Electromagnetism
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Lecture – 70
Boundary conditions on magnetic field

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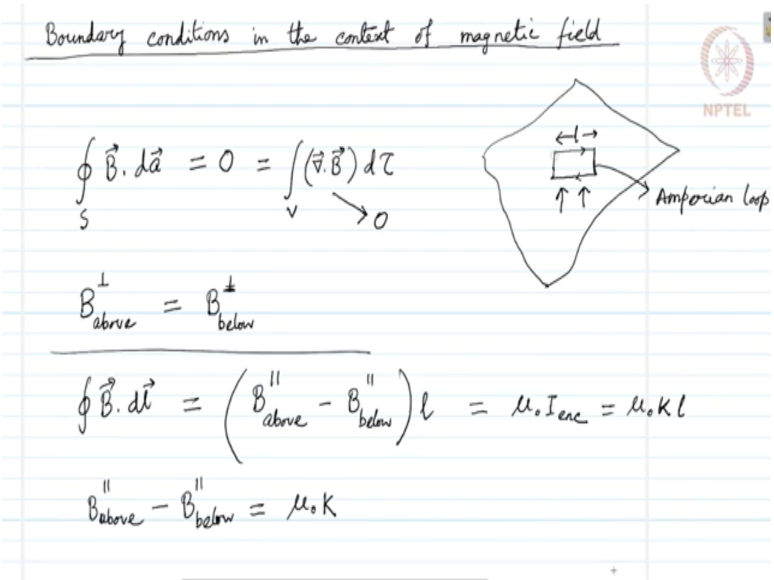

Let us consider the Boundary condition in the context of magnetic field. So, just as electric field suffered a discontinuity across a surface charge a magnetic field will suffer a discontinuity across a surface current. Let us draw this kind of a plane on which we will assume a surface current. And we will see that the surface integral over a closed surface B dot da this quantity across a surface current goes to 0 because this quantity is equal to the volume integral over the volume that is enclosed by this surface divergence of B d tau.

So, the divergence of B is always 0 therefore, the surface integral B dot da the flux of a magnetic field over a surface enclosing a volume will always be 0. And if we have this kind of a surface on which there is a current K we can consider a box on this surface and there is also a part of this box that extends below the surface.

So, this surface actually see this box actually is bisected by this surface on which the current is flowing just like the Gaussian pillbox that we considered earlier. So, we have a surface that encloses a volume namely the surface of the box. Therefore, we will have with very thin box we can write that since the surface integral over this enclosed surface will always be 0, we can write that the perpendicular component of the magnetic field above is equal to the perpendicular component of the magnetic field below.

This is always going to be the going to be true. How about a tangential component? For a tangential component we do not need a box here, we need something else. Let us get rid of the box and try to draw an Amperian loop and the direction of dl is this direction. So, here it is this direction, this length of this Amperian loop is l it is also very thin these small arms they are much thinner than what I have drawn and this rectangle is bisected by the plane carrying the current. So, let us assume that the current is flowing in this direction. So, our Amperian loop is a cross section across which the current is flowing.

In that situation we can write on this Amperian loop closed line integral of B dot dl that will be nothing but B parallel above minus B parallel below, although the parallel times l. Although the parallel component of the magnetic field is a vector because we have considered a particular direction of the magnetic field, here we are talking about only one parallel component of it.

So, we have written it in terms of a scalar and that is equals mu naught times the total current enclosed which is mu naught K times the disk the length of this box l provided our surface current is uniform.

Then we can write down that the parallel component of the magnetic field above the plane minus the parallel component of the magnetic field below the plane that will be mu naught times K. Hence we can clearly see the discontinuity in the magnetic parallel component of the magnetic field above and below the plane.

Remember in case of electro static field we had the discontinuity in the perpendicular component and the parallel component was continuous; here it is the other way around. The perpendicular component of the magnetic field is continuous and the parallel component experiences a discontinuity.

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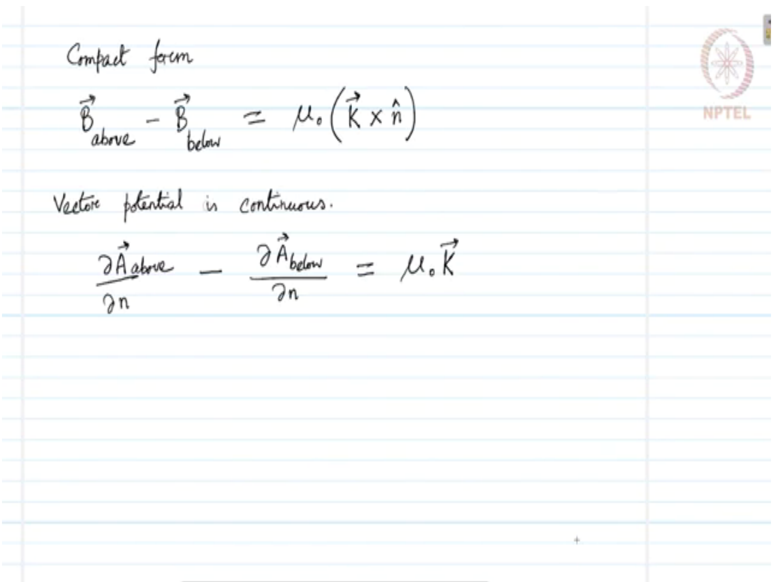

Now, if we try to write everything in one vector form the entire boundary condition that we worked out in one vector equation then we can write in a compact form, B vector above the plane minus the magnetic field vector below the plane that equals mu naught times the surface current density cross n cap, where n cap is normal to the surface that is under consideration.

And we have some discontinuity in the magnetic field. How about the vector potential? The vector potential because it is a potential and by differentiating it and in by taking the curl of that quantity we are going to get the magnetic field. Therefore, the vector potential must be continuous across any boundary. Every component of the vector potential will be continuous across any surface current density, there is no other possibility, but the normal derivative of the vector potential is not continuous. So, we can write the discontinuity in the magnetic field in terms of the normal derivative of the vector potential in the sense in the in this in the following way.

Del A above del n minus del A below del n, this quantity equals mu naught K. And this equation does not say anything about the vector potential itself rather it expresses the discontinuity in the magnetic field that we have already derived.