Electromagnetism

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Lecture – 81
Maxwell's correction to electromagnetism

Hello. So, we discussed the Electromagnetism before the advent of Maxwell. And we have seen the Faraday's law how ah time varying magnetic field leads to some curl in the electric field; and we have seen how that has modified the equations relevant to electromagnetism. And we have written down all four equations relevant to electromagnetism before Maxwell. Now, what made Maxwell modify these equations in the context of electromagnetism? Let us consider that situation.

So, there was an inconsistency, in order to fix that inconsistency, reconcile everything properly. Maxwell has introduced a new term in the Ampere's law, let us see what kind of situation led to that inconsistency.

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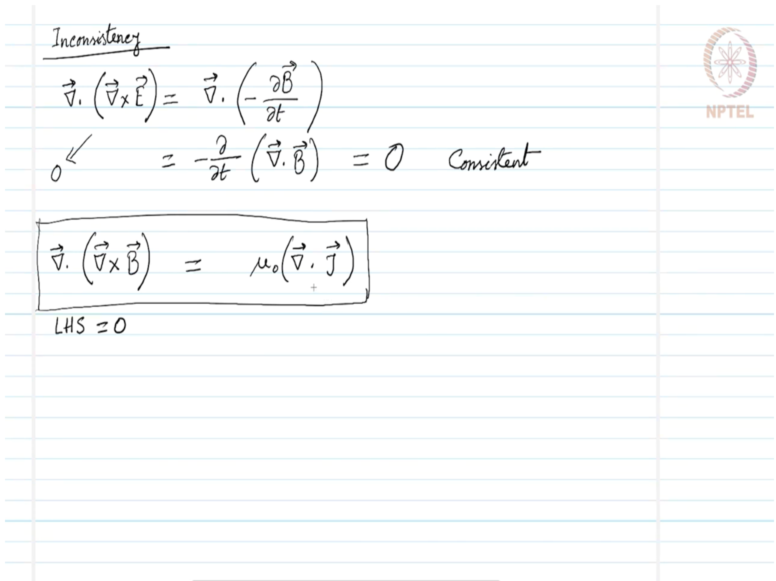

So, if we consider the curl of the electric field, that is given by minus del B del t. Now if we take the divergence of this quantity then, its this which is nothing but the divergence of minus del B del t. And that is since the divergence and the time derivative they commute, we can take the time derivative out.

So, it becomes the negative time derivative of the divergence of the magnetic field and the divergence of the magnetic field is 0 so, this quantity has to be 0. But now divergence of a curl is always 0. So, this quantity on the left-hand side must be 0, on the right-hand side we have found this to be 0 so everything is consistent so far . How about considering the other way around? If we take the curl of a magnetic field that is mu naught times the current density. Now, if we take the divergence of this quantity, that is mu naught times the divergence of the current density.

And so, divergence of a curl is always 0, the left-hand side must be 0. But what is the reason? Sorry, please delete this part delete where I have said the left-hand side must be 0. So, we have a divergence of curl here and that equals to the divergence of current. Now, we can see that the divergence of curl is 0 so we have the left-hand side to be 0, but there is no reason that the divergence of the current density would go to 0, this may not be 0.

Do we have an example where the divergence of the current density is not 0? Let us consider 1 example, let us consider the case when a capacitor is being charged.

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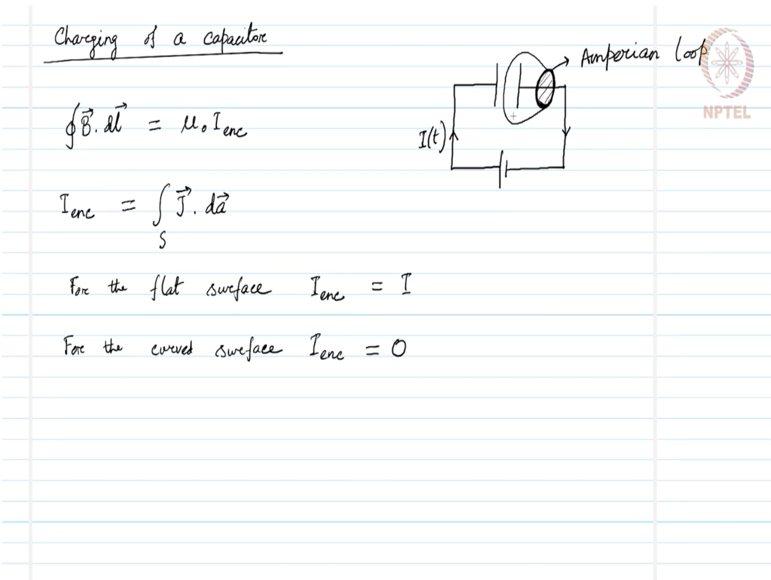

