Introduction to Classical Mechanics
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Lecture 2
Generalized coordinates and degrees of freedom

Hello students, let us continue from where we left last time. So, in the last lecture we discussed about the constraints that space time symmetries impose on the kind of forces that we can have. We saw, for example that time cannot appear explicitly in the forces and that is true if your system is isolated from all other things, if only if you are if your system is not influenced by something external, if that is not the case, then of course the forces can be time dependent but if you have isolated system, then the forces which are acting on individual particles within the system are not dependent on time.

Also you saw that the forces cannot depend on the absolute locations of individual particles, they can only know about the relative separation between the constituents and similarly you saw that the fact that physics does not change in going from one inertial frame to another inertial frame, implies that those forces cannot depend on what the velocities are in a given reference frame, they can only depend on the differences in velocities of different particle, so that was our first experience with the use of symmetries of space and time and we will have more occasions to do these things later as well.

Now, today we want to proceed by looking at the coordinates that we have used more carefully. So, till now I have been whenever I have been speaking of where the particles are located I always assumed that the coordinates are Cartesian coordinates and for us there was no reason to use any other coordinate and that is why we use Cartesian coordinates. So, now we want to talk a little more about coordinates and that is what the plan is for this short video.

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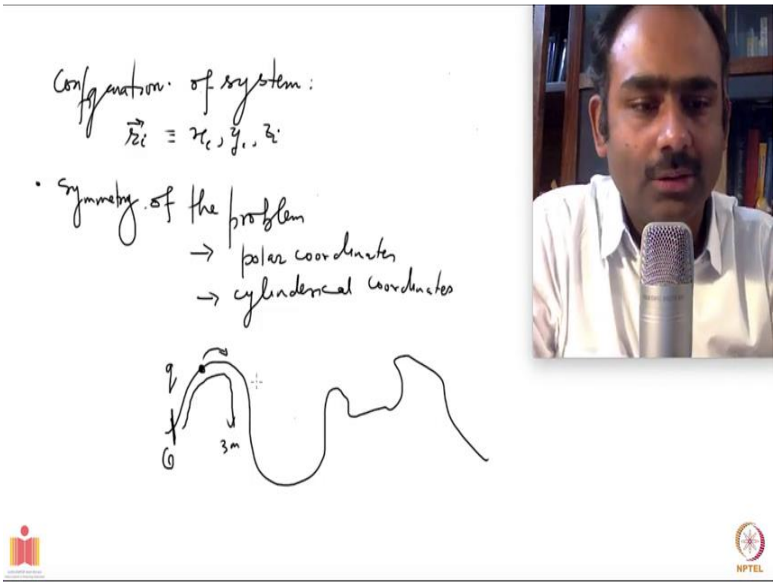

So, they said just now is that till now, we have used the Cartesian coordinates, so if I am specifying the configuration of the system, I was using the coordinate ri and by this ri I mean xi, yi and zi. It is clear that there may be situations when using Cartesian coordinates may not be the best thing to do for example if you are looking at, let us say a planet which is going around sun in a circular orbit it is clear that it is not such useful thing to do to use xy and z components to tell where that planet is located, it will be more natural to use the polar coordinates.

So, you would use for example the radial distance is away from the sun, the polar and azimuthal angles they will be more natural coordinates because they know about the symmetry of the problem, see the force which is acting between the sun and the planet is central meaning it depends only on the distances between these two objects, the sun and the planet. So, you want to use the co-ordinate where this symmetry becomes apparent and clearly using your polar coordinates will be more useful.

So, one reason why you may want to use a co-ordinate other than the Cartesian coordinates is purely the symmetry of the problem may dictate for example use polar coordinates or it may say you should use cylindrical coordinates. So, that is one simple way of understanding why you might be interested in different coordinates is another reason why you would like to use coordinates other than Cartesian coordinates.

