Introduction to Classical Mechanics
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Lecture 37
Rigid Body, Degrees of Freedom

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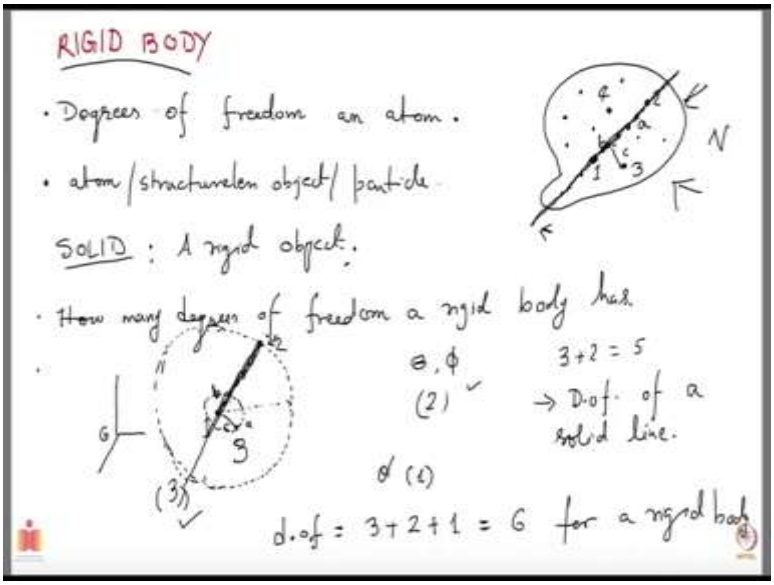

Till now in this course we have covered several topics. We talked about oscillations we talked about two body problem; where we looked at Keplers problems and another things. Now, we will move on to our topic which is Rigid Body Dynamics. And here we are going to start asking very simple things about the rigid body, and that is the plan for this video today. But, even before I talk about a rigid body let me talk about an atom.

I want to ask how many degrees of freedom an atom has; I am asking this because however rigid bodies are made up of atoms. So, let us ask what are the degrees of freedom of an atom? Now the answer depends on what you mean by an atom. If you mean by an atom is structureless thing, which has no internal structure; there are no parts to it then it has 3 degrees of freedom.

You need to tell is where that guy is located in this space time in this space. So it will have 3 degrees of freedom. But, if you want to view your atom as something which is made up of a nucleus and an electron then the degrees of freedom changes immediately. It has more degrees of freedom compared to structureless things.

But, let say if you want to view your atom as something as an electron and nucleus as far as nucleus is concerned you want to allow for more structure to it. That is you want to tell that nucleus has protons and neutrons, then the degrees of freedom have are even more compared to what it would be for just a nucleus and an electron. You may want to go further and you say it is not just protons, neutrons and electrons I know further that protons and neutrons are made up of course of gluons then the degrees of freedom further increases.

So, it really depends on what you mean by an atom, which is going to decide how many degrees of freedom that atom has. So, in this well we are talking about rigid bodies, when I say an atom I mean a structureless quantity, which has 3 degrees of freedom. Usually, to be more precise I would instead of an atom, I would say a point or a point particle and all these three will mean the same thing.

So, for me when I am saying an atom it is a structureless thing; structureless object or part or by a particle you always mean a point like particle. That is the meaning of a particle. Now, whether you should treat your atom as a particle or something made up of a nucleus or nucleus and an electron or something with more details regarding its nuclear structure.

Depends on what you are looking at, depends on the scale which you are doing physics. For example in some case it may be completely pointless to even think of what the structure of atom is. So its really determine by the problem you are looking at and you cannot say a priory whether you should look at an atom as a point of point like object or not.

Now, similar things you can talk about say about the solids, so let say you are given a solid. Now, how many degrees of freedom does this solid have? Let say it is made up of n atoms now how many degrees of freedom does it have? Well, as I said just now it will be depend on the details. But, let say I start to think of this solid as made up of atoms which are point like, without any structure.

Now, how many degrees of freedom does this solid have if it has n such atoms? Well still depends on what you want to see about that solid. Let say you are interested in only throwing it up in air or in space and you just want to see how it moves around. If you are interested in only that much, then you can regard your solid as made of particles or atoms whose distance from each other are fixed, that is what you can regard it as.

But, if you are interested in taking up the solid and hitting it up; then that description which I just said is not appropriate. Because you cannot heat up such a solid it will not absorb any energy; there is no way you can give energy to such solid. I mean you cannot give energy by hitting it up because there is nothing internal which can change. So, a description of a solid as a set of particles whose distances are fixed, is not suitable for a studying its properties under thermal exchanges.

