Introduction to Classical Mechanics
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Lecture 1
Introduction- Symmetries of space and time

Hello students, this is a course on Analytical Mechanics or Analytical Dynamics and this is targeted towards students in their bachelor final bachelor years like BSC or and also first year MSC, this will also be useful for students who are doing their BTech in different Indian universities. The goal of this course is twofold, one we want to develop the language that will enable us to comprehend the world around us and its details and what we are really interested in is the details at the fundamental level.

And the second, we would like to learn to appreciate and cultivate the thought process that leads to the formulation of questions in their ever increasing cleaner and deeper forms which will lead us to a deeper and much more beautiful forms of understanding of the world around us. So, that is the goal at a deeper level and of course as we go around we will learn we will be learning things in this subject. So, let me start by recollecting what we already know, what you all have seen in your physics courses.

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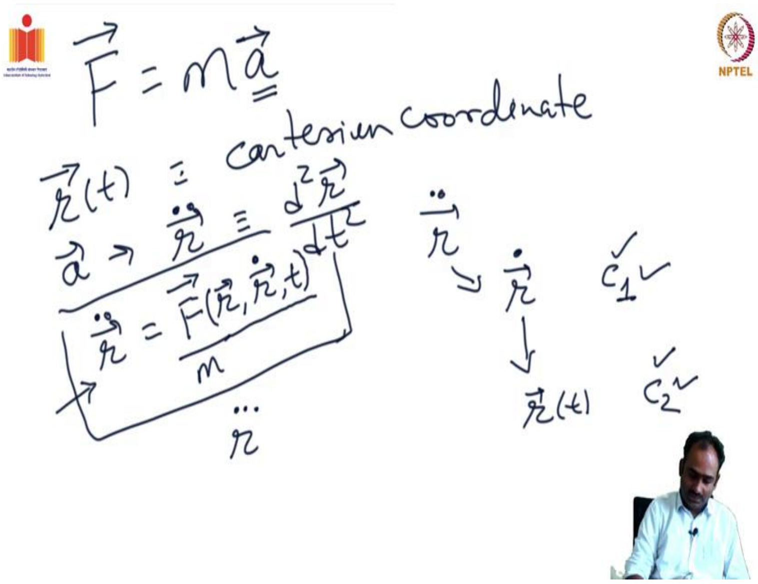

So, I start by writing this very familiar equation, that is the Newton's second law which says that the acceleration which is a vector quantity, so let us say we are imagining a particle which has mass M and it is subjected a force F, then it will accelerate according to this relation that is your Newton's second law. Let me write it in a slightly different form which will be useful to ask further questions, so instead of writing a for acceleration, let me introduce the following notation I write vector r at any time t to be the Cartesian coordinate of the particle with mass M.

So, r would be the Cartesian and instead of writing a I will write r double dot and these two dots signify that I am taking a second derivative with respect to time, that is what r double dot means. So, with this I can write r double dot as force or mass, let me also specify the arguments that enter into the force, the force could in principle depend on where the particle is at the time t, so it depends on it could depend on the r, it could depend on velocities how fast that particle is moving, it could depend on other things and it could also depend on time.

Now, when you look at this equation, what comes to your mind? There are several things which we can think of, one is at a very philosophical level you see what this equation is saying is that if you tell what are the forces this particle is experiencing if you tell that I can tell what the acceleration is.

Now, if you know the acceleration at any point of time, then you can integrate that equation and get the trajectory you can tell where the particle would be at any later time t if you solve this equation and also on the top of it you have to specify initial conditions where it was at time t equal to 0, how fast it was moving at time t equal to 0, that information will be sufficient to tell all this.

So, coming to the philosophical thing, it is non-trivial that there are such laws, it is a non-trivial statement that we can predict what will happen later, that is the philosophical thing but let us say we agree upon that universe is describable by laws that what can more be asked from this equation.

You can ask, why it is second order? Why only derivatives which are second order D 2 T over DT square involved in this relation. Now, that has to come from your experimentation experiences, you see when you say it is second order in a time derivative it means that the solution will involve two constants of integration. You see when you integrate it first you will get down from r double dot to r dot and you will have a constant of integration let us call it C1 which cannot be determined by this equation that is a physics input.

Then you integrate once more you arrive at r that is what you are looking for but then at this stage you get one more constant integration C2. So, two constants of integration you will have which you typically know that they are the coordinates at any time t at any initial time, let us say t equal to 0 and the velocity of the particle at the same time you could use them to fix what C1 and C2 are, but I mean why should this be a second order differential equation?