So, we draw a circuit here, its a parallel plate capacitor and its connected with a battery like this. So, the capacitor is getting charged, its a closed loop. During the charging of this capacitor, a current I that is the function of t flows into this circuit and when the capacitor is fully charged, there is no current flowing in the circuit. The capacitor does not allow any current anymore.

Now the current is flowing through the wire, but not through the capacitor because in between the capacitor with plates, we have some dielectric material that does not allow any current to flow through it. And now let us consider an Amperian loop like this. For this Amperian loop, we can write that the closed integral of B dot dl.

This can be given as mu naught times the current enclosed using Ampere's law in integral form. And the current enclosed in this case is the surface integral of J dot da where da is the area cross-section of this ah current carrying wire. Now this is valid for any surface. We can consider this loop so which area do we consider that is enclosed by this Amperian loop?

We can consider the flat area here, but we can also consider the area of a balloon like this. That is also allowed and so, if we consider the flat surface shaded here, then the current enclosed is for the flat surface is the amount of current flowing through the wire.

But if we considered the curved surface like the balloon, then it does not enclose any current whatsoever because this curved surface is going through the capacitor and inside the capacitor there is no current even when the capacitor is getting charged. So, the current is 0 and that brings in an inconsistency. What is the closed integral of B dot dl? Now Maxwell fixed this inconsistency. How did he fix this? The problem is that if we consider this equation here, this one the divergence of a curl is always 0.

So, the right-hand side should also be always 0, but if the right hand side is not 0 of this equation we have to make it 0. Maxwell tried to make that right-hand side 0. How did he do that? He did that by applying the continuity equation.

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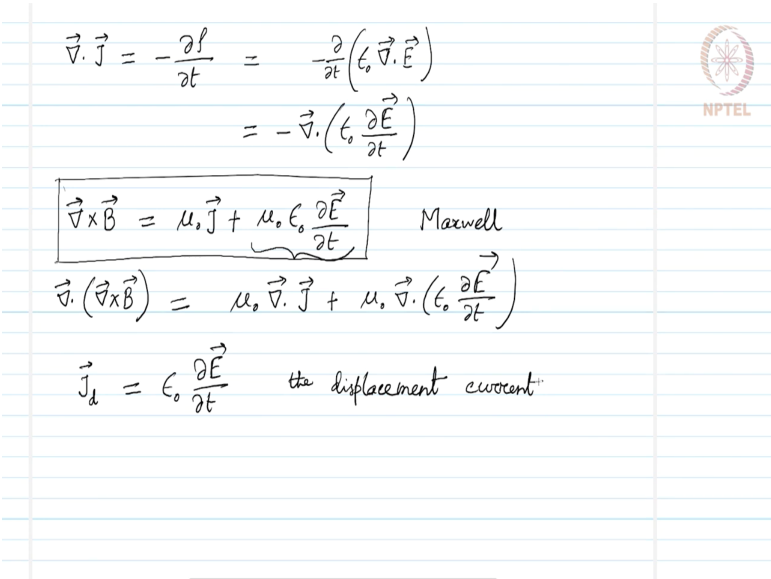

So, on the right-hand side, we have the divergence of the current density which is according to the continuity equation, minus del rho del t. And rho using Gauss law can be given as epsilon naught divergence of the electric field minus del t of that comes here. That means, this quantity is equal to minus the negative divergence of epsilon naught del E del t; because the time derivative and space derivative they commute with each other.

Now if we have this, then now if we write the curl of the magnetic field B as mu naught J plus mu naught epsilon naught del E del t. And now if we take the divergence of this equation, that will give us divergence of the curl of B, which is mu naught times the divergence of J plus mu naught epsilon naught times the diverge. So, the divergence of let me take epsilon naught inside not outside.

So, we can write mu naught times the divergence of epsilon naught del E del t. Now from the continuity equation, we can clearly see that the right-hand side becomes 0 and the left-hand side is also 0 because it involves the divergence of a curl. Now, this equation, this equation is consistent with every physical principle. And this was due to Maxwell. This is the way Maxwell fixed this inconsistency.

