Introduction to Classical Mechanics
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Lecture No: 01
Two-body problem, Conic Sections in Polar Coordinates

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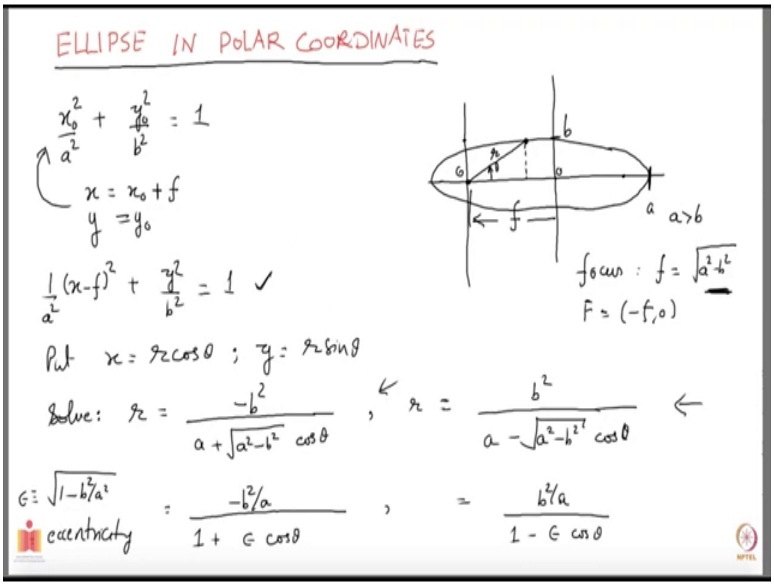

Recall what the equation of an ellipse is which is centered at the origin. If you remember it is x square by a square, y square over b square this is equal to 1. So, that is the equation of an ellipse and as you can see that if you scale the x and y coordinates appropriately, you can absorb the, a and b into the coordinates themselves and that will become an equation of a circle. So, we can turn ellipse, an ellipse into a circle by scaling the coordinates, x and y coordinates appropriately.

I want to, so let me draw this ellipse first, that is our x axis y and so, this is our ellipse which has focus here and there, and the focus is at, so this length from here to this place it is not looking in the symmetric but it is okay, this length if I call f this is at a square minus b square, square root. Because, a is going to be greater than b for us this is why? And this this point is a.

So, this is 0 that is a and this point is b, just fine and our focus f is at minus f comma 0 so, I am looking at this one right now, it is good. Now, I will call these x and y coordinates as x0 and y0. So, these instead of calling them x and y, I will call them x0 and y0 because I want to shift my coordinate system to here. So, I want to shift the y axis to this point, so my new origin is here at the focus which means that you should do the shift of x axis to new x according to this relation.

So, I can say if an x naught is minus f your new x is 0 so, which is correct so, you origin is here now. And to get the equation in polar form, all of you have to do is substitute this in here and then use the polar transformation so, also let me write down this so, you get x minus f where f is given by this relation whole squared, 1 over a square plus y square over b square is equal to 1.

Next you put x is equal to r cos theta and y is equal to r sin theta so, what we are doing now is you are measuring things from this origin, new origin this is your origin let say the distance here is r and if you drop it this horizontal distance is r cos theta and the vertical is r sin theta That is all you have to substitute and then you solve it and you get two solutions, you get r equal to minus b square over a plus a square minus b square, square root cos of theta that is one solution, and the other solution is r is equal b square over a minus a square minus b square, under square root this entire thing multiplied by cos of theta.

That is what you get, we can, we can rewrite this as the following, minus b square over a and you can write it in nicer form then what I am going to write here but, it is not necessary for me to do that, you get 1 plus epsilon cos of theta, I will define the epsilon right straight away and here you get, b square over a 1 minus epsilon cos of theta, where epsilon is 1 minus b square over a square.

And it is called eccentricity, let us look at this if b is equal to a then epsilon is equal to 0 and eccentricity would be 0 and your ellipse will transform into a circle. So, it is not at all eccentric, it looks same in all the directions, this is what you get. Now, let us ask, so these are the polar forms of your ellipse.