Could Newton have chosen this to be a third order differential equation, could he have written something involving three derivatives, that is one thing you can ask and it's not a very difficult thing to answer.

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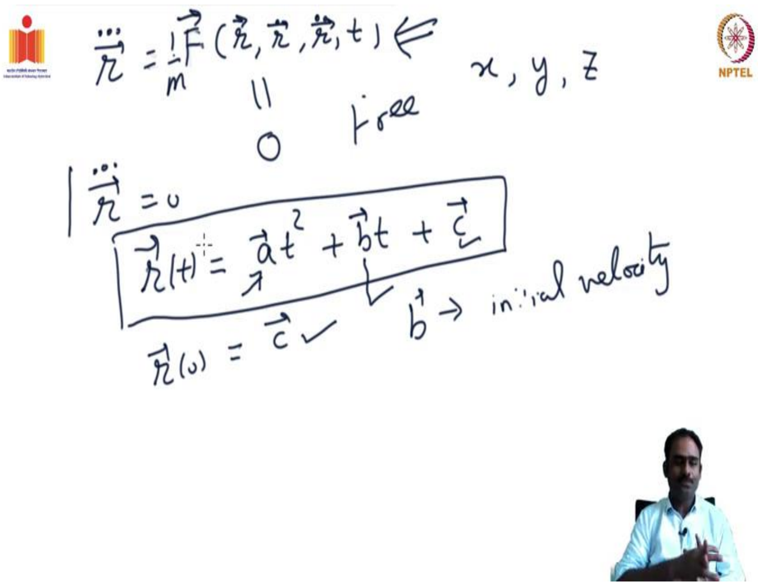

Imagine we could, then it would imply that I have a relation like this, I am keeping up to the second or second time derivative here, so all derivatives lower than r triple dot I keep on the right-hand side and the definition of force. Let us see what happens if you try to write down such an equation, let us say we take a particle which is free there is no force acting on it so I can put down the F to be 0 let us say I take a particle which is free, by free I mean no forces will acting on this one, then you have r triple dot 0.

Now, if you write down the solution of this equation, this will involve three constants of integration vector a, vector b, and vector c, remember this is this equation for example here it is not one equation it is three equations, there is one equation for each Cartesian component, so one equation for rx, one equation for r instead of rx I can say x, y and z.

So, these are the three, so you should read it as x triple dot as 1 over F FX Y triple dot as 1 over m FY. So, these are really three equations, so each vector equation becomes three equations. So, coming back to this, if you solve it you will get such a result after integrating and clearly if you choose time the particle to be at rest at time t equal to 0, so you put r of 0 that would be your C.

So, this you can fix by telling where the particle is located at time t0, very good. Similarly, the B you will be able to fix by telling how fast it was moving or what the velocity was at time t equal to 0, you take one derivative this C will be gone this will leave only B behind and this will involve t put t equal to 0 this term is also gone so you have only B left, so you will be able to fix B by initial velocity and as you can see here that this a you will need to specify, for example what the acceleration was at time t equal to 0 before you could really tell what r at any time t would be.

So, this constant will not be get will not be determined by telling just the coordinate and the velocity at time t equal to 0 you will also need to specify the acceleration at time t equal to 0 but you already know from your experience that is not how our nature works, you just need to tell these two things where and the velocity, you do not need to specify acceleration to know where it would be at any later time t.

Which means that your equations should be second order differential equations and these are ordinary differential equations and they cannot involve terms which are derivatives in time more than two cannot have second order third order or fourth order. That is one thing about the equation which was our force law.

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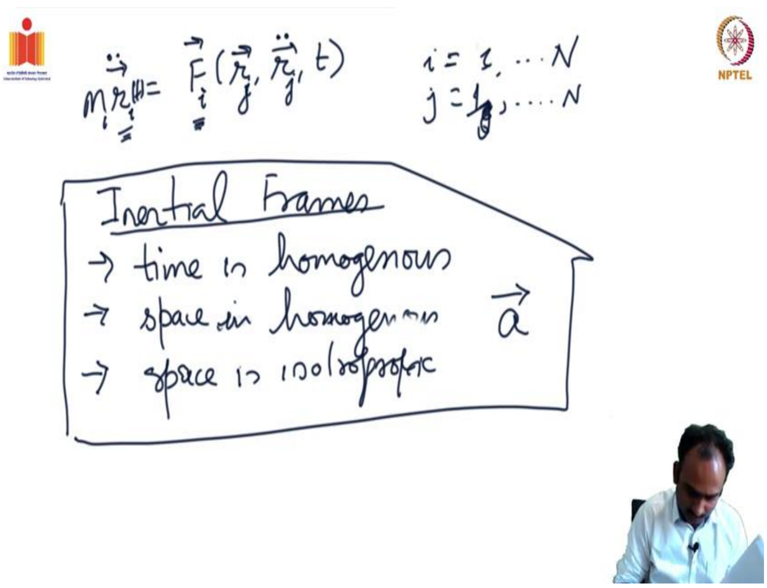