But if that is not your concern and you just want to ask how it moves around as a whole then you consider those distances to be fixed. And you can regard this, okay this is not, yes that is correct solid, you can regard it as a rigid object. So, by rigid I mean the distances between different particles of the object are not going to change with time; and that is what we mean by a rigid and as I said this may or may not be suitable for the problem you have at your hand.

So, let say we are interested in rigid objects. The first question we would like to ask is how many degrees of freedom a rigid body has; so that is our question. How many degrees of freedom a rigid body has? So there are many ways in which you can do this and I am going to do it in following manner. Now, suppose this is the rigid body which is given to you, the one which is marked here.

Now, the way I am going to count the degrees of freedom for this rigid body, is by building it up from scratch. So, I am going to assemble all the points which are in there, this is made up of several points, n points let say. I am not taking a continuous thing, but I am taking a discrete things. So, let say it made up of n particles and its distances of these particles from each other are given. So, I am going to now build this up from scratch and doing so I count the degrees of freedom.

Let us choose anyone of these particles which we like. Let us say this one. Now, suppose I want this body to be move to a new location, and I want this point, this particle to be here. So, I have some origin some system of coordinates and I bring that particle to here. So, I am imagining taking only one particle and putting it to here; so I want this particle to be here. How many degrees of freedom does this have?

Meaning how many numbers actually you specify, how many numbers I need to tell where this is located; ofcourse 3. So, 3, 2 tell where this guy is. Now, let us take any other particle, let say this

one. I want to bring it here because I am building the body, and this distance is given to me, it is already fixed. Let say this guy is a distance a away from this one let us say this is particle number 1, this is particle number 2.

So, I have already bought my particle brought my particle number 1 to the right location. Now, I want to bring in my particle 2. Now, particle number 2 is at a fixed distance a away from particle number 1, so let me construct a sphere of radius a around particle number 1. So, that is my radius a and we can construct a sphere around this.

Let us try doing this because I have been drawing ellipse, is becoming an ellipse instead of a circle. So, this particle number 2 could in principle be anywhere on the surface of this thing and it will be still a distance away and it will still be the part of that rigid body. Because all that particle has to do is b away distance a away from particle number 1 because you can take this body and turn it and move it around and the location of particle number 2 will change.

Its orientation will change, but the fact it is a distance a away from is not going to change. So it is it can leave anywhere on this surface. So, to tell that I want the particle number 2 to be here I have to specify two things. Remember this is sphere, not a circle, I have specified two things. I have to specify two angles which will fix the location of this particle.

So, you have to specify for example the theta and phi, the polar and (())(12:02) angles for in this sphere. So you choose a direction and if you specify those, then you will have told where this guy is located. So, this is next two, first we had three numbers for first particle then for the particle number 2, I have used up two numbers. Not used up, but I need the two numbers to specify where it is. So, as far as this body is concerned, right now I know about these two particles but, there are lot of n minus 2 particles still remaining, their total of n.

Now, let us look at all these particles which are on this line all these particles. How much how many degrees of freedom is going to be taken up by telling where this particle is or where that one is. Well, nothing you do not need anything, you do not need any number to specify this because they are all going to be completely determined where they are just because you have fixed the particle number 1 and 2.

I hope that that is clear there is no, there is nothing really in your hands. These particles cannot be anywhere else if your particle number 1 and particle number 2 are fixed they have to be on this line. So, let say this guy was at some distance b away from the first particle; this is distance b away from this particle first particle. It cannot be anywhere else it has to be on this line a b distance of a it is fixed.

So, there is nothing you need some for this, so our degrees of freedom still remain the same as far as our all these particles are concerned. So, if you draw a line connecting point number 1 and point number 2; then all the particles on that line they are completely specified, their locations by specifying these 3 numbers and these 2 numbers.

So, 3 plus 2 is 5 and clearly that is the degrees of freedom of a solid line. This entire line is completely fixed; all the particles on that line are completely fixed. If you specify these five numbers; so line has 5 degrees of freedom. We are very close, we have built up a whole object but I have at this location now this this line, this line is same as this line here, these two are same things. Now let see how much information is require to bring in another points. Now, let say some point here, some distance c away from that line; and let see how much freedom this point c has.

Now once you have specified this line, this is the direction in which all these particles are oriented and point number 1 is here. And you are talking about a particle, which is a distance c away from the particle number 1 or let say from that last line. Now, if you just tell me that this is this is let us call it as particle number 3, let us call is called particle number 3 and the distance is c, so this is particle number 3.

Now, this particle number 3 if you tell me that it is distance c away from this line then it could be anywhere on the circle, who centralize here, see with this distance fixed you can draw a circle around this point, let me draw it, try to draw it. This is a circle which is going into the plane of this your laptop or screen. So, in principle this particle could be anywhere on the circle, and it will have the same distance c.