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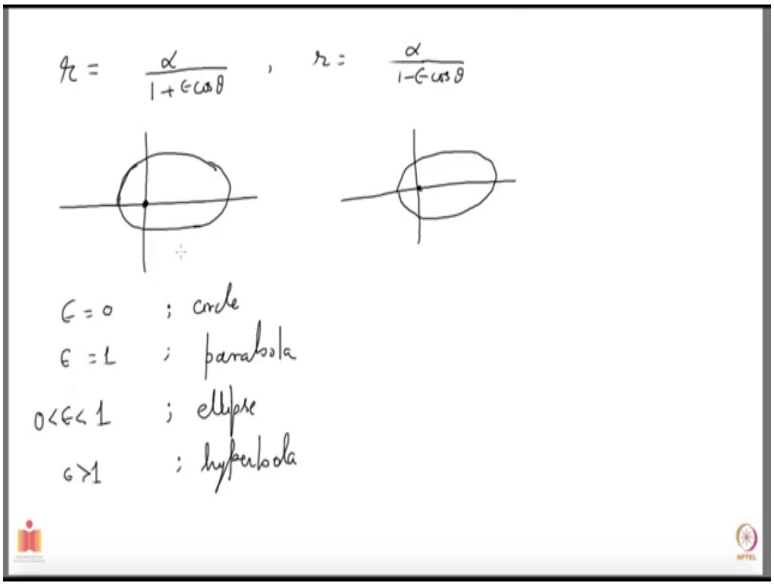

And we can simply write down without worrying about what is in the numerator of course that can be related to other quantities which characterize an ellipse. So, you have the following from 1 minus epsilon, r is alpha over 1 plus epsilon cos theta or r is equal to alpha over 1 minus epsilon cos theta. See, you have two numbers which characterize the ellipse a and b here. But, this epsilon does not know about a and b separately, only knows about the ratio.

So, if you had let say taken a and b to be something, if make a 10 times and b 10 times, epsilon still remains the same, but then you have the other thing which is in the numerator which knows about other variable also because it has it does not of a square, it has a. So, that way you can determine both a and b if we know epsilon and what you have in the numerator.

So, let us ask what these ellipses looks like and I will tell you what they look like, so these ellipse ellipse looks like and that is how we have construct, we started with this, this is one of the focus and this also looks identical to the one I have drawn just now. We get the same thing so, it is clear if you put epsilon to 0, you get a circle as I said just now, a minute back, if you put epsilon to 1 you get a parabola, let us go back and see.

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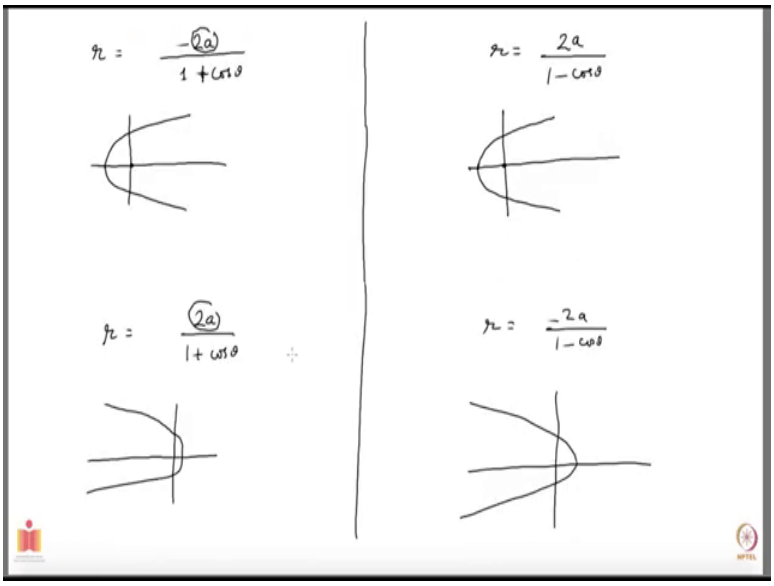

See here, this is 1 plus cos theta or 1 minus cos theta and something in the numerator, so epsilon 1 we give a parabola and epsilon between 0 and 1, will give you an ellipse and you, if you put epsilon greater than 1 we will get a hyperbola that you check. This is all I would remind you about ellipse is but, encourage you to use a some programming languages to once and while do calculations using them I will show you a small calculation which are did before preparing in this lecture.

So, I have solved this equation here, of this one and obtaining these results, these two in Mathematica, and as I said before you do not have to learn for exam but, it will be nice if you do so.

