Electromagnetism
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Lecture – 83
Maxwell's equation in matter and the boundary conditions

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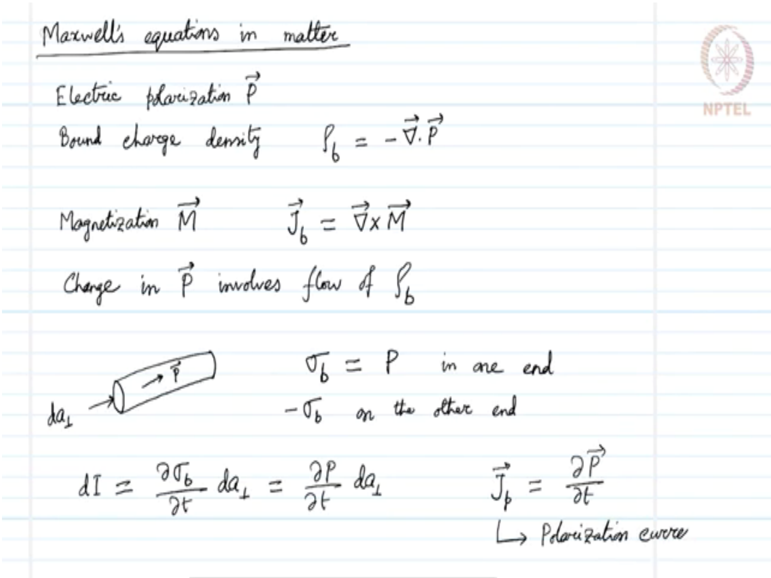

Now, let us consider the Maxwell's equations in the presence of matter. Maxwell's equations are complete and correct as they stand, but when we have a polarized material under consideration, then we have to be careful about this. Why? Because, if we have an electric polarization P. Then, we have certain bound charge density.

How do we get the bound charge density? The volume bound charge density is given as the negative divergence of the polarization. Similarly, if we have a magnetic polarization that is

Magnetization M, we have a corresponding bound current density where the volume current density is given by the curl of this magnetization.

But there is one new feature to consider, when we have non static polarization. If we change the electric polarization that involves flow of the bound charges. So, let us consider a small tiny chunk of polarized material like this. Here is an exposed area, there is another exposed area and let us consider the polarization vector in this direction and let this be the direction of the perpendicular area. If we have this kind of an arrangement then the polarization introduces a surface charge density, surface bound charge density that can be given as the polarization itself, the magnitude of the polarization itself under this configuration where the area and the polarization are in the same direction.

And so that is in one end. And on the other end it would be minus sigma b, due to this polarization. Now, if we increase this polarization then the bound charges are going to increase as well; that means, that will as be similar to a flow of current. If amount of d I current flows due to this infinitesimal increment in the polarization, then that can be given as del sigma b over del t times d a perpendicular which is nothing, but del P del t da perpendicular.

So, this current density can be given as so we call it the polarization current, and its given as del P, del t ok. After introducing the concept of polarization current ah. So, we know that this has nothing to do with the bound current. Bound current is due to magnetization and polarization current that we are talking about is due to electric polarization.

Now, this polarization current after introducing this polarization current we have some change in the total current, but that must be consistent with the continuity equation; that means, the conservation of electric charge.

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$$\vec{\nabla}\cdot\vec{J}_p = \vec{\nabla}\cdot\frac{\partial\vec{P}}{\partial t} = \frac{\partial}{\partial t}\left(\vec{\nabla}\cdot\vec{P}\right) = -\frac{\partial\rho_b}{\partial t}$$

$$\rho = \rho_f + \rho_b = \rho_f - \vec{\nabla}\cdot\vec{P}$$

$$\vec{J} = \vec{J}_f + \vec{J}_b + \vec{J}_p = \vec{J}_f + \vec{\nabla}\times\vec{M} + \frac{\partial\vec{P}}{\partial t}$$

Gauss Law

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{\epsilon_0}\left(\rho_f - \vec{\nabla}\cdot\vec{P}\right)$$

$$\vec{\nabla}\cdot\vec{D} = \rho_f \qquad \vec{D} = \epsilon_0\vec{E} + \vec{P}$$

So, we can write that the divergence of this polarization current J p, that is equal to the divergence of del P, del t that we can now flip this operators del and del t time derivative and space derivative. So, del t of the divergence of P, must be equal to according to the continuity equation, minus del rho b, del t.

So, the charge density can be separated into two parts, rho equals rho f, plus rho b. We knew this from earlier and the current density can be separated into actually three parts ok. So, let us write the charge density as rho f, minus the divergence of the polarization which gives us the bound charge bound volume charge density. For the current density, it becomes J equals, J f plus J b bound current density plus J p the polarization current density.

So, it split into three parts which essentially gives us J f plus, the curl of magnetization that represents the bound volume current density, plus del P, del t. This gives us the polarization current density. And once we have this the Gauss law, would take the form the divergence of electric field is one over epsilon naught rho f minus, the divergence of the polarization. So, this may be written as the divergence of the electric displacement vector equals rho f. Where the displacement vector is given as epsilon naught times e plus the polarization vector.

