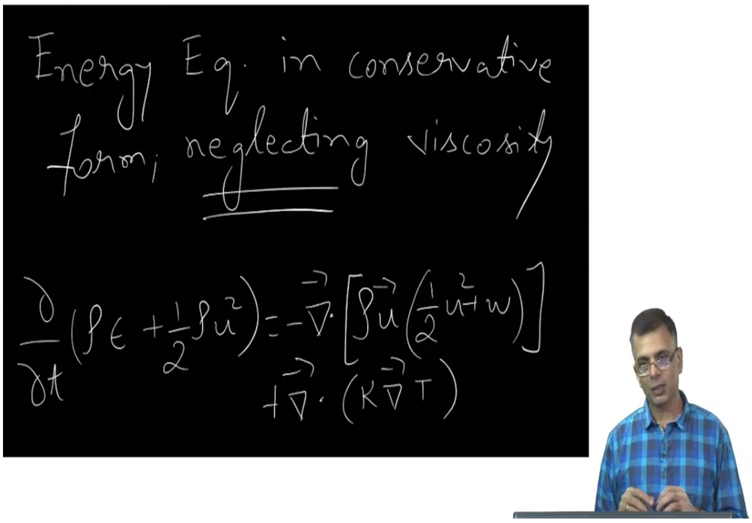

Simply by analogy with these other equations; it is useful in conservative form, but neglecting viscosity. For the time being, at least for writing it down like this; I do not want to be bothered about viscosity about viscous dissipation; this is what we focused on the last time.

But for now in order to write down the energy equation in a neat form; I would much rather neglect viscosity; knowing fully well that if I wanted to add viscosity, I would I would simply write this down on the right hand side; that is it really. So, in exactly in going by this analogy; I can write down the conservative form of the energy equation as energy density; this is what I am going to write down now right.

So, the I will write this right of the flux of the same quantity; if I wanted to I could also write, I could also add a; I will tell you what this means and like this ok. This is the energy equation in the in conservative form, so clearly what I am implying by this is that this is the energy density. And because I have got a u here; this is the flux of the energy density and this is another thing that we will come to in a second right.

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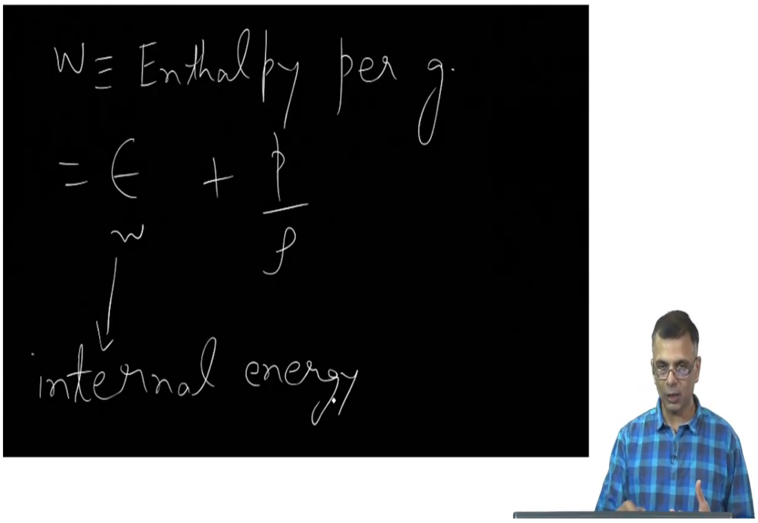

Now, what I am really saying by writing this down is that this quantity W that I wrote here; this quantity W that I wrote here is W is the enthalpy per gram or per unit mass. And this is equal to the internal energy plus p over rho; this would be the internal energy of the gas depending upon how many ever degrees of freedom it has, you would have you know the formula would differ slightly.

So, this is the enthalpy per gram; you can think of this entire thing as something internal to the gas right; so that would be this ok. So, this is the internal energy right here and this guy is a kinetic energy density of the ok. So, you can see how I am able to you know then; how I am able to simply write down the energy equation just like that simply knowing; simply going by the general form of, simply going by this general form ok.