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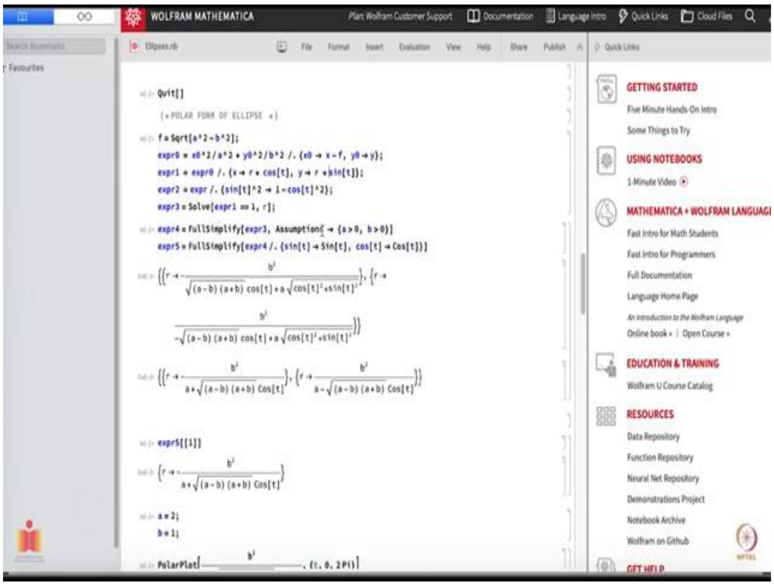

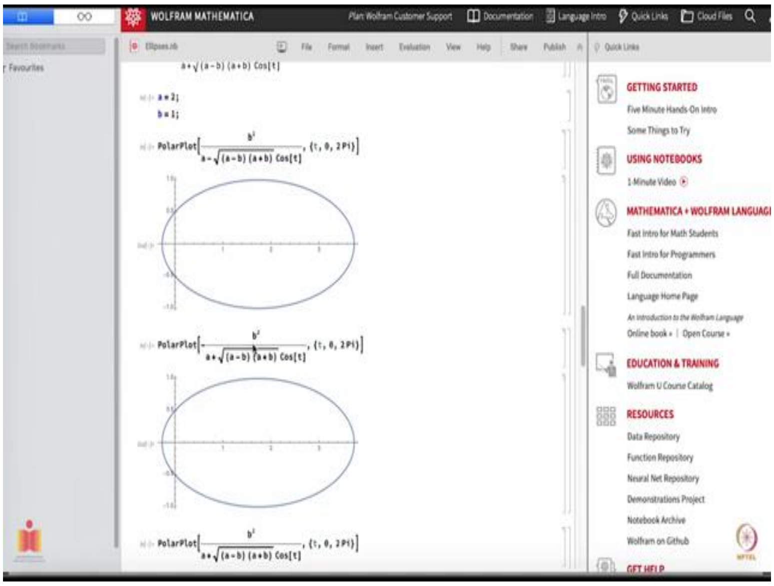

So here, I have written program which is nt necessary in the written in the best way but it works. So, I have defined my f here is a square minus b square, square root that is the distance of focus from the origin. Then I am calling something expression expr is expression 0, and I say x0 square over a square plus y0 square over b square. Which is what you had here, f0 square plus y0 square and both are divided by a square and b square and then you have 1. So, this is what I am writing here?

And then let us go back and then this slash dot means I am going to substitute so, this slash dot just in here. Thus, the syntax I am going to substitute x naught to be x minus f and y naught to be y and whatever you get in that I again further substitute x to be r cos theta and y to be r sin theta, and t theta is being called t right now and this cos and sin, these are user defined so, these are not really cos and sin functions in mathematical, they have to have capital C and capital S.

Then I define an a new expression in which I will take the previous one and substitute sin square theta to be 1 minus cos square theta and then I am whatever I have got, I am solving that expression so, we say if you write down the equations on the sheet, on on the piece of paper, you will find that what we have to solve is the entire thing which have written above that equal to 1, that is your equation for the ellipse.