Now, let us move to something even deeper, let us say we have established that indeed if I solve this Newton's equation I will be able to tell the future trajectory, imagine you have n number of particles lots of particles in your system and each of them is let me put an index I to emphasize particle number I not necessarily I here, it could be the force could depend on the coordinates of all the particles.

So, let us say I runs from 1 to N, you have n number of particles in your system and J also runs from 1 to n, sorry this is 1. So, what this equation is saying is that the acceleration of ith particle at time T would be determined by what this what force it is experiencing and that force depends on the coordinates of further particles and their velocities and time t, that is the most generic thing which you can write.

Now, let us ask can we say something more without going into the details of the system based on our some general understanding of how the wall is. One thing you clearly know from your education for previous years that if I do an experiment now, whatever results I get I will get the same results if I were to do that experiment, let us say 100 years of from now or 1000 of years from now, meaning it does not matter when you do that experiment, this is this can be put more formally as saying that the time is homogeneous.

So, what I am saying is, there exists frames of references in which time is homogeneous which means you whether you do experiment now or 1 billion years from now the results will be identical if you have arranged everything identically in these two different times, that is one thing, so there are frames special frames call inertial frames in which time flows in a homogeneous manner time is homogeneous, that is nice.

There is also another fact which you definitely know is that if you take your experimental setup whatever experiment is happening you take this thing and instead of doing it here, let us say you do it a million kilometres away from here, or billion kilometres away whatever results you will get here, it's the same you will get by doing the same experiment millions or billions of kilometres away.

So, this location is in no manner special compared to any other locations all locations are equivalent, meaning if I take my setup and translate it to another location this way, that way, this way whichever way you take it if you do a translation by some amount a, a is a vector if you if you translate everything by amount a, your results will not depend on the choice of a you do the experiment wherever you wish to and this is more formally said that said as the following.

So, you say that space is homogenous, all locations are equivalent that is what this sophisticated sounding sentence means. And that is the property of a special kind of a frame which we call inertial frames, there is still more one thing which you know is that if you take your experimental setup and now I am not putting it somewhere and not taking it in a different place, all I am I want to do is take this experimental setup and reorient it.

So, instead of let us say it was something was going from here to there something was being measured here there were forces that you take the entire set up and rotate it in this way or this way or whichever way you like. Doing this rotation will also not change the outcomes of your experiment.

All orientations are equivalent and this we say more formally as that space is isotropic, isotropic meaning equivalent in all directions. So, I say space is isotropic, space is isotropic. So, you see what we have written down is really nice, we are not telling about what will happen to a particle when it is in such a frame, we are telling things about how time behaves in such frames.

See generally when you talk about inertial frames, you will be told that if there is a particle and it is not acted upon by any forces if it is at rest or if it is in your motion it will remain so that is how you say but now we are describing inertial frames not by appealing to what happens to a particle but rather saying what is the nature of time in such a frame and space nature of space in such a frame.

And this will be very useful for us in understanding our laws of nature. So, we have, let us put it in a box and we will try to make slightly more mathematical statements of based on it. Now, you say that is all nice, you have something about some understanding of inertial frames. Now what can be what more can we say now?

Well, one thing it is clear that the system of your particles which is let us say we are looking at an inertial frame, then that system of particles should somehow know that they are in an inertial frame, they have to know the time is homogeneous in this frame that space is homogeneous that it is isotropic.

