Introduction to Classical Mechanics
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Lecture 51
Sleeping Top

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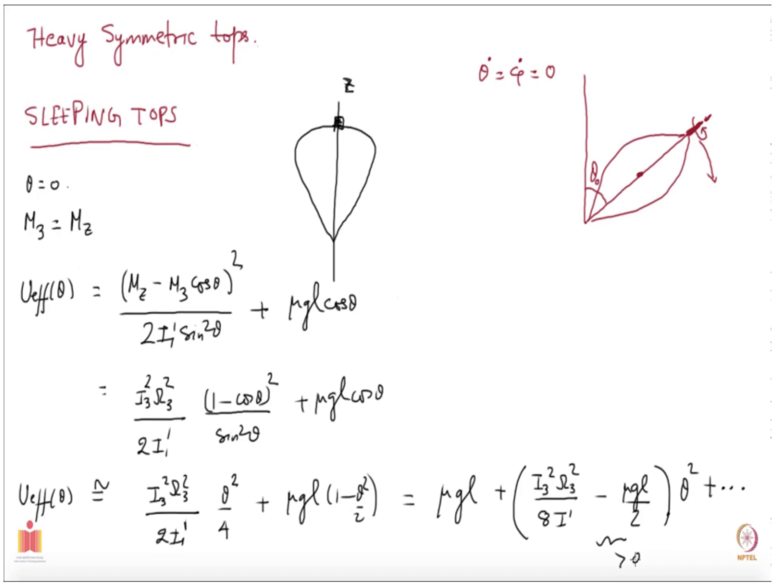

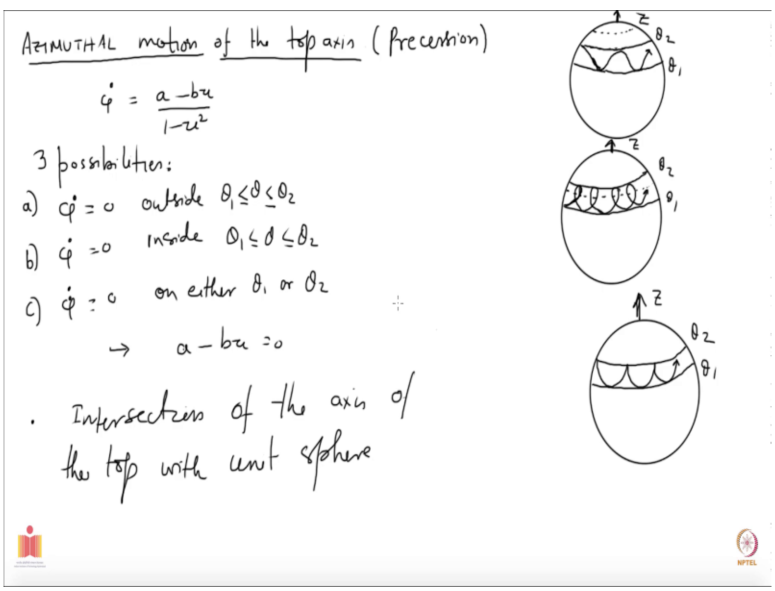

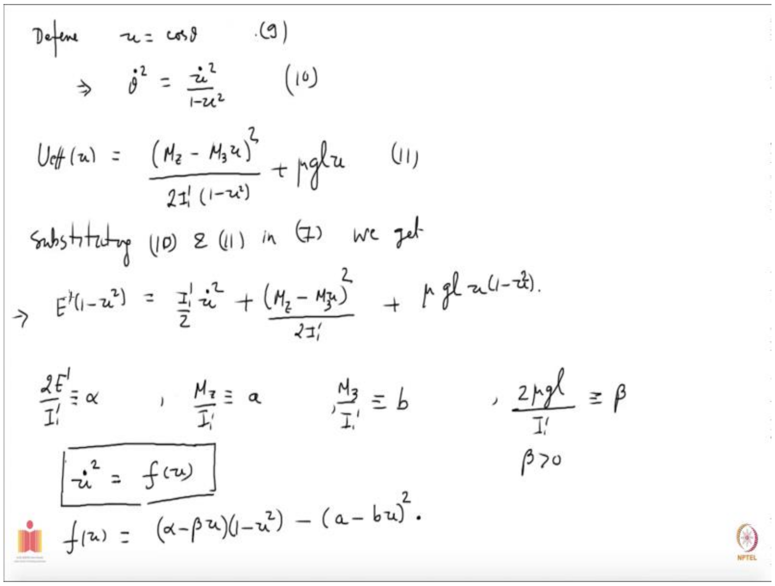

Let us, continue with our discussion on symmetric tops. We obtained the following results last time. So, we saw that the top axis can I mean make these kinds of motion which were projected on a unit sphere. So, there are these three possibilities corresponding to phi dot being outside the theta 1 and theta 2 the limiting values of theta 1 and theta 2 or it could be inside or could be on one of the limits of the theta.