So, that is what we want to solve, so I have solved this and simplified a bit, and after simplifying I get the following result. This, so, it gives me two results, r equal to minus b square over a plus this thing times cos t and r equal to b square over a minus distinct times cos t, I have just extracted out the first piece I could also removed the curly brackets but I have just copy pasted next. So, I am putting a equal to 2, b equal to 1. We wanted to choose b to be smaller than a that is why I have chosen these numbers.

And I do a polar plot which I was talking last time, I have just taken this piece and putting here and I vary theta from 0 to 2pi and that is what you get. Next I have taken this piece, the second one the second r and plotted it here, you see we have a minus sign here and then this is a second piece, it gives it the same ellipse as I said.

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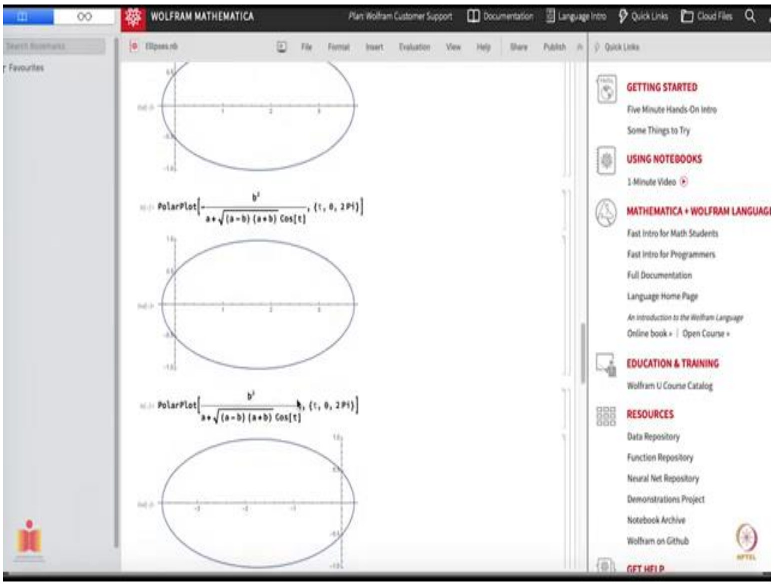

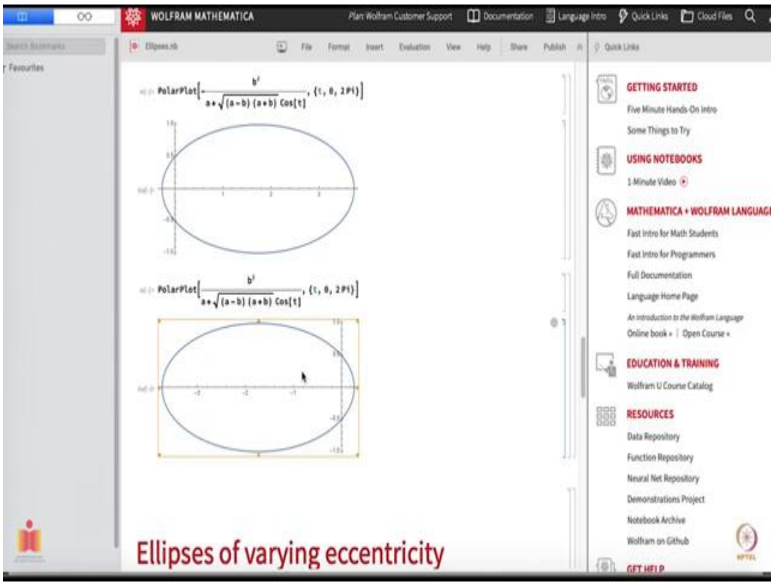

Next, what I have done is, I have taken the expression which we have here, and just remove the minus sign, and plot it and you see, you get the ellipse but, this time it is, its focus is here and the ellipse is going to the left hand side of going to the right as here. That is the only difference so, if you change this sign this is what you get.