So, we have done this earlier and it doesn't change at all. Well, whatever we have done earlier that remains here in the context of Gauss law, let us consider Amperes law now.

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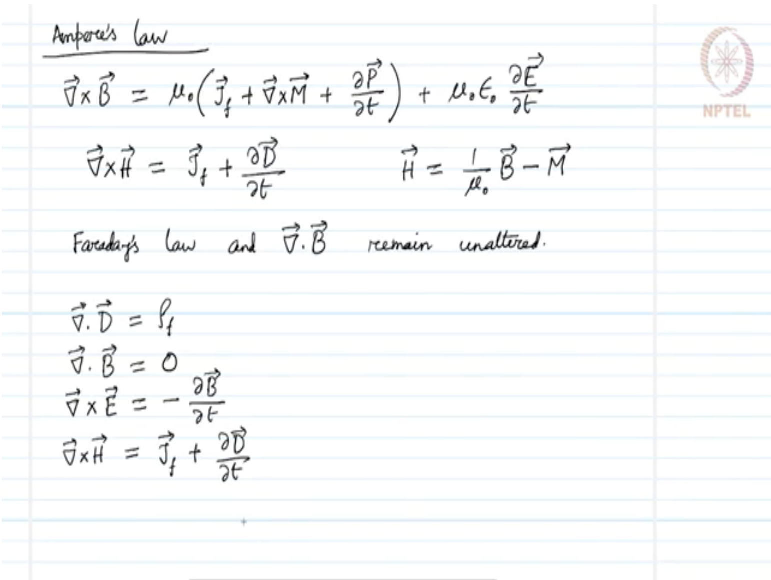

Curl of B, will be mu naught times the sum of all type of current densities that is the free current density, plus the bound volume current density that is the curl of the magnetization, plus the polarization current density that is del P, del t plus of course, the displacement current term mu naught epsilon naught del e, del t.

Now, in terms of the H field, we can write curl of H equals J f plus del D, del t where H is given as 1 over mu naught times B minus the magnetization vector. And D vector is the electric displacement vector. And we know that the Faraday's law and the divergence of B, these two laws remain unaltered.

So, if we write down all the Maxwell's equations in the presence of matter. We can write it in the following way, the divergence of the displacement vector is the free volume charge density. The divergence of the magnetic field is 0. The curl of the electric field that is Faraday's law is unaltered minus del B, del t. And the curl of the magnetic H field is J f, the free current density plus del D, del t the time derivative of the displacement vector.

Now, let us consider the boundary condition with Maxwell's equations, in the presence of matter.

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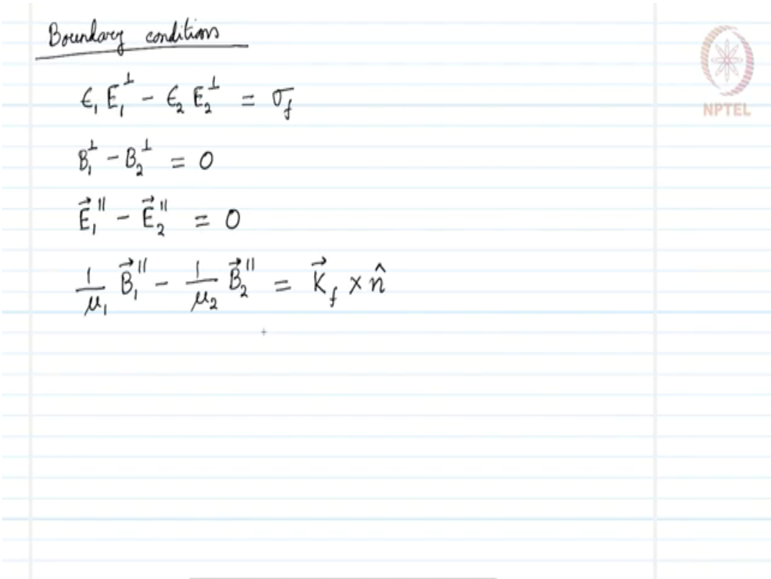

So, we have four fields under consideration E, B, D and H. And all these fields would have discontinuity across a surface charge density and surface current density. So, we can use just like earlier a surface with charge density and current density and we can use Gaussian Pillbox and Amperian loop across that surface and find out the discontinuities. And if we work all these things out, we will find that the electric field and magnetic field across the boundary can be expressed the discontinuity in that can be expressed as epsilon 1, E 1 perpendicular component minus epsilon 2, E 2 perpendicular component.

So, 1 and 2 are two different sides of that surface charge and current density. This equals the surface free charge density. B1 perpendicular minus B2 perpendicular remains 0. E1 parallel component, minus E2, parallel component that becomes 0 and 1 over mu 1, B1 parallel minus

1 over mu 2, B2 parallel that gives us the free current density on the surface cross the direction perpendicular to the surface.

So, these are the boundary conditions in presence of a material, in presence of. So, it's at the interface of two different materials and at the interface we have some charge density or current and or current density. With this these are the boundary conditions that we have.