Electromagnetism
Dr. Nirmal Ganguli
Department of Physics
Indian Institute of Science Education and Research, Bhopal

Lecture - 62
Straight line current: Curl of the magnetic field

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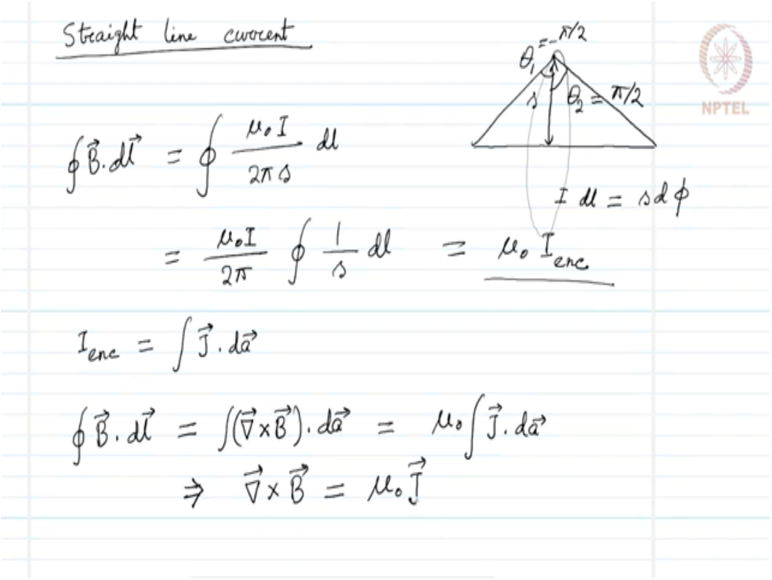

Straight line current

$$\oint \vec{B}\cdot d\vec{l} = \oint \frac{\mu_0 I}{2\pi s}\, dl$$

$$= \frac{\mu_0 I}{2\pi} \oint \frac{1}{s}\, dl = \mu_0 I_{enc}$$

$I\ dl = s\, d\phi$

$$I_{enc} = \int \vec{J}\cdot d\vec{a}$$

$$\oint \vec{B}\cdot d\vec{l} = \int (\vec{\nabla}\times\vec{B})\cdot d\vec{a} = \mu_0 \int \vec{J}\cdot d\vec{a}$$

$$\Rightarrow \vec{\nabla}\times\vec{B} = \mu_0 \vec{J}$$

Now, let us consider a straight line current. On a straight line, we have seen the situation we have already worked out the example where we have considered a very long wire and at a distance s from this wire we have found out the magnetic field.

So, the magnetic field the magnitude of that magnetic field was mu naught I over twice pi s where s is this distance. Distance of this wire from the point of observation, and this is valid for very long wire by putting the appropriate values of theta 1 and theta 2; theta 1 that was this angle that we set to minus pi over minus pi by 2 and theta 2 is this angle that we set to pi by 2 by doing this we obtained this expression for the magnetic field at this point; that means, that this magnetic field corresponds to an infinitely long wire.

And if we have that then can we find out cyclic integral over B dot d l that is cyclic line integral over B dot d l if we try doing that, we will have to write down this and if we do this in cylindrical coordinate system except s everything else is constant here in under the integral. So, that will come out mu naught I over twice pi integration cyclic line integral over 1 over s d l and what is the result of this if we perform this we will get the result will be mu naught times the current enclosed by this loop that we are considering by the line that we are considering.

So, the line would be something that circles around this wire. If we consider that kind of a line B will be constant over that and when B is constant over that the line element on this line d l that can be expressed as. So, what is changing the phi angle is changing here. So, this would be s d phi if we put in place of d l s d phi s and s will cancel.

So, integration will be over twice pi sorry integration will be over d phi which will result into twice pi twice pi twice pi will cancel and we will be left with mu naught times the current enclosed by this loop. So, this answer is not dependent on the distance of our point of observation or the radius of the loop in this case when we have this we can also write that the current enclosed is equals the surface integral over the volume current density.

Now, on this expression cyclic line integral of B dot d l if we apply the Stokes theorem we can write down that cyclic line integral of B dot d l over a closed line that is equal to the surface integral over the curl of B. Stokes theorem gives us this and this quantity equals mu naught times surface integral over J dot d a and these two quantities would be equal provided the integrant is equal; that means, we will have from this curl of B equals mu naught J that is something very interesting result that we have obtained.

In case of electric field we have found that curl of the electric field was 0 electric electrostatic field and in case of magnetic field magneto static fields curl is nonzero it is proportional to the volume current density that is something very interesting we have obtained with a simple example of infinitely long straight wire carrying a steady current I.