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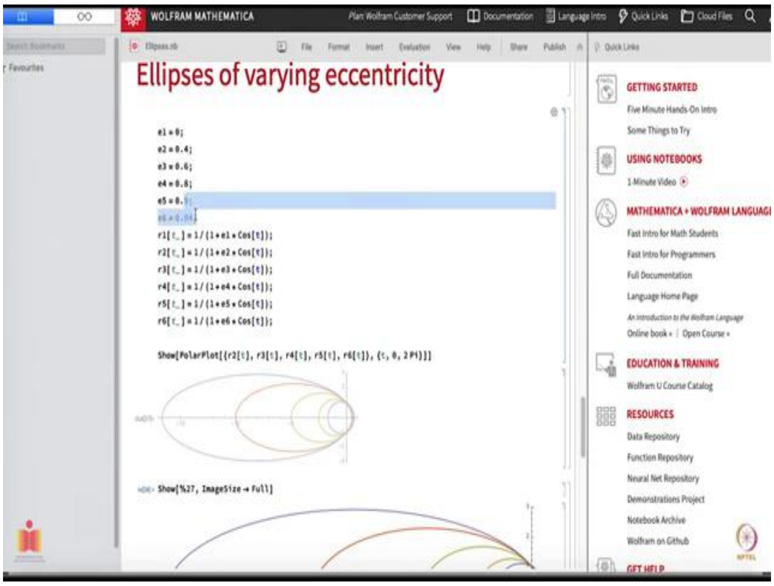

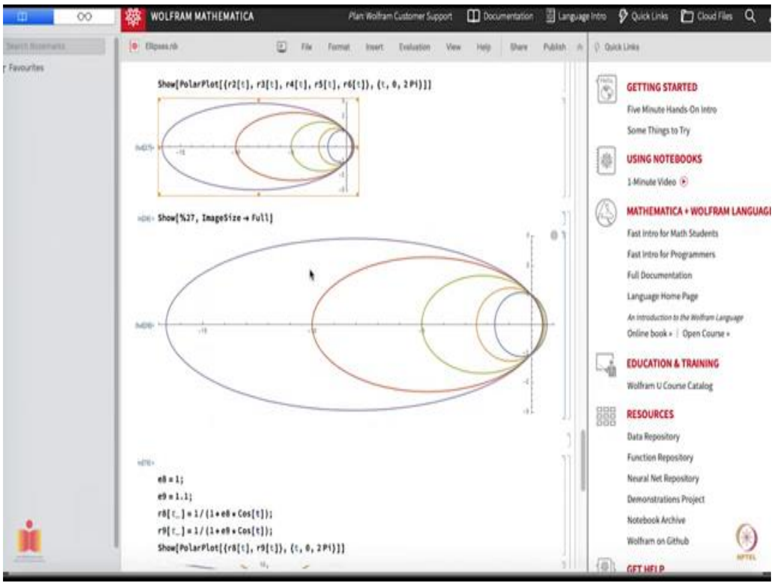

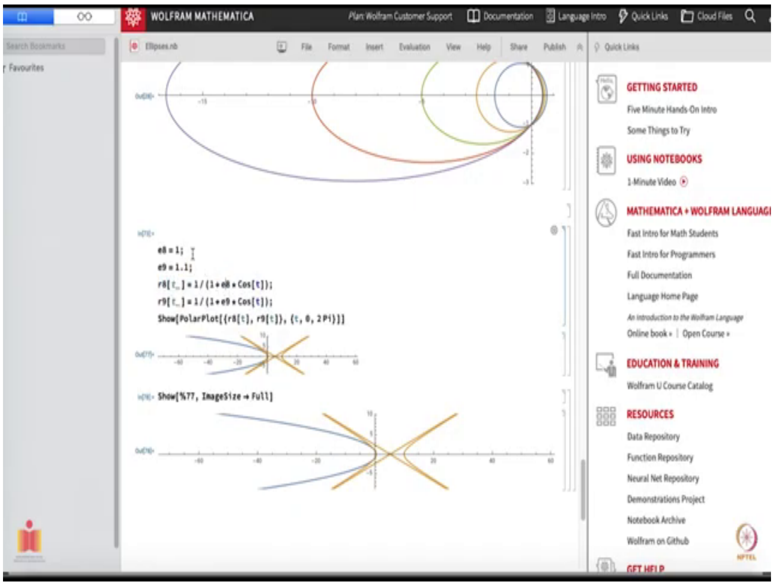

Before we leave ellipses let's look at how a ellipses change as you change the eccentricity. This is again a very crud way of doing things but, is it to understand if you have not already learned programming, a mathematical programming so, I have chosen different values of epsilons eccentricity is here. So, I have chosen e1 to be 0, epsilon 2 to be 0.4, epsilon 3 to be 0.6, 0.8, 0.9 epsilon 6 to be 0.94

And if you take epsilon to be 1, you should get an parabola so, this is very closed to a being parabola so, let see. And here, I am just plotting r I have defining r1 to be 1 over 1 plus epsilon cos t. And these different values of e1 e2, 3, e3 and e6 they will control, the eccentricity of ellipses.