Imagine you have one particle and that particle is moving along a specified curve, meaning it can go let us say it can go only imagine a thread here imagine a thread been given here and you say that your particle can only trace this thread, so can always be on this thread and you have already told the way in which that thread is placed.

And there may be certain forces acting on this particle it might be getting pulled and pushed by other things but it still always remains on the thread. Now, is using Cartesian coordinates useful thing to do? Well, that would be and overuse of things because you do not really need to tell where, I mean to tell where that object is located, you do not need to tell three numbers you do not need to tell what the x-coordinate is what the y-coordinate is and what the z coordinate is, clearly it is an over specification what you need to tell is how far along the curve it is from some chosen point o.

So, you choose let say, let me draw it, it will be probably easier to understand, let us see you are given some such curve and you have your particle sitting here and this guy is moving along the curve. Now, if I want to tell where it is, let us say I choose some point as a reference point o and if I tell, go along this curve for three metres and you will find the particle that one number 3 metres is sufficient to tell where it is, you do not need to tell me 3 numbers, you do not need to tell me x, y and z.

So, clearly if you are studying such a system you do not need to use Cartesian coordinates, let us call the distance along this curve to be q that is a notation I will use, this is your first instance of using a coordinate which we call a generalized coordinate, in this case it is still a distance but we will see more examples where a generalized coordinate may not even have the dimensions of length. So, I hope you understand that it may be more natural to use other coordinates rather than Cartesian coordinates. I will give you another example, for that I should go to next page which is here it work yes, look at this very commonly used example.

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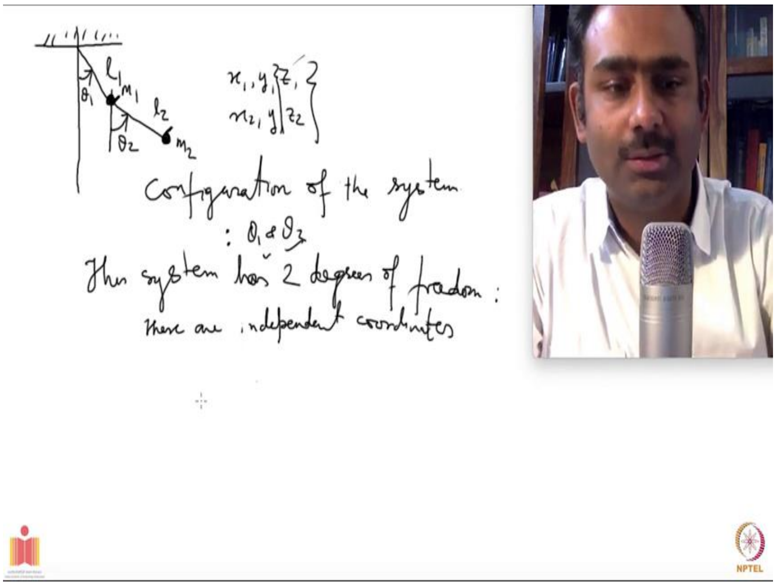

So, there is some ceiling and from the ceiling you have suspended one pendulum and from this pendulum this bob this mass you have attached another pendulum. Now, my system is these two masses, I am assuming the strings to be massless, weight less, massless and they are just there to support the masses. Now, I ask how many numbers you should tell me to tell exactly where those two particles are located, now you may say we need to specify, let us call this particular 1 and particle 2 so they have the masses 1 and 2.

So, very naively you might think that I need to specify x1, y1, z1 for the particle number 1 x2, y2, z2 for the particle number 2 but again this is an over specification you do not need those many numbers. Let us say these two masses are really doing whatever they do in the plane in the plane of let us say this whiteboard, so there are always in the plane, clearly you do not need the z1 and z2 they always remain the same, they always remain 0 if you choose them to be.

But even then you do not need x1 and y1 x2 and y2, let us see what you really need to specify, if you tell me this angle theta 1 if you tell me that one I will know where this guy is, I will know that because this length is not going to change, so I have to tell you the length if I do not tell you the length, you cannot know where it is.

