Introduction to Classical Mechanics
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Lecture 3
Virtual Work

So, last time we were talking about degrees of freedom of different systems and in particular we talked about a particle that is constrained to move on a given smooth surface, let us perceive that example further and let us say that we give the surface we tell we specify the surface on which the particle is constrained to move and there may be other forces which are acting on it, so it might be getting pulled gravitationally or electromagnetically by some other things but it always has to live on that surface. And we will investigate this in some detail, so let us go to.

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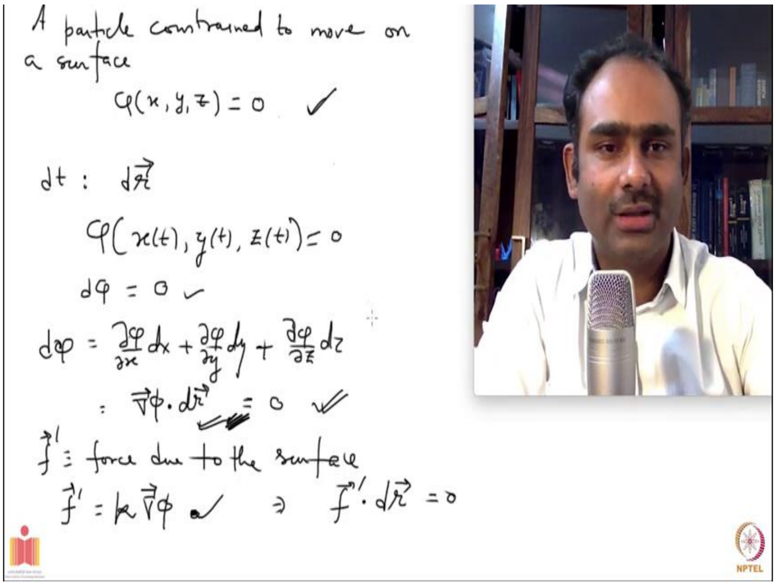

So, a particle constrained to move on a surface and let say the equation that describes the surface is given by this relation f x, y, z equals to 0. So, that is the surface this is one equation, so the particle has a free particle has 3 degrees of freedom and because of this one equation it will have 2 because 3 minus 1 is 2, so 2 degrees of freedom for this system for this particle and that is the equation. So, let us say in some time interval DT the particle gets displacement by an amount dr so in some interval DT it gets displaced by dr and that displacement has to be on the surface.

Now, how are the displacements dx, dy and dz, how are the constrained? So, they have to be related to each other so that they so that the particle stays on the surface. So, to find that how they are related to each other we can write phi so this is telling that whatever that particle does whenever it goes its coordinates at every point of time t should satisfy this, which means if I take a total time derivative of this that has to be 0.

So, d phi, let me use the same kind of phi as before d phi over dt should be 0 or even or simply let us say d5 is 0, that has to hold true. Now, d phi is the following, so you take the partial derivative of phi with respect to x into dx plus partial derivative of phi with respect to y dy plus del phi over del z dz this has to be 0.

Now, this you can write as gradient of phi dot dr what are the components of dr? dx, dy and dz these are the components, so that is what your dr is and gradient of phi and the components are del phi over del x, del phi over del phi y and del phi over del z and this entire thing should be equal to 0 because of this.

That is good, now let us ask what are the forces that are acting on the particle due to the constraint due to the surface, see if the particle is constraint on the surface and the surface is smooth the force on it can only be in the direction normal to the surface, I hope that is clear? See the surface is smooth and there is no, it is smooth so there is nothing which can give a force in the horizontal direction or let us say in the tangential direction, tangential to the surface, that cannot happen that is what you mean by smooth if something is not smooth then it can give you a force which is along the surface, tangential to the surface, if it is smooth it cannot.

So, whatever that force is, let us call it f prime, so f prime force due to the surface, that is the constant force that f prime will be proportional to gradient of phi because gradient gives the direction normal to the surface and f prime is normal to the surface so there will be some constant K involved proportionality constant involved in here and this will be the relation between the force and the gradient of phi, that is good.

