Introduction to Classical Mechanics
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Lecture-4
Virtual Work (rigid body)

In these lectures, whenever we were talking about constraints, we were imagining a surface over which particles are constrained to move. But that is not the only way in which constraints can arise. There are several other ways in which they can appear and 1 of the very familiar once is what you see in a rigid body. Okay, so in a rigid body, the distances between the particles which constitute that body is fixed. So, that is a constraint. A simpler version of that would be, imagine 2 particles which are let us say connected by a thin rod.

Okay, and the distance between these 2 particles always remains the same. That rod is just to make the visualization simpler. So, that is an example of a constraint. So, I will take this example and we will calculate the virtual work. And we will see that even though the forces on either of the particles does not give a nonzero virtual work, but if you consider the pair as a whole, then the virtual work is 0. So, that is what we want to see in this video.

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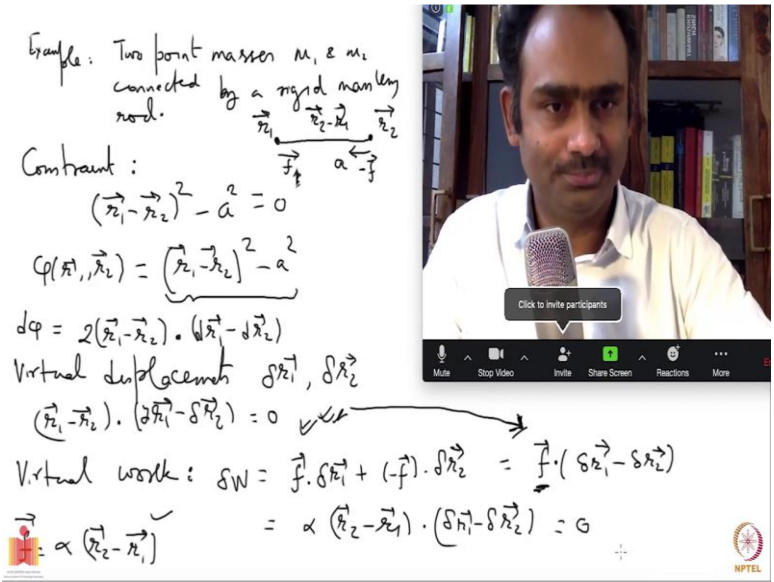

So, we, our example is, so I have two-point masses m1 and m2. And they are connected by a rigid massless rod. The system is really the 2 particles, the rod is just a fiction to help imagination. Okay, so what is the constraint we have? Constraint is that let us look at the two particles, m1 and m2, and the distance between them is let us say a. So, the constraint is that r1 so I am saying with respect to some origin that you choose, the location of this guy is r1. Okay. And the location of that guy is r2.

So, the constraint is r1 minus r2, that is this vector. Or you can see r2 minus r1 Square, that square gives you the length of the separation between r1 and r2 is equal to a square or minus a square equals 0. Okay, that is our equation of constraint. So, if I want to write down in terms of phi, which we had last time, that the equation of constraint, I would write the function phi of r1 and r2, there is no time dependence here, so the distance is fixed.

You could imagine another case in which the distance is changing with time in a specified manner. That would also be a constraint, a time dependent constraint, but this one is time independent. So, phi r1, r2 is r1 minus r2 square minus a square. Now, let us look at the differential of phi as we used to do earlier.

So, d phi would be d of this quantity, which will give you immediately 2 r1 minus r2 dot d of r1 minus r2, which gives you d r1 minus d r2. So, our virtual displacements will satisfy. So, if I write delta r1 and delta r2 as the virtual displacements, as you already saw in previous videos, that the virtual displacement has to satisfy, instead of d r1 and d r2, I put delta r1 and delta 2 and equate the d phi to 0. So, this becomes r1 minus r2 dot with delta r1 minus delta r2.

And this is what you should have as equal to 0. So, that is the constraint you have on the virtual displacements. Because say, if you take delta r1 to be something the delta r2 is fixed by this relation. Okay, that is good. Now let us find out the work done in a virtual displacement. So, let us find out the virtual work. Okay, so let us look at these particles. If r2 is pulling with force f1, then r2 is being pulled by the same force f1 but in the opposite direction. So, the force in 2 is minus f1. Let us remove 1 there is no, there is no need to put a 1 here, so I will just remove the 1.

So, the force on r1 is f, the force on r2 is minus f, that is the situation you have here. And let us calculate the virtual work delta w or whatever you want to call it. So, the force is f on r1, it is getting displaced by delta r1 plus force on point 2 is minus f, is getting displaced by r2. Which is you take out the fs common f dot delta r1 minus delta r2. I hope everything is fine till now. Now, delta r1 minus delta r2 is constrained by this relation. So, I take this one and put in here. Okay, that is what I do. So, this becomes f dot.