So, there were these three possibilities which we looked at last time. If you look at the last possibility where phi dot 0 on theta 1 and theta 2 this is a case which happens when let us say you start your top in the following fashion. So, let us say you have, let us say this is the vertical and this is the axis of the top, so you can imagine you have a top like this here and there is let us say some metal part is coming out and this has been fixed here.

So, you can imagine that you can bring an external device which could rotate which could attach to the axis and this this device can make this top rotate about it, about this axis. So, the top starts spinning about this axis at the fixed angle whatever theta is here, so it is spinning around this axis. So, the initial condition is that you are theta dot is 0 and also your phi dot is 0, because that is all fixed, the motor is not letting anything this device is not letting these change. So, it is fixed there and start spinning.

Now, question is what happens when you remove this external agent. What happens is that it will immediately start dropping down. So, this will start falling the top meaning the theta will start increasing. You should convince yourself that it has to decrease and not increase. Because that will that will not be allowed by the energy conservation.

So, that is one thing so it falls down, it comes down and then again goes up and then comes down and again goes up and that is what is this possibility this. So, this corresponds to this setup. So, that is one thing now today I wanted to talk about sleeping top. Let me, by sleeping we mean that the top which seems to be doing nothing, meaning it axis remains wherever it is and does nothing that will be a sleeping top.

So, imagine we are looking at theta equal to 0. So, the top is in a vertical position. So, here you see our vertical and let us say here is our top, something here and if it is made to spin in this position, then clearly theta is 0 and my M3, which is the component of M along the third axis of the body will be same as Mz because M3 and this Z they all coincide and our E prime if you calculate this will be 0. Because the entire energy rotational energy it will equal the rotational kinetic energy due to along the third axis.

So, it will is going to come out to be 0 but not so important anyway for us. Now, let us look at the effective potential energy. Which we wrote down some time back. Where is it? Here this is in terms of u the equation 11, but I want in terms of this theta, so this is the expression we had let me write it down again.

Our U effective of theta is Mz minus M3 cos of theta the entire thing squared divided 2 I1 prime sin square theta and then you have the gravitational term mu gl cos of theta, you can now as you know that Mz and M3 are same you can pull out and M3 which you can equate to I3 half I3 omega 3. So, if I pull out M3, it will become M3 square because of the square and you will have half I3 omega 3 square and that, there should be no half M3 is just I3 omega 3.

So, you have I3 omega 3 both of these squared and then you have 2 I1 prime coming from the denominator. Then you have 1 minus cos theta squared over sin square theta plus mu g l cos theta, I want to check whether, I have an equilibrium position at theta equal to 0. So, I will expand it around theta equal to 0.

So, U effective theta for small theta would be I3 square omega 3 square over 2 I1 prime. So, cos theta you can approximate to 1 minus theta square over 2 which will cancel this one and you will have here let me check, yes you will get theta 4 over 4 theta square. So, you will have sin square theta as theta square and you will have theta to the 4 over theta's power 4 over 4, which will give you theta square by 4 and then plus mu gl cos theta is 1 minus theta square by 2.

So, 1 minus theta square by 2. In this I want to write as mu gl plus I3 square omega 3 square these two factors here then I have a theta square in these two terms. So, which I take common and you have 4 8I prime, 4 into 2 is 8 I prime that is correct and then you have mu gl over 2, over 2 times theta square plus higher order terms.

So, this is the constant piece I am not bothered by that and whether their equilibrium at theta equal to 0 is stable or not is determined by this coefficient. So, if this is positive that would mean that the potential energy increases as you increase theta and then that would imply that equilibrium is stable. So, we have to ensure that this is greater than 0.

