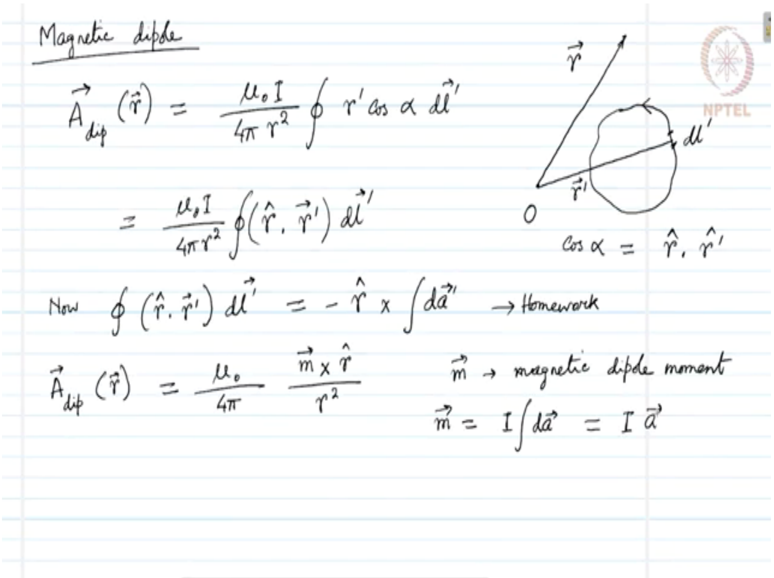
Electromagnetism

Dr. Nirmal Ganguli
Department of Physics
Indian Institute of Science Education and Research, Bhopal

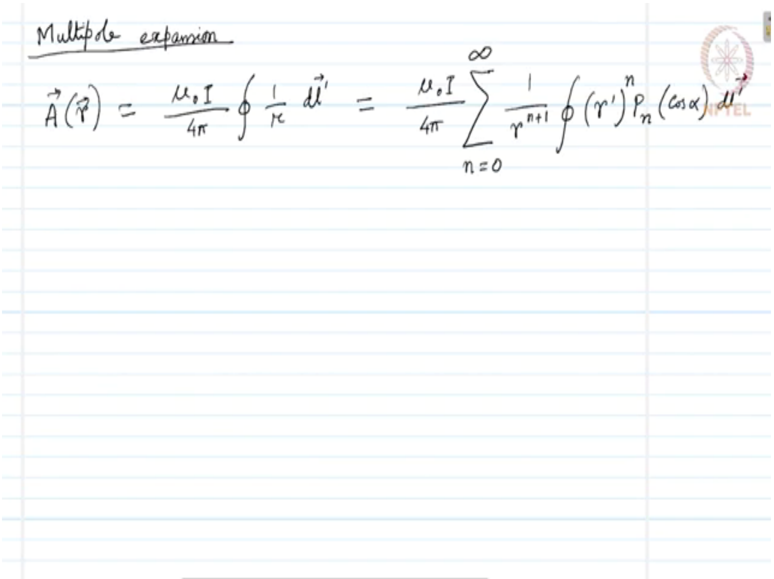
Lecture – 72
Multipole expansion of the vector potential

(Refer Slide Time: 00:32)



Now, if we have a vector potential, can we expand it in terms of Multipoles? Yes, that is possible.

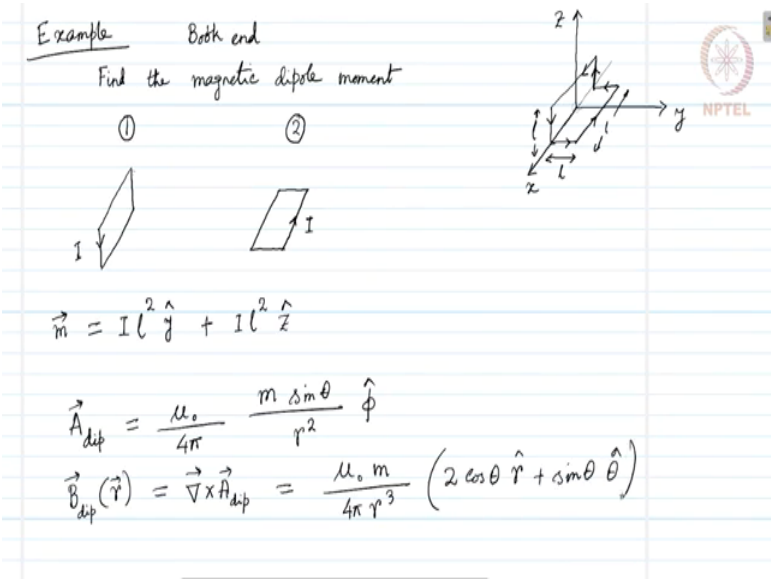
(Refer Slide Time: 00:44)



Just like the case of electrostatic potential, we could expand the magnetic vector potential also in terms of multipole. So, how does the multiple expansion look like in the context of magnetic vector potential? This can be expressed as mu naught I over 4 pi closed integral over 1 over r dl prime. This is the vector potential which is mu naught I over 4 pi sum over n equals 0 to infinity for multipole expansion, 1 over r n plus 1 closed integral over r prime power n legend a polynomial P n that is a function of the cosine of alpha dl prime vector. This is the multipole expansion.

After defining the dipole moment, the vector potential due to dipole and the multipole expansion, let us see an example where we find the magnetic dipole moment of a current carrying loop.

(Refer Slide Time: 02:38)



So in library, you can see book ends; book ends are shapes like this. So you its the other way around. If you have a book like this; rectangular book, you can put it with a book end like this and the bookend will ensure that the book never falls down. So book end is a 3-dimensional structure, we can draw it in the following way in x, y, z Cartesian coordinate system. Let us consider this is x-axis, this is y-axis and this is z-axis.

So, we can have this is the vertical part of it and this is the horizontal part of the book end something like this we can have. We have a book end here and we consider that a current I passes through this bookend. So our loop, current carrying loop has this book end kind of a shape and this current is passing through the shape.

If we consider this length to be l, then also this length is l all sides have length l, this length is also l things like that and it carries a current I. So we want to find the magnetic dipole moment for this kind of a configuration ok. So, how do we find the magnetic dipole moment? Let us try to split the problem into 2. So we have a loop that is of a 3-dimensional shape, we want to split this into 2 loops of 2-dimensional shape. Let us see if we can do that.

The first loop would be like the vertical part of this loop what we have this and the current will be along this direction I and the second loop would be like the horizontal part of it and the current on this is also I, that is along this direction.

And if we split this into 2, then the problem becomes very easy we just multiply the current with the area that is l squared and for the first loop its in y cap direction and for the second loop the magnitude of the dipole moment is the same I times l squared and its in the z cap direction. So, that is the magnetic dipole moment for this entire system. Now, let us see the situation whether the magnetic dipole moment is independent of the choice of origin or not.

Let us work that out in spherical coordinate system that will help us. So in spherical coordinate system let us consider a dipole at the origin pointing along the z-direction if we have a dipole if you have a magnetic dipole at the origin pointing along the z-direction, then we can write for that the magnetic vector potential due to the dipole is mu naught over 4 pi m sin theta over r squared phi cap; m is the dipole moment.

And with this situation, the magnetic field due to this dipole can be given as simply curl of the magnetic vector potential which is mu naught m over 4 pi r cubed twice of cosine of theta and this is along r cap plus sin theta along theta cap. This will be the magnetic field due to this kind of a dipole and this expression is same as the electric field due to a dipole. An electric dipole of course, in that case and in this case its a magnetic dipole, but the expression remains unchanged.