So, this length of this first string does not change, let us say l1 and look at the particle number 2, so this is another vertical line, so what I am saying is from the bob number 1 you draw a vertical line, tell this angle theta 2 and you will know where this M2 is because the length l2 from this to this is also going to be specified to you.

So, you see this to specify the configuration of this system, you need only two numbers theta 1 and theta 2 let me write it down configuration of the system, and this one needs only theta 1 and theta 2. So, this is another example of generalised coordinates, so instead of 6 or instead of all this 6 or even 4 by if you tell z1 and z2 are fixed, you still realise that the constraints of your system impose restrictions which leads to the fact that only two coordinates are independent.

So, what we say is that this system has two degrees of freedom, two because two numbers here which means that these are the two these are the independent coordinates. Now, see you may say I do not want to use theta 1 and theta 2 I want to use some other combination of theta 1 and theta 2 that you can do of course but that will not change the fact that you still you need only two independent quantities two independent numbers.

So, whatever transformations you do, you still realise that only two independent coordinates are required, the minimum number is two and that is what is called the degrees of freedom of this system or any system which you are looking at. Let us look at a few examples, my plan was to give you a few examples as an exercise, they are very very simple and I will write them down you think about them and then one after the other I will tell you the answers.

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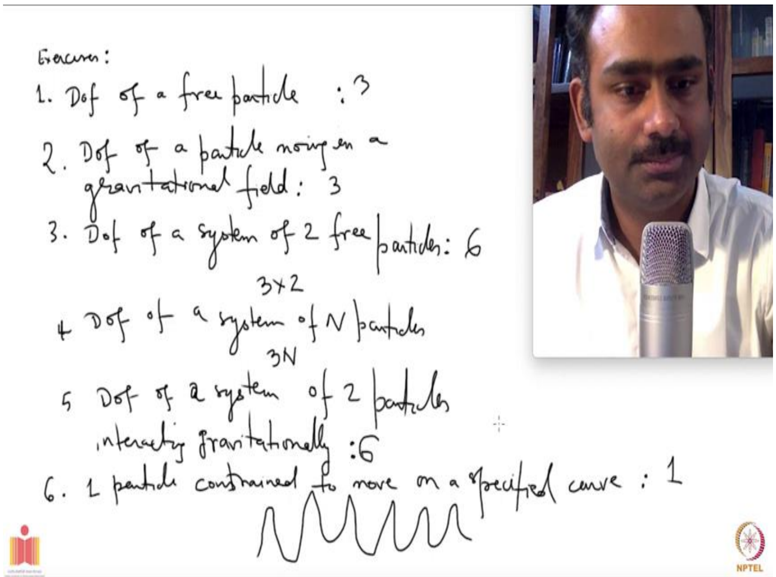

So, let us do that I will go to next page, so some small exercises or problems. So, number 1, so first is how many degrees of freedom a free particle has? So, degrees of freedom of a free particle, by free I mean a particle on which there are no forces acting, it is free. So, how many freedom how many degrees of freedom it has? 3, you need to tell me three numbers x, y and z or r theta and 5 whatever you like but you need to tell me at least three numbers and then only I will be able to tell you where it is.

So, that is 3, now how about this, what are the degrees of freedom of a particle that is moving in a gravitational field? What do you think the answer is? So, while I am writing you please think about it, so degrees of freedom of a particle moving in a gravitational field, and what you think your answer is? This could be gravitational field of one planet or let us imagine there are hundreds of planets around sun everything and then this guy is moving, how many degrees of freedom it has?

Well, the fact that there are forces acting on this is not going to change, how many numbers you need to specify its location it still remains 3, I hope you understand that part, so degrees of freedom still remains 3, another very simple problem. Imagine now you have two particles both of them are free and they do not interact with each other, so two particles non-interacting there are no forces on it on them, how many degrees of freedom?