Now, from here I can immediately see that this result which we found earlier tell us something about the nature of the force of constraint, so what we do is we put this thing this equation in here in this one and we immediately see that f prime dot dr is equal to 0, what is f prime dot dr? That is the work done by the force of constraint in moving the particle by amount dr and we see that work turns out to be 0. So, this is one thing we have noticed about the force of constraint.

That is nice, note however that the phi here did not involve time explicitly, does no explicit dependence on time in here, meaning I am imagining a surface that does not change over time, let us relax that condition and say that the surface is changing with time, you can imagine, for example a particle which is sitting on the floor of the left and the lift is going up or down whatever, let us say it is going up with some velocity V. Now, this surface on which the particle is lying is not staying put here but it is going up, so there is a explicit time dependence it has to move with velocity V and let us see in that case what happens.

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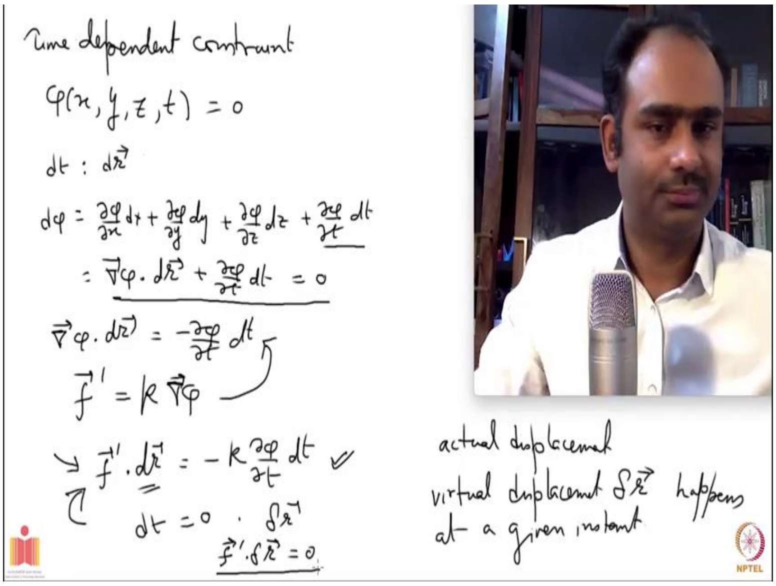

So, we go to the next page and here so I want to put time dependence time dependent constraint and inside the time dependence is explicit not implicit. So here, let us say I have a surface which is given by this relation, so some phi which depends on time is well and let us ask can I say something similar or something similar about the work done which we saw in the last page, I mean why I am interesting in the work done, we will come to that later and not today in this video, but let us for the moment just ask what is the work done by the force of constraint when there is an explicit time dependence.

So, as before I just write down d phi so let us say again in time dt my particle moves or gets displaced by amount dr, then d phi would be del phi over del x dx plus del phi over del y dy plus del phi over del z dz plus now there is time dependence as well del phi over del t dt this was not this term was not there earlier, this as before is gradient of phi dot dr plus delta phi over delta t into dt and which will be 0 d phi is 0 because the particle is constraint on that surface.

From this I can write down gradient of phi dot dr equals minus del phi over del t into dt. Now, in this case also the force will be normal to the surface, which means it will be proportional to gradient of phi. So, imagine your lift is going upwards the particle is constrained to move on the surface it is going somewhere it is getting displaced with time but whatever happens the forces are always perpendicular to the surface.

So, again my f prime would be proportional to the gradient of phi some k times gradient of phi if I substitute this in here in this relation I get f prime dot dr equals so I have gradient phi to be f prime over k, so there is a k in this denominator which I take to the other side and it becomes minus k del phi over del t dt, let us see if it is okay, it is f prime over k is gradient phi, that is correct.

So, now in the present case this is the work done, so when particle goes from wherever it was to the other location in time interval dt, that is the displacement dr the work done by the force is not 0, it is a non-zero force because there is a non-zero right hand side which is not surprising if you think of this example of the left when the lift floor is going up that particle is going upward, so some work is being done because of the force of constraint.