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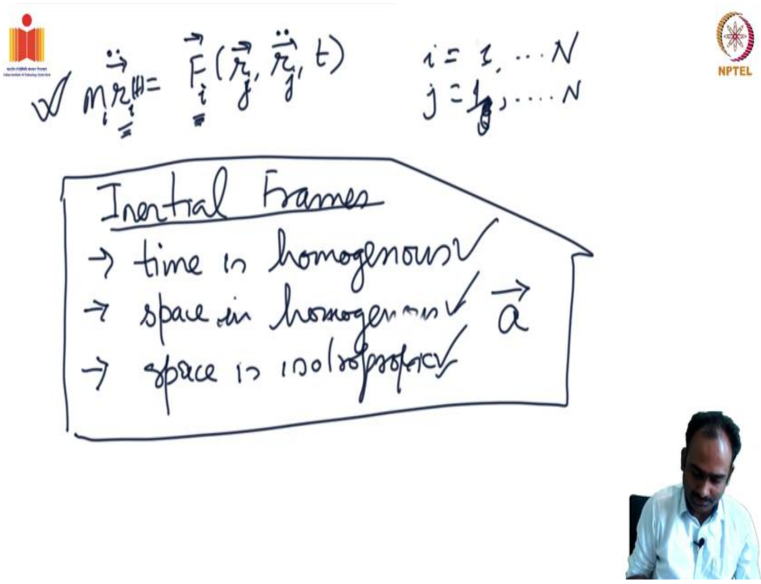

What part of the system knows about these things and clearly let us write down our equation of motion, so we had let us go back here I think, here this thing, now if I am saying that each particle is going to accelerate according to the forces, then it has to be those forces which will know about the frame, there is nothing else on the left hand, left hand side is just the acceleration of that particle and how much to accelerate and in what manner is told to it by the forces which is on the right hand side. So, the right hand side has to know about this information which we wrote here, so let us have a look at this, should I write let us write it again.

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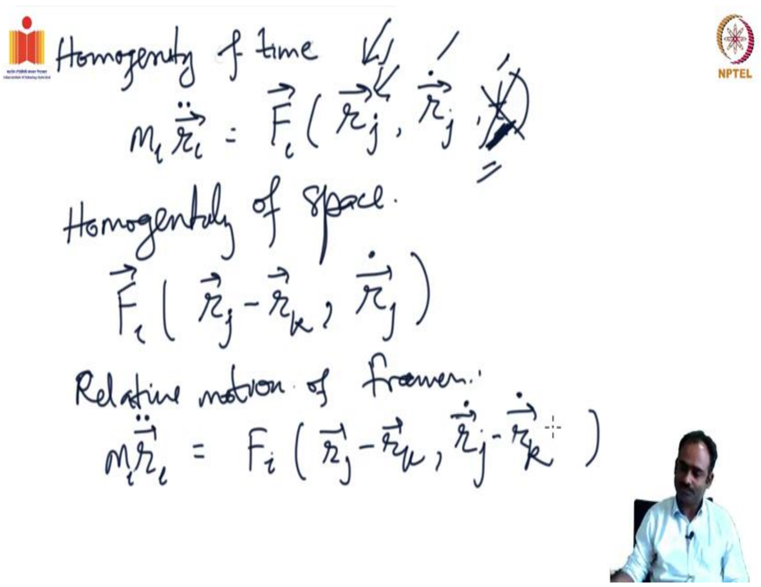

So, I want to now talk about first homogeneity of time, these are so mi ri double dot is force on i th particle which depends on where each particle is located at that time I am suppressing the t here I could have put a t here as well which I am suppressing for now j and t, that is what we wrote earlier. Now, let us see what homogeneity of time has to say about the forces.

Now, imagine you are doing the experiment now, and all the forces are behaving which are way they behave and you get the accelerations of each of the particles at this time. Now, as I said you can do the experiment thousand years later and you should expect everything the same, meaning all the particles will move around in time in exactly the same manner but that can happen only if the forces do not change over time.

If after doing if I do starting experiment 1 million years later and because of this time dependence forces change and everything will change, the evolution of the system will not be the same if the forces depended on time, so clearly this t cannot appear in the force. Now, remember I am talking about a system which is insulated from or isolated from all other things the system is self-contained there is no there are no external forces on the system whatever forces individual particles are experiencing they are due to all other particles in the system, that is very important otherwise what I am saying is not true.

So, that's nice we have said that the forces in an isolated system cannot be function of time that is already a progress, let us see what homogeneity of space implies. Again, let me write it down force on that particle, now you see, here we are saying that we need to know where exactly each particle is located, so let us say the system of two or twenty particles it does not matter look at two of them, let us look at the force on a particle here, I am saying the force on this is determined by the particle here, there, there, there, there, it really depends on r j where j runs from 2 to n let us say this is particle number 1.

Now, because of homogeneity of space if I take the entire set up and move to some other location if I do a translation by a vector so I let us say just translate everything along x axis nothing should change, whether I experiment I do here or take the entire box put somewhere else nothing should change which would mean that the forces should not care about where this guy is, where that guys, where that guy I mean what are the exact locations of those other particles at time t.