So, you think about it while I write the question, degrees of freedom of system of two free particles. Answer, well to tell where the first guy is you need to specify three numbers to tell where the second guy is again you to specify three numbers, so in total you have 6. So, each one of them has 3 degrees of freedom times the number of articles you have which in your case is two, if you have 5 degrees of freedom of a system of n particles clearly that would be three times and n that will be three n.

Now, what if you have two particles but they pull and push each other through coulomb force, meaning the particles are not free but they are interacting with some forces, two of them what is the degrees of freedom of the system? Well, as before this interaction cannot change the number of coordinates you need to specify, it still remains 6.

So, degrees of freedom of a system of two particles interacting, let us say gravitational or whatever force you like it still remains 6. So, usually students get confused when you say that there are certain forces acting on them and then they may get wrong answers. How about this, till now all our particles were free to move in space and it was quite easy to answer how many degrees of freedom they have.

Now, imagine you have one particle, this we already talked constrained to move on a specified curve, then what then only one, because it is curve does not matter what the shape is, it is one you tell choose a reference point and you tell the distance along the curve you should go to find that particle and that 1 number that distance is the degree of freedom, so that is one it is interesting to see that in a classroom if you ask this question and you draw this curve and say the particle is moving along this, quite a big chunk of the classroom will tell that this system has 2 degrees of freedom because they think that is going up and down somehow.

So, it is in a plane and they will think it is 2 degrees of freedom but no it is 1 and you know why I am saying 1. How about a particle constrained to move on a plane, if it is on a plane let us say, I put X and Y coordinates system, then clearly the degrees of freedom will be 2 you can tell how much should go along X, how much along Y and you will find the particle so 2 degrees of freedom.

And clearly if I take a surface, whatever the shape of the surface is, if its surface again the degrees of freedom is 2, and why is that so? That is something you should think about, I will give you one exercise which you should try to do yourself and we will end with that one.

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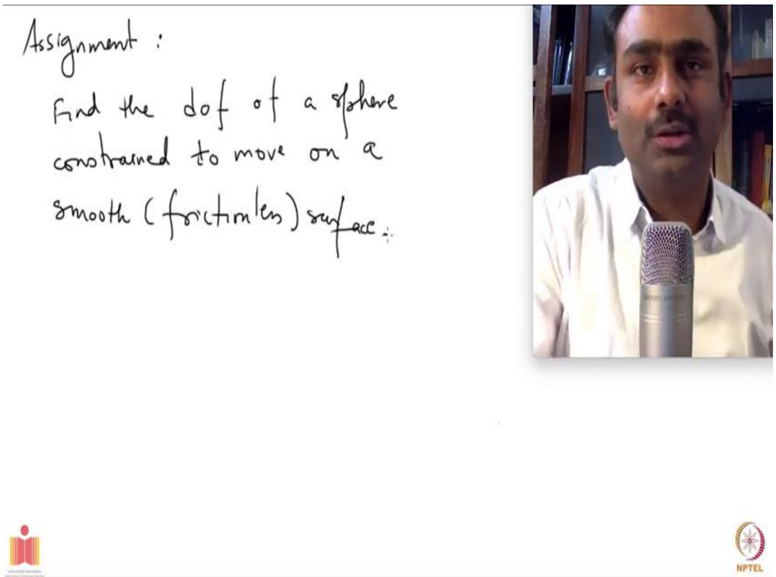

Yes, so here is an assignment for you, so find the degrees of freedom of a sphere constrained to move on a smooth surface, so imagine you have a ball and this can only move on a smooth surface. Now, pay attention to the word smooth, there is no friction at all, 0 friction so you should first think in what manner this ball is going to move, can it roll and then you find out how many degrees of freedom you need to specify to where that guy is, find the degrees of freedom of a sphere constrained to move on a smooth surface, by smooth I mean frictionless surface. We will meet next time and look at the solution of this one, hopefully you would have done it by then and we will continue our discussion of analytical mechanics, see you.