Introduction to Classical Mechanics
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Lecture 16
Matrices, Forms, and all that

Last time we started looking at some of the mathematical quantities that we will require in this course. And mostly we were looking at matrices. So, we will do a quick recap of what we talked about last time and we will continue talking little bit more on this. Basically what we are doing is we are preparing ourselves for a discussion of oscillations, small oscillations.

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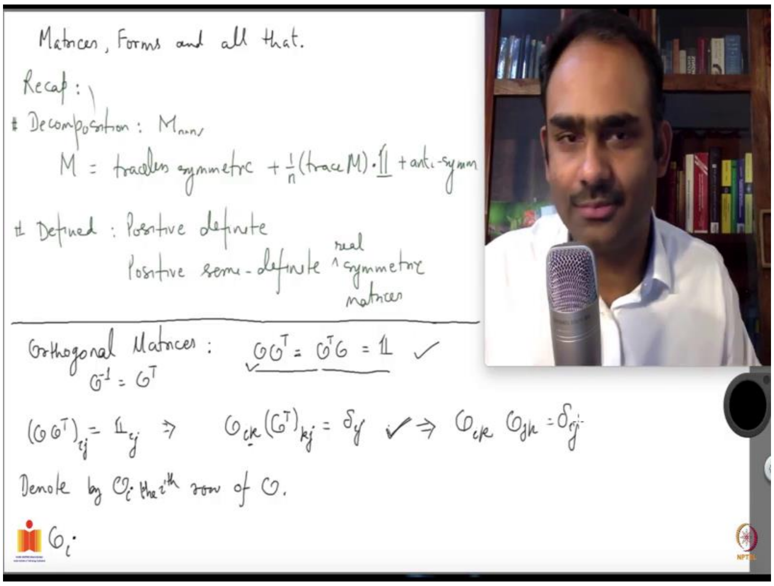

So, as we wrote last time matrices, I will add forms now, we will talk about them today and all there. Let me do a quick recap. Let us try a colour this time for recap. So, last time we saw that we can decompose any n cross n matrix, any n cross n matrix into a traceless symmetric part, a part which contains the trace which is proportional to identity and antisymmetric part. So, that is what we talked about. So, let me write it.

Decomposition, this you can always do. So, you if you have a matrix and which is n cross n, then you can do the following. You can write M as a traceless symmetric part plus trace of M which is a number. You just add the entries which are present on the diagonal times the identity matrix and we had to divide by 1 over n and this n is dimension of the matrix, plus an antisymmetric place. This you can do.

Then we also defined what are positive definite and positive semi definite matrices. So, for harmision case and real case. So, we also defined, we also defined, positive definite and positive semi definite matrices. Let us call, let us only talk about symmetric matrices, real symmetric. You recall that if you have a positive definite matrix and if you take it and sandwich between any two real vectors, if that vector is non-zero it will produce a positive number.

And a positive definite matrix is semi definite matrices which will produce either positive values or 0, never a negative number. That is good. So, let us come back to what we want to talk today. Now, as you can, already anticipate that because we are going to deal with symmetric matrices that is why I am talking about then here. We will be (run) running into orthogonal matrices because I mean if you want to diagonalize a symmetric matrix, you do so by orthogonal transformation.

So, let us briefly talk about orthogonal matrices and then we will move on to Forms. So, what is an orthogonal matrix? A matrix O is called orthogonal if you take the transpose of it and multiply with it, either from the right or from the left in whatever order you do, you get identity. That is the definition of an orthogonal matrix. And that is fine. Which means that the inverse of O is same as the O transpose. That is what this relation means.

When you take O multiply with O inverse, you should give identity. And this what is happening here. Now, I want to write down this definition of orthogonal matrix using indices. So, let us say, I take this piece O, O transpose is equal to identity and I on the both sides look at the element Ij. So, what does it become? It becomes Oik O transpose k j let me be more, even more careful in writing this.

I will write O transpose and we are talking about kj element of the transpose matrix and this will be delta i j because that is what your identity matrix is, this is good. Now, if I denote by so, let me denote by Oi, now see Oi is not an element of the matrix O because any element will have two indices i, j, k, l whatever you want. So, it will always have two indices like here.

So, clearly this is not an element of your matrix. So, there is no confusion. So, I am denoting by Oi, ith row of the matrix O. The ith row of matrix O. So, you have a n cross n matrix and it has several rows in it, so look at the ith row and that is what I am denoting by Oi. Now, let us see what this relation is saying about these rows, row vectors. So, let us look at this. This becomes Oi dot, I should have written this as sorry, before I Oik Ojk equals delta ij.