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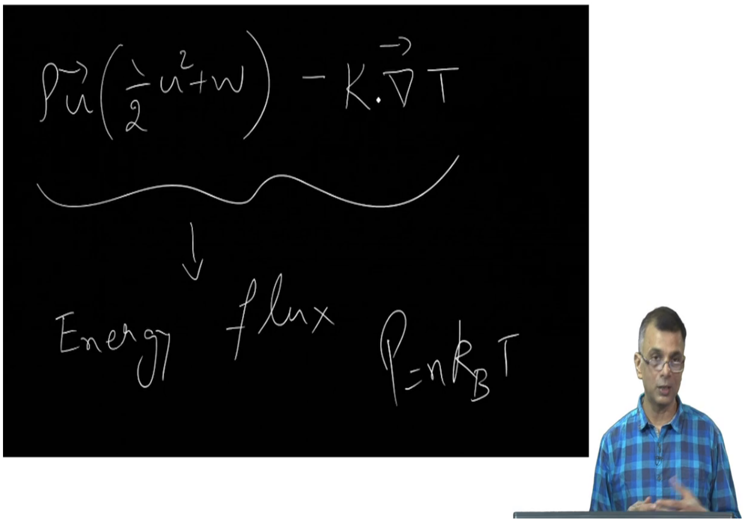

Now, coming to right; so the other thing to keep in mind is that. So, going by the same token, the quantity rho u times 1 half; u squared plus enthalpy density yeah; minus sorry, this should not have; this should not this is just a coefficient. This entire thing is the energy flux; again this is nothing new right.

So, this has to do with the kinetic energy of the bulk flow and this has to do with whatever is internal to the fluid. Remember, the enthalpy is this and this quantity that we have written a new; the T is the temperature and you would be relating T to pressure and density using maybe a P equals n k T; you know P equals this kind of a formula. So, this would be the gradient of temperature and this would be the conductivity.

So, energy can be carried; you know can be transported due to temperature gradients as well. You may or may not want to have this and if you have this; then you need an equation of state like so. Because that is yet one more variable; you only have three equations and there is one more variable; so, you need to tie things together right.

So, right so this is all I really wanted to say at this point; you it is possible to write down an energy equation in a conservative form in a form like this and that is this yeah; it does not include viscous dissipation. If you wanted to include viscous dissipation, you would do so on the right hand side here yeah.

It does not include other forms of energy generation; for instance suppose you know this was you know a flow of chemical you know; so suppose there were chemical reactions, as the fluid was flowing there were exothermic reactions; that too that is not a dissipation, that is a generation of energy. And you will be able to you know account for that on the right hand side just like that ok.

Just like you are wanted to include viscous dissipation, you do the same thing on the right hand side ok. So, that is it really this is yet another way of writing down the energy equation ok.

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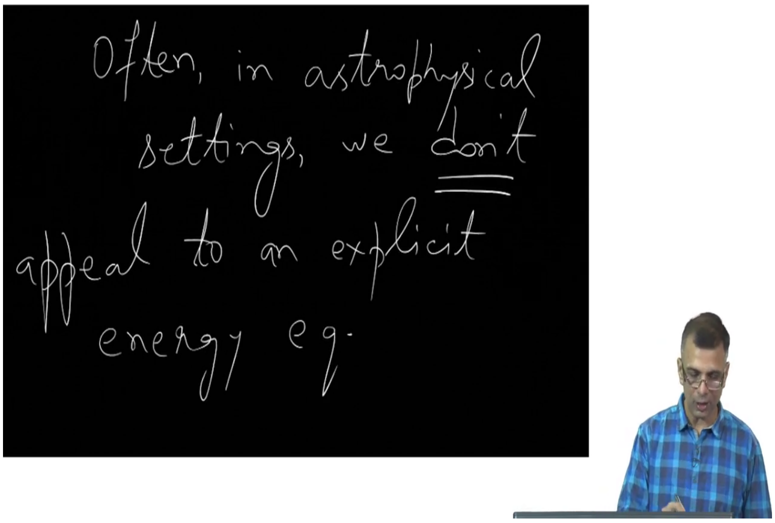

So, I wanted to focus a little since and one final thing before we leave this topic. Often times in astrophysics, since this course is oriented towards astrophysical settings; we do not appeal to an explicit energy equation of the form that we just wrote down, instead we just use something like P equals.