Yeah, before I do that, let me, I jumped a little bit, up to here it is good. Now, let us look at the force. The force is in this direction, Okay, and what is this direction? This is just the direction of r2 minus r1. This is r2, this is r1 if you subtract, that is the, that is this vector. So, our force f, that was fine here. If you look at the force f, that is something proportional to r2 minus r1.

The proportionality will depend on the strength of the force, but the direction is given by this vector which I am writing here right now. Now, I substitute this fact in here in this. And I get alpha times r2 minus r1 dot delta r1 minus delta l r2. And I hope you have already realized that this is 0. And this is 0 because of this relation. You see r1 minus r2, which is minus of r2 minus r1 and then the dot product of, dot product with these displacements, that is 0.

So, if you look at the forces in this system in pair, then the virtual work done in any virtual displacement is 0. So, this is another example that we know of now. And clearly if you are looking at a rigid body which is just a collection of particles whose distances are fixed, the same argument which are used here, you can use their, make pairs and you will realize that immediately the virtual work done in any virtual displacement would be 0. Okay, that is good. We have now some familiarity with some other cases.

Let us go to next thing which I want to tell you. Okay, before I proceed further, let me tell what kind of constraints will be considered in this course. So, we are going to restrict ourselves completely to only some specific kinds and they are of the following nature. So, whatever constraints you have, they should be, we should be able to write them in the following form. So, I should be able to write the functional forms of them.

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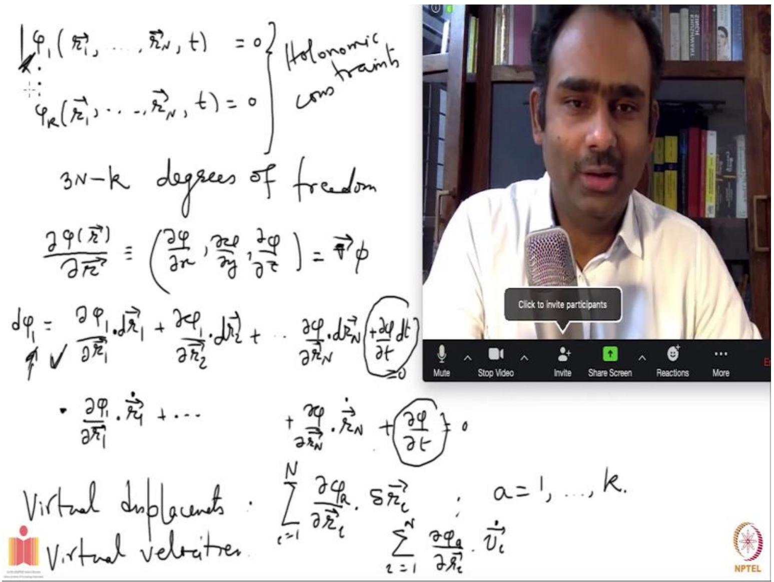

So, I should be able to specify let us say you have k number of constraints, so, I should be able to write phi 1, r1, let us say you have n number of particles r n and the constraint may depend on time okay. Similarly, we will have total number of k and I mean k independent. It should not happen that the constraints are dependent on each other okay. So, they should be really independent constraints research. And these are the only ones which we will assume that they are presenting this course.

Now, if you have such constraints, we say that these are holonomic. So, we say that the constraints are holonomic. If your constraints cannot be put in this form, meaning you cannot write down functions specifying your constraints then they are not holonomic. There may be, there are different kinds of non-holonomic constraints, we are not bothered with them, we will only restrict ourselves to these. Okay, note that I have allowed for time dependence in here, because this time dependence is fine for us.

Now, I will tell you a little more about what kind of constraints would be non-holonomic. At least one set. So, let us say let us say we are given these k number of constraints, which means that my system now has three n minus k degrees of freedom. Because you have n number of particles, which is clear because I am using r1, r2 and rn. So, if total number of particles, each of them has three because of XYZ, and then I have specified k constraints for you. So, you will be able to eliminate k of them and you are left with three n minus k degrees of freedom.

Now, let us take the differentials of these k functions and write the differential forms. So, if you look at first one, phi one, I will be able to write d phi 1 which will be 0 as the following, just like what I did before. Maybe I, let me introduce a slight notation, before I write this, I want to define. So, let us say you are given some function phi, which is a function of some vector r, which could be a position vector.

Then if I write del over del r, what I really mean by this notation is delta phi over delta x, delta phi over delta y, delta phi or delta z and this must be very familiar to you, this is nothing but the gradient of phi. And this is a very handy notation. So, sometimes I will use gradient to phi or sometimes I will use del phi over del r, this notation you can find in for example, in (())(14:16) or other books, some other books. So, now if I want to write d phi I can utilize this notation which I told you just now.