Now, I am trying to do a polar plot of several of this so, I have r2, r3, r4, r6 they are all have to be included within a curly bracket here. And I am plotting from 0 to 2 pi and I should put a show here so, that can show all of them together. And this is what you get. The same image I have zoom now and this is what you get so, you see even when are very closed to 0.1 your epsilon is 0.9 or 0.94 you are still not getting a highly elliptical thing it is, you know you say it is what you are getting

So, this is how this eccentricity grows, and let see what happens if I put a epsilon to 1 you get a parabola and also epsilon greater than 1 will get hyperbola and that is what I have done here. So, I have define another r8 and r9, you have e8, e9 which I have taken to be 1 and 1.1, and you see this is what you get. So, it is blue thing is the parabola here, and then you have the hyperbola. Okay the two parts of the hyperbola here you can see.

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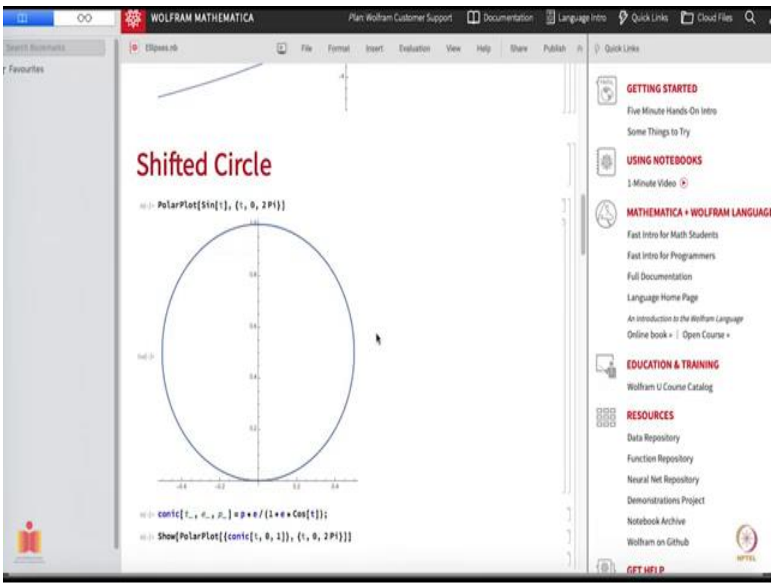

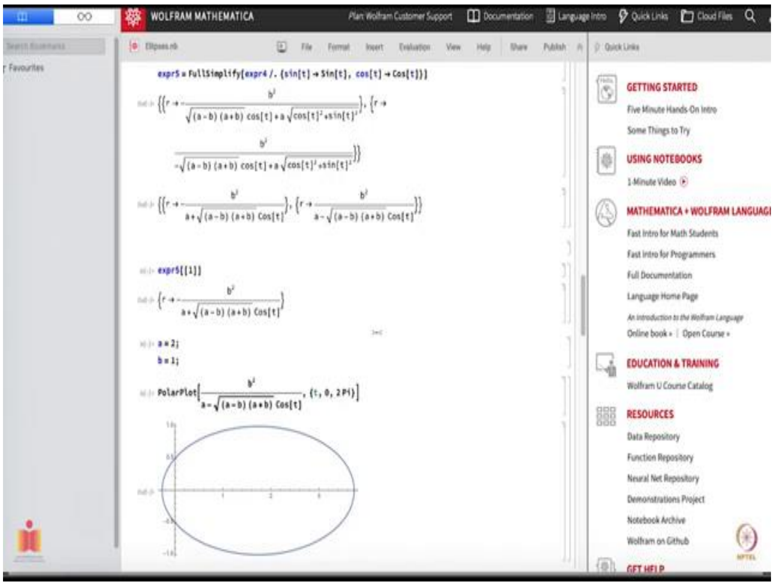

So, now you know what the form of a conic section is, in polar cadence it is just r is equal to alpha over 1 plus epsilon cos of theta, good that is where we will stop and now we will continue with our caporal problem.