But what it should care about is what is the separation between these end that, we are talking about the forces on this, so separation between this and that, separation between this and that, this and that, this and that because everything should give the same answer if I translate from here to there meaning the forces cannot depend on the locations of all the particles but rather only on their differences. So, I will write r j minus r k and k and j and k runs over all the particles t anyway was already gone.

So, you see we have been able to put much more restriction on the nature of on the arguments that can enter into the force instead of having this full freedom here, now we have much more restricted form t is gone and the coordinates cannot appear directly they can only appear in differences, that is good how about the homogeneity, homogeneity we have taken care of let us talk about the isotropy or even before I do isotropy let me so let us talk about will come to isotropy later, let us talk about something else.

There is something more we know about nature is that if you are doing an experiment in one inertial frame whatever that frame is I will not go into details of how you find our initial frame that I believe you already know, if you are given an inertial frame and you take another frame which is moving in with the constant velocity with respect to this frame, then both the frames are inertial whatever physics you see in the first frame is exactly what you are going to see in the second frame, physics will not change by moving from this inertial frame to another frame which is in a real in a which is in a uniform motion with respect to the first one.

Now, I want to put this information into our equations of motion meaning it will be clearly the force which will know about this, so this let us call it relative motion of frames, so what will happen is this will be now already you had rj minus rk from the homogeneity of space time was already gone. Now, if you go to another frame in which is moving with some velocity V with respect to the first one, all the particles whatever their velocities were at time t in the frame number one will change to the velocities what they had plus the velocity of this frame.

So, all of them will get all the velocities at time t will get added by this fixed amount which is the relative velocity between the two frames. So, clearly because the physics is not going to change the force should not care about what the velocities are in a given frame. What are the velocities of each of the particles in that given frame, what it should care about is only the relative velocities because when you go from one frame to another frame which are moving with respect to each other, the relative velocities are not going to change because when you take the difference the additive part drops out.

So, which means that I should not have rj dot but I should have r sorry rj dot minus rk dot time was anyway gone. So, these are the constraints which we have been able to put on the forces and let me give you one example which is very familiar to you, so that you can see that indeed the forces which you already know are of this form.

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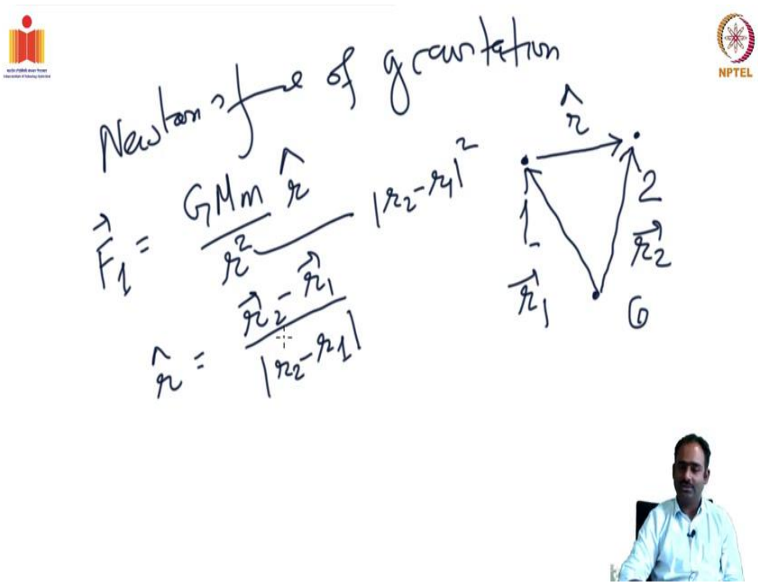

So, let us go back to over Mister Newton, Newton's force of gravitation, what is it? So, let us say you have two particles, particle number 1, particle number 2 some origin o and I say this is the coordinate r1 that guy is at r2 these are the vectors I want to know the force on the first one, clearly the force on the first one will be directed towards two because it is being pulled into two, it's the gravitational force.

So, the force on particle number 1 would be GMm over r square it says inverse square law you know and then you have a unit vector where this is the unit vector I can write down or hat more clearly for you it will be r2 minus r1 you see this is r2, r1 if you subtract this is what you will get over r2 minus r1 modulus that is the magnitude of it, so that is why this is a unit vector and this r is just r2 minus r1 modulus square.

You see the force depends only on the differences in the positions of these two parts, it does not care where these guys and where that guy is individually but only on the differences here you also do not see any time and in this force, there is no velocity involved. That is all for this video and we will start talking about I believe more about the coordinates which we should use in studying different kinds of systems, what kind of coordinates are suitable we will have a long discussion on that next time, okay see you.