

**Electromagnetism**  
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**Lecture – 76**  
**A tutorial on the magnetic dipole moment**

Hello, now we discuss tutorial 7. In this tutorial, we are going to discuss a problem of involving magnetic dipole and another simple problem; very simple problem for you to solve. We will just state the problem.

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Problem 1: Magnetic dipole due to spinning charged sphere



A spherical shell, of radius  $R$ , carrying a uniform surface charge density  $\sigma$ , is set spinning at angular velocity  $\omega$ . Find the magnetic dipole moment of the spinning spherical shell. Show that for points  $r > R$  the potential is that of a perfect dipole.



So, let us move on to the first problem the first problem is about the magnetic dipole due to a spinning charged sphere. So, you have a spherical shell of radius capital  $R$  and that carries the





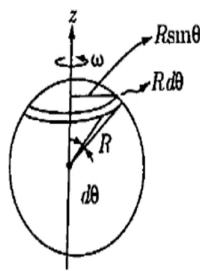
uniform surface charge density  $\sigma$ . Now this sphere is set spinning the way earth spins about its axis at an angular velocity  $\omega$ .

And you are supposed to find the magnetic dipole moment of this spinning spherical shell and you are supposed to show that for points small  $r$  that is the usual spherical coordinate  $r$  when it is greater than the radius; that means, if you have a point outside the sphere, the potential for this spinning sphere is that of a perfect dipole.

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**Problem 1: Steps for the solution**

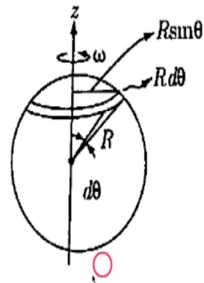
- Step: Take a ring on the sphere
- Step: Find the total charge on the ring
- Step: Find the current due to angular motion of the ring
- Step: Calculate magnetic moment of the ring



So, what are the steps involved in solving this problem? You are supposed to take a ring on the sphere first the ring. That is shown here in the picture and find the total charge on the ring. And then when you have this angular motion of the ring, you are supposed to find the current due to that motion of the ring, then calculate the magnetic moment of the ring. Here you can pause the video, try solving the problem yourself and restart the video if you need hints.

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Problem 1: Solution



Total charge on the ring of radius  $R \sin \theta$ ,

$$dq = \sigma \text{Area} = \sigma 2\pi R \sin \theta R d\theta$$



Let us solve the problem total charge on the ring of radius  $R \sin \theta$ . So, you can see that this radius is  $R \sin \theta$ . The total charge can be expressed  $dq$  as  $\sigma$  that is the charge density uniform surface charge density times the area of this ring. The radius is  $R \sin \theta$  and this width is  $R d\theta$ . So,  $R \sin \theta$  times  $R d\theta$  is the area of sorry  $2\pi R \sin \theta$  times  $R d\theta$  is the area of this ring. And if we multiply that with  $\sigma$ , we will get the elemental charge  $dq$  on the ring.

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### Problem 1: Solution (continued)

So the current due spin of the sphere on the ring is given as,

$$I = \sigma\omega R^2 \sin\theta d\theta$$

Magnetic dipole moment of the ring is

$$d\vec{m} = I(\text{Area})$$

Area of the ring  $\pi(R \sin\theta)^2$

$$d\vec{m} = I(\text{Area}) = \sigma\omega R^2 \sin\theta \pi(R \sin\theta)^2 d\theta \hat{z}$$

Total magnetic dipole moment

$$\vec{m} = I(\text{Area}) = \int_0^\pi \pi\sigma\omega R^4 \sin^3\theta d\theta \hat{z}$$
$$\vec{m} = (4/3)\pi\sigma\omega R^4 \hat{z}$$



And once we have this elemental charge on the ring, the current due to the spinning sphere coming from this ring is  $I$  is given as  $\sigma\omega R^2 \sin\theta d\theta$ ; that is obvious. Now the magnetic dipole moment that is given us current times the area. So, elemental magnetic dipole moment is  $d\vec{m}$  and that is current times the area.

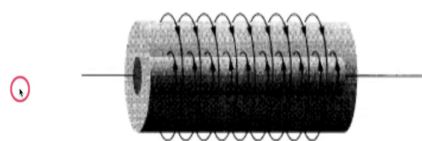
Area of the ring is  $\pi R^2 \sin^2\theta$  and  $d\vec{m}$  is given as  $\sigma\omega R^2 \sin\theta \pi R^2 \sin^2\theta d\theta \hat{z}$ . It is along the  $z$  cap direction. So, the total magnetic dipole moment that can be found by integrating  $\theta$  from 0 to  $\pi$   $\sigma\omega R^4 \int_0^\pi \sin^3\theta d\theta \hat{z}$ , it is along  $z$  cap direction. So, the total magnetization magnetic moment of this spinning sphere would be  $(4/3)\pi\sigma\omega R^4 \hat{z}$  along the  $z$  cap direction.

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### Problem 2: Coaxial solenoids

Two long coaxial solenoids each carry current  $I$ , but in opposite directions, as shown in Figure. The inner solenoid (radius  $a$ ) has  $n_1$  turns per unit length, and the outer one (radius  $b$ ) has  $n_2$ . Find  $\vec{B}$  in each of the three regions:

- 1 Inside the inner solenoid
- 2 Between the solenoids
- 3 Outside the outer solenoid



Solve yourself

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Now, let us come to the simple problem. Here we have two long coaxial solenoids, each carry a current  $I$ , but in opposite directions as shown in this figure. The inner solenoid that has radius  $a$  has  $n_1$  number of turns per unit length and the outer solenoid with radius  $b$  has  $n_2$  number of turns per unit length.

So, we know we have already worked out how magnetic field behaves inside the solenoid and outside the solenoid, we are going to apply that in this problem. So, in this problem you are supposed to find the magnetic field in three regions. One is inside the inner solenoid, two between the solenoids and outside the outer solenoid. That is what you are supposed to find from this problem.

And this problem is a simple one, just apply the expressions for the magnetic field in case of solenoid; infinitely long solenoid you must remember, otherwise the magnetic field would

have some other kind of effect. But here we are ignoring those kind of effects of a finite solenoid; infinitely long uniform solenoid. So, the problem is rather simple and solve this problem solve this problem yourself. That is the end of this tutorial.