So, I have to take partial derivative with respect to all the coordinates. So, it becomes delta phi, phi 1, okay delta phi 1 over delta r1 d r1 plus delta phi 1, so I am looking at first constraint still delta r2 d r2. So, they have to be dot products here, because you are dotting two vectors and delta phi over delta r and dr end. And you will have a time derivative term as well, let me write down maybe that also delta phi over delta t dt and this all should be equal to 0.

Now, if I divide by dt, I can write this also another relation, which now constraints the velocity. So, delta phi 1 over delta r1 dot r1 dot, so on and so forth, plus delta phi over delta r end r n dot plus delta phi over delta t, this is equal to 0. So, that is fine. But if I and if I want to turn these into virtual displacements and virtual velocities, I will just drop this term, you remember, this is what we talked last time. So, these 2 you drop and your relation will become the constraint which virtual velocities and virtual displacements should satisfy.

That is one thing, so let me write in shorthand. Your virtual displacements. they satisfy delta phi over delta r i dot, so I am using the notation which is for virtual displacements, and now, i runs from 1 to n for all the particles. And I put a label a because there are a number of constraints, a runs from 1 to k. And for your virtual velocities, you will have summation delta phi a over delta r i into vi dot. That is good.

Now, why am I interested in only holonomic constraints? Okay, the reason is that if you have holonomic constraints. I jumped a bit again. I just should have made one more remark here. So, see the constraints, let us say you are not given constraints in the form of functions which I wrote, which I wrote here. Let us say you are given constraints in the differential form. So, you are given constraints in this manner. So, you are given k equations involving the differentials.

Now, if you are given a set of constraints in the form of differentials, it is not necessary that you will be able to integrate them and write them as functions. So, there may not exist integrating factors which will allow you to turn them into functions. Now, if that is the case, then your constants are not holonomic, because we need them to be written in terms of functions. So, they will not be holonomic constraints and that is why for your constraints to be holonomic and if they are written in differential forms, you should be able to find integrating factors which can turn these ones into these ones.

So, let us say we are given holonomic constraints. Now, why are we so much interested in holonomic constraints? The reason is simple. If I am given holonomic constraints, then I can eliminate the dependent ones okay and write down or specify my system only using coordinates which are independent of each other.

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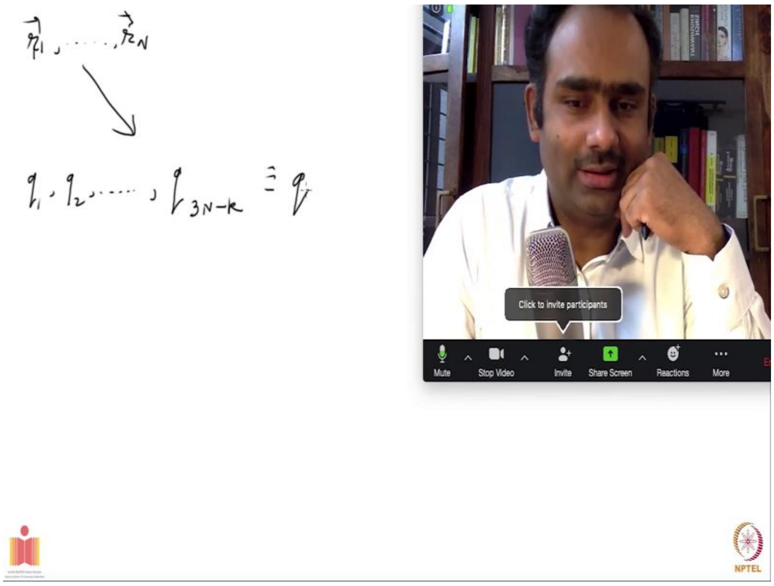

Which means, if I am given holonomic constraints, then I will move from r1 to rn, I will not use them, instead, I will use q1, q2, so forth q, how many, I am saying the qs are the independent coordinates. So, you remember there are total of three n minus k independent coordinates. So, let me call this set as q. So, I will be able to phrase my problem in terms of these independent coordinates and that is why I am interested in holonomic constraints you.

If your system is not holonomic, this you will not be able to write down. So, now, we are almost there to start writing down a very important principle, which is called the d'Alembert principle, that will be the subject of next video. And from the d'Alembert principle we will be able to write down very nice form for the equations of motion. And what we have discussed here will be utilized there because you will see that the system will be described using these independent coordinates which are called generalized coordinates. And that is the plan for the next video. Okay, see you in the next video.