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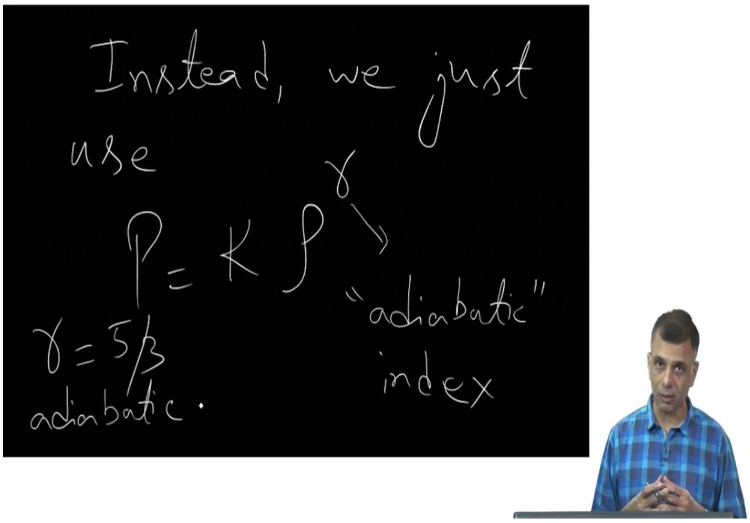

Now, this K is not the same as the other K that appeared in the conductivity where this would be an adiabatic index. And I deliberately write down the adiabatic; you know in quotes to imply that this need not be always 5 thirds; for always 1, as would be for a isothermal gas; it can be anything in between.

But the point is you can relate temperature and sorry; you never mind temperature, for the time being we are not introducing temperature; we can relate pressure and density like this. In which case you eliminate the actual need for an explicit energy equation and everything all the information about the evolution of energy is enclosed in this index right here ok.

And if gamma equal to 5 thirds; adiabatic in other words the system is closed and any PDV work that you do appears in internal energy and vice versa. And if gamma is 1, then it is isothermal and in principle it can be anything in between. And it is this gamma which tells you everything about the energy processes energy; dissipation processes.

So, many times you know in astrophysics as I said; we do not really appeal to an explicit energy equation instead we use something like this. And it is just the way people do things; it is cheap, you can call this a cheap energy equation instead of writing down an explicit energy equation; I get away. Well, from mathematical point of view; you can see if you tie down these two variables like this, there is no scope to have one more equation like an explicit energy equation.

Otherwise, you would be over determining things; so that is one way of looking at it. The other way of looking at it is; all the physics of energy dissipation or generation or whatever is hidden in this index right here. So, this would be a polytropic law; as you know and the polytropic index, I should not call it an adiabatic index; I should call it a polytropic index because that is what is used in the literature many times.

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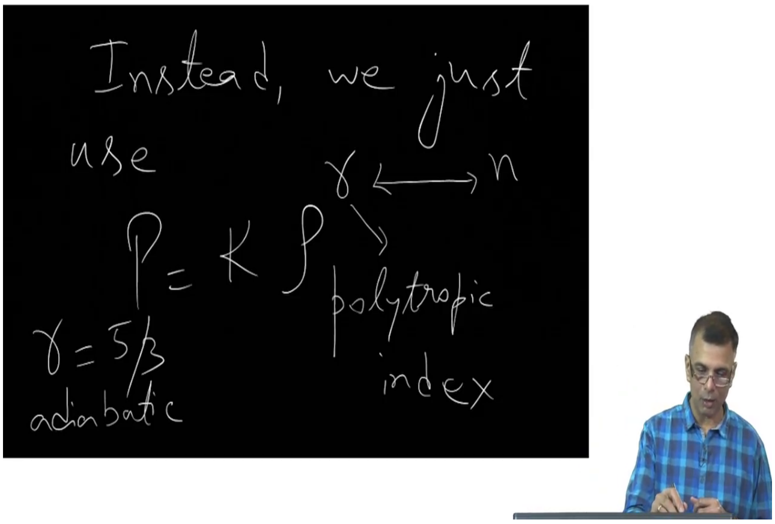

So, the only thing is often; it is the polytropic index is not denoted by gamma, it is denoted by n ok; so like so. So, all the physics of energy generation or dissipation what have you is encapsulated in this polytropic index right. So, that is; that for the energy equation and I thought I would next start talking about this was kind of where we wrapped up the last time.

And the next thing I want to start talking about is go back to the Navier Stokes equation right and talk a little bit about boundary conditions. How the inclusion of the viscous term now we go back to viscosity again and how the inclusion of the viscous term alters things greatly; especially with regard to boundary conditions. So, we will stop here for the time being.

Thank you.