Now, there is something you can see for some reason which will come to you later, for some reason I am interested in getting the work done to be 0 I am interested in those forces were the work done will be 0 but clearly this is not the case here but notice if I start looking at not the actual displacements which happen over time interval dt.

So, this is let me be more clear this is the actual displacement I want to imagine another kind of displacement I want to be in I want to look at a displacement that happens, okay let me put it differently imagine a displacement at a fixed time, so you have to imagine at particular time and on that time itself you have to imagine a displacement.

So, let me write down, so I am thinking of a virtual displacement because all actual real displacements will happen in time but now I am interested in something that happens at the same time some talking about virtual displacement I will denote it by delta r to distinguish it from dr and that virtual displacements dr happens at a given instant.

Now, if you are hearing this for the first time this might be very confusing because I am asking you to imagine a displacement which happens at a given instant but the moment you start imagining it your brain will trick you and not because it knows all the displacement happen in time you will always start thinking something changing with time.

So, if that is happening to you do not worry you do not have to imagine it really all you have to do is think mathematically put the time interval dt to be 0 because I am saying that displacement virtual displacement has to happen at the same time and not the displacement by sorry delta r. So, imagine your lift is here at time t, and what are the displacements that can happen?

At that time it can happen on the surface of the lift but any real displacement will not happen on the surface with the same value of z because in any finite interval of time or even infinite interval of time, this particle will have some displacement along the z direction as well, that will be the case for the real displacement but I am talking about the virtual one and displacement will be on the surface.

So, the real displacement and the virtual displacements are very different but do not worry about imagining it, just put dt to 0 and think of a displacement which is allowed at that time so that is what we are talking about. Now, this relation then becomes f prime dot delta r equals 0 that is the relation. So, this say that under the virtual displacement f prime does 0 work.

Now, it is clear that not all forces will give you 0 virtual work, so let us call this work as virtual work not everything will give you 0 virtual work, if your surface was not smooth if it had friction, then whatever displacement you imagine from here to there even at given at one instant because of the friction will be horizontal component to f prime and you will get a non-zero work in going from here to there. So, not all forces will give you 0 virtual work for what we are interested in is the force which will give us 0 virtual work let see what else I wanted to say this one.

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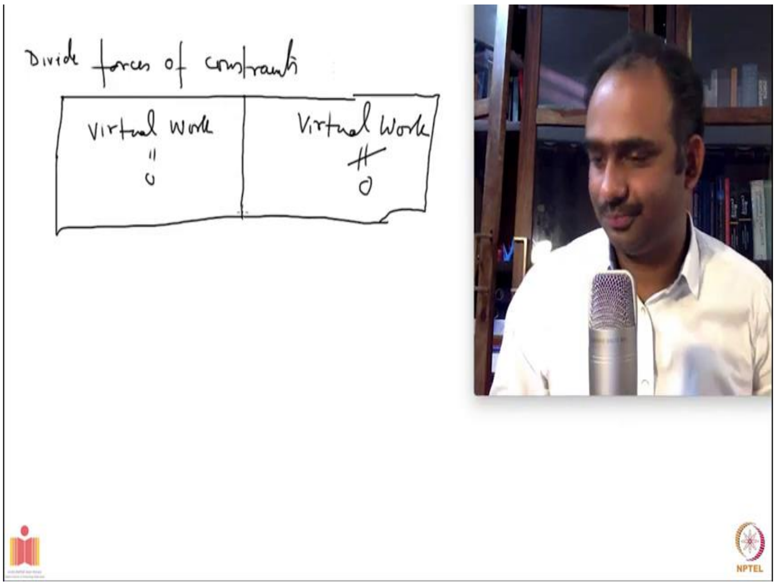

So, what we have just to summarize we can divide the divide the forces of constraints into two varieties, one which do 0 virtual work and another which do some virtual work. So, right now it is not clear why I am talking about those things and why should that be important but as you will see this will be very useful and we will try to get equations of motion using concept of virtual work and as I mention friction is out if you want the virtual work to be 0 and now onwards, we would be only interested in those kinds of constraints which gave the virtual work to be 0. So, in this course will not have those constraint forces for which virtual work is not 0. So, we will talk more about such things in the next video, see you then.