So, this term here that represents a current density. We can write that as J d, its called the displacement current equals epsilon naught del E del t. After adding this, Maxwell's term to Ampere's law, the Maxwell's equations get modified, let us write down.

So, the equations relevant to electromagnetism get modified and all four equations are now called Maxwell's equations. Let us write down those Maxwell's equations.

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Maxwell's equations

1. $\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$

2. $\vec{\nabla} \cdot \vec{B} = 0$

3. $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

4. $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

The first equation becomes in its differential form, the divergence of electric field equals the volume charge density over epsilon naught. The second equation is the divergence of magnetic field is as always 0, the third equation is about the curl of electric field that is Faraday's law which gives us minus the rate of change of magnetic field with time.

And the fourth equation as modified by Maxwell gives us curl of B equals mu naught times the volume current density, plus mu naught times the displacement current that is epsilon naught del E del t.

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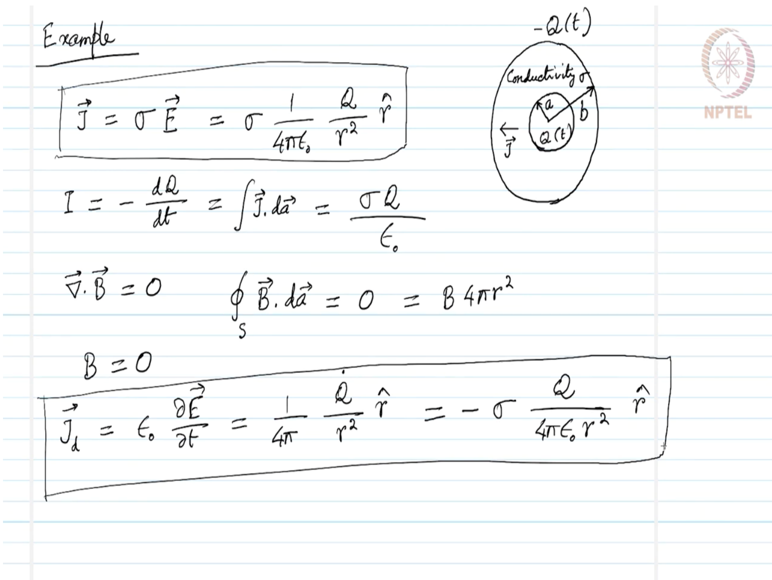

Now let us consider an example. After finding after writing down the Maxwell's equations. Imagine two concentric metal spherical shells; one is big and the other one is small with a common center here. The smaller one is with radius a and the bigger one has radius b. And the inner one with radius a, that carries a charge Q that is a function of time, its not constant over time. And the outer one that also carries a charge exactly of same amount, but opposite in magnitude that is minus Q.

The space between them is filled with some ohmic material with conductivity sigma. So, we will have a radial current flowing from the inner cylinder to the outer cylinder and this current can be given as sigma times the electric field. And if we work out the electric field using Gauss law for this simple system, we can find out that that current density would be 1 over 4 pi epsilon naught sigma times this times Q over r squared along r cap direction.

So, we can write I the total current equals minus d Q dt which is integration over J dot da and that becomes sigma Q over epsilon naught. Now, this is a spherically symmetric configuration that we have at hand. So, the only direction the magnetic field can point is radial. Then, we can write the divergence of B that equals 0. And that means, closed surface integral over B dot da that also goes to 0. Now if B was uniform over the surface, then it would have the magnitude of B times 4 pi r squared.

Now what would be 0? B must be 0 in this case. Otherwise, this integral cannot be 0. So, how could B be 0? In this situation, there is a current so, current should develop some magnetic field, but there is no magnetic field in this situation. How could that happen? That could happen because if we consider the displacement current then, that is of the amount epsilon naught del E del t which is nothing but 1 over 4 pi and we have the expression for the electric field, from that we can write the time derivative of Q as Q dot over r squared r cap.

This is the expression for the displacement current and this is exactly equal and opposite to the current that we have minus sigma times q over 4 pi epsilon naught r squared. So, and in r cap direction. So, the displacement current density is exactly equal and opposite to the real current density that we have, the current density volume current density that we have calculated earlier.

Compare this equation and this equation and that tells us why there is no magnetic field. The displacement current exactly cancels the conduction current and therefore, we expect no magnetic field in this example.