But, if you want to tell that I want no particle to be here or there then you have to tell one angle. To tell where you are on a circle, you need to specify one angle; let me call it theta prime because I have already used up a theta here. So, you need one more number to tell where you are

on that circle; that is nice. So I know how to also bring in this point. Now, any other point do I have still have some freedom?

Let say I have fixed, once I have fixed point number 1, point number 2 and point number 3. Can I do anything else, can this point 4 be anywhere else, if I have fixed location of 1, 2 and 3. And the answer is no, it cannot be anywhere else, it has to be here only; and this point has to be here only, these are all fix now. Now, once you have specified the locations of these 3 points, all other points in the rigid body are completely fixed you are not left with any freedom.

So, it means that to specify your rigid body, where it is or equivalently to specify all these n points where they are. If they are all rigidly connected, you need to tell me the following numbers. I mean the following these many numbers, so you need 3 to specify the coordinates of first particle which is 3. Then the coordinates of second particle 2; all the particles on that line are fixed. Then for any third particle whichever you like you need one more that angle on the circle.

And there is nothing else left, all particles are completely fixed in space now, and this is fixed, so you have 6 degrees of freedom for a rigid body. Now, sometimes it will happen that your rigid body would be let us see what has happened; just a second, we are here.

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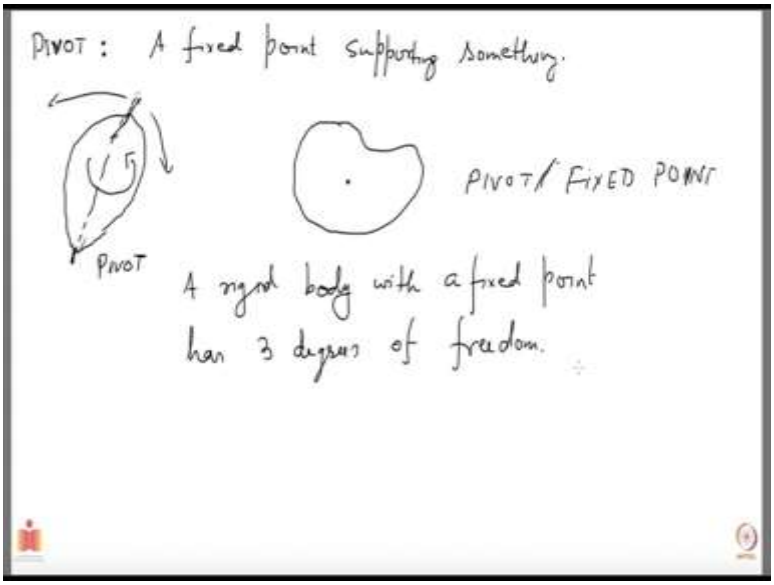

So sometimes you will have your rigid body which will be pivoted, so there will be a pivot let me write this word pivot. Sometimes things appeared more difficult just because the words are unfamiliar and I remember not knowing this word pivot and being scared of this.

So, pivot let me write down the dictionary meaning from Cambridge I think. Pivot is a fix point supporting something that turns or balances something like this; a fix point supporting something, that is sufficient for us. So, imagine for example you have let say, what you say, let say a top, and this is its nail. But you fixed the nail to be here it cannot move anywhere other from this location.

But other than that the top can be do anything, so it can let say this axis, this is the axis of the symmetry axis of the top. So this axis can go there, can go there with this point still remaining fix. The top can turn, all these things can do; but this always remains point is a pivot. You may not necessarily have cases where the pivot is like this you may have a fix point somewhere in the body. So, you can fix a point somewhere in the body and then you ask about its motion.

With that point always being there stationary; that could be a pivot. Pivot will be again a pivot or more easily I will just say fixed a point. So, for this ofcourse you cannot imagine like putting some wire or stand something that will not because then it will immediately fix many other points. So, you have to just imagine that somehow that point is holding there.

Okay that is fine now you may ask how many degrees of freedom does rigid body has; does a rigid body have if it is pivoted, if it has a fix point. And clearly it will have 3 degrees of freedom because remember when we are talking about…when we started a building it up, you need 3 degrees of freedom for one point so that is those 3 degrees of freedom that I am going to take away.

So, if it is not immediately clear, you please sit down for a while and I am sure you will understand why it has 3 degrees of freedom. So, a rigid body with a fix point has 3 degrees of freedom. One way to think about this would be you just imagine your origin to be here. And then run through the argument which I gave for the general case, and you will arrive at 3 degrees of freedom. So, will stop here in this video and will continue about this subject in more detail in the next video.