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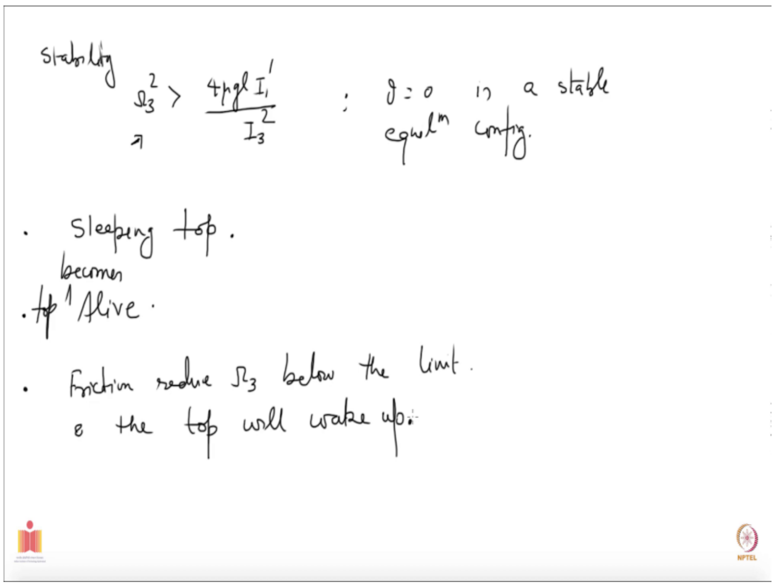

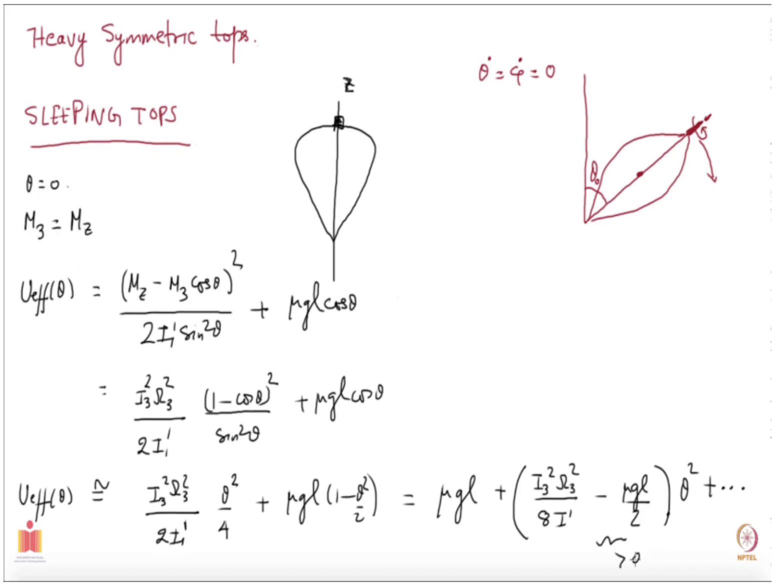

Which means that if or condition for stability is omega 3 square greater than 4 mu gl I1 prime over I3 square. Let us go back. So, this is omega 3 square and this entire thing should be greater than 0. So, I bring it to the other side and this is what I get. So, if this is satisfied then theta equal to 0 is a stable equilibrium configuration.

So, if you start your top in theta equal to 0 direction. So, again imagine you have your heavy top and it is being made to spin by some external device, which is holding it in the theta equal to 0 position and when you gently remove this device and depending on what omega 3 has been given to it. If it satisfies this condition, then it will just remain spinning at theta equal to 0. So, that (())(10:37), then we will say that, then we will say that the top is sleeping. Sleeping meaning its axis is doing nothing. It stays put at theta equal to 0, no precision, no nutation nothing is there that is called a spinning top.

But what will happen in reality is that after a while because the point which is the fixed point which is in contact with something. It will because of the friction there or even be with the friction with the environment air let us say, the spin will start decreasing with time and the moment spin gets below this value omega 3 the moment it gets below this value the top will become alive, top becomes alive meaning it will no more remain at theta equal to 0 its axis will start doing all the things that we have seen before which we have learned here.

So, that is what for the sleeping top let me just write down about this, so friction will reduce omega 3 below the limit mentioned above and the top will wake up. So, I think I would stop the discussion of tops with this, there is lot more literature that is available on tops and gyroscopes and I would encourage you to look up a in at some of these textbooks which have little bit more of discussion on this and also you can see more references in them and which will lead you to much more discussion on the tops. So, I will finish this discussion here and we will take up other topics next time.