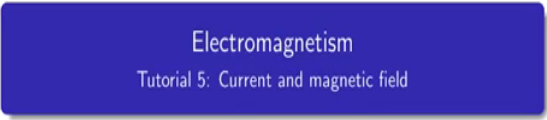



**Electromagnetism**  
**Dr. Nirmal Ganguli**  
**Department of Physics**  
**Indian Institute of Science Education and Research, Bhopal**

**Lecture - 61**  
**A tutorial on currents and magnetic field**



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Nirmal Ganguli

Department of Physics  
Indian Institute of Science Education and Research Bhopal

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So let us consider the next tutorial that is, tutorial 5 and its about current and magnetic Fields.

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**Problem 1: Current**



A current  $I$  flows down a wire of radius  $a$ .

- If it is uniformly distributed over the surface, what is the surface current density  $K$ ?
- If it is distributed in such a way that the volume current density is inversely proportional to the distance from the axis  $J = \frac{k}{s}$ , then find the  $J$  in terms of  $I$ ?
- Step: Use definition of surface current density  $K = \frac{I}{L_{per}}$ .
- Step:  $L_{per}$  is width of surface element perpendicular to the current?
- Step : Find out the  $L_{per}$  of the present case.

①



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The first problem here that we discuss is about current. So, we have a current  $I$  that flows down a wire of radius  $a$ , is a cylindrical wire that we consider and its of radius  $a$ . And if it is uniformly distributed over the surface of the wire, what is the surface current density  $K$ ? That something you are supposed to find out.

And in the next scenario, if it is distributed in such a way that the volume current density is inversely proportional to the distance from the axis; that means,  $J$  is  $k$  over  $s$  this small  $k$  here is just a constant, it is it has nothing to do with the surface density and then find  $J$  in terms of  $I$ . So, what are the steps involved in here?.

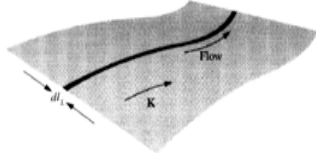
Use the definition of surface current density that is  $K$  equals to  $I$  over the perpendicular length. And the perpendicular length  $L_{per}$  is the width of the surface element that is perpendicular to the current and find out the  $L_{per}$  that is perpendicular length for the present

case. Now you can pause the video and try doing this and then when you are done, you can restart the video.

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Problem 1: Solution


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$$K = \frac{I}{L_{per}}$$

For the wire of radius  $a$ ,

$$L_{per} = 2\pi a$$

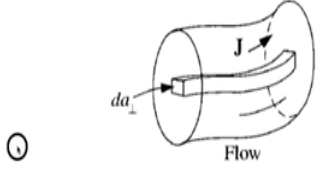

$$K = \frac{I}{2\pi a}$$


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So, this is a picture of the surface current. In case of surface current  $K$  flowing like this, and this is  $d l_{perpendicular}$  that is the length along the perpendicular direction to the current. The current is flowing in this direction and  $d l_{perpendicular}$  is this much, the width of this black strip here that is  $d l_{perpendicular}$ . So,  $K$  is  $I$  over  $L_{perpendicular}$  and for the radius of area  $a$ ,  $L_{perpendicular}$  becomes twice pi times  $a$ . The circumference of the cylinder, and if that is the case then  $K$  can be given as  $I$  over twice pi  $a$ . That is simple.

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Problem 1: Solution (continued)



- Step: Use the definition of current in terms of current density.
- Step: Integrate current density of surface.
- Step: Use cylindrical coordinate for the integration over surface.


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And in case of a volume current the area perpendicular area is given if the current flows in this direction and this is the volume element through which the current is flowing then the perpendicular area element can be given shown like this here that is  $da_{\perp}$ .


And the steps involved in calculating the volume current density are the following, you can use the definition of the current in terms of the current density and integrate the current density of the surface. And because there is a cylindrical symmetry to this problems, use of cylindrical coordinates for the surface integral would be particularly useful.

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Problem 1: Solution (continued)


$$I = \int_s J \cdot da$$
$$J = \frac{k}{s} \hat{z}$$

In cylindrical co-ordinate surface element perpendicular to  $z$  axis is  
 $da = s ds d\phi \hat{z}$

$$I = \int_0^a \int_0^{2\pi} \frac{k}{s} s ds d\phi$$
$$I = \int_0^a \int_0^{2\pi} k ds d\phi$$
$$k = I/2\pi a; \quad J = I/2\pi as$$


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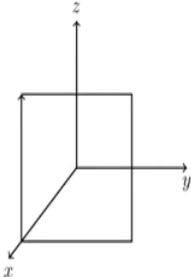
So, you can perform the integral this way  $I$  becomes surface integral of  $J$  dot  $da$ . And  $J$  is given as  $k$  over  $s$  along the  $z$  cap direction, and in cylindrical coordinate system, the surface element perpendicular to  $z$ -axis that would be  $s ds d\phi \hat{z}$ .

So, if we perform this dot product and integrate it over  $s$  and  $\phi$ , then we will have  $s$  integral limit from  $0$  to  $a$ ,  $\phi$  integral limit from  $0$  to twice  $\pi$  and the integrand is this, and the  $s$  here and  $s$  here these 2 cancel and we are left with integration over  $k ds d\phi$ . So,  $k$  can be with when we have this kind of an expression, we can perform the integral and find the value of  $J$ .  $k$  is not necessarily the surface current density. So, you can ignore this part here.

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**Problem 2: Magnetic field**

Suppose that the magnetic field in some region has the form  $B = kz\hat{x}$ , where  $k$  is a constant. Find the force on a square loop (side  $a$ ), lying in the  $yz$  plane and centered at the origin, if it carries a current  $I$ , flowing counterclockwise, when you look down the  $x$  axis.




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And then let us move on to the next problem that is related to magnetic field. Suppose, that the magnetic field is in some region  $B$  its given as  $B$  equals  $kz$  and that is along  $x$  cap direction where  $k$  is constant. And in that region, we have a square loop of side  $a$  on length  $a$  and that is carrying current, its laying on the  $y$   $z$  plane and centred at the origin. If it carries a current  $I$ , flowing counter-clockwise, then when you look down the  $x$ -axis, the current is counter-clockwise. Ignore the direction given in this picture. Then, can you find out the force on this current carrying loop?

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Problem 2: Solution



- Step: Calculate force on each arm and sum it.
- Step : Use expression for the force on current carrying wire due to magnetic field.

Force on the top and bottom wires

$$\vec{F}_b + \vec{F}_t = I \int_b (d\vec{l} \times \vec{B}) + I \int_t (d\vec{l} \times \vec{B})$$

$\vec{L} = a\hat{y}$  for the top and  $\vec{L} = -a\hat{y}$


Using these relations

$$\vec{F}_b + \vec{F}_t = I(a\hat{y} \times \hat{x}ka/2) + I(a(-\hat{y}) \times (-\hat{x}ka/2))$$

$$\vec{F}_b + \vec{F}_t = -Ika^2\hat{z}$$

Similarly you can also calculate force on the wire left side and right side.

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So, here the steps involve first to calculate the force on each arm of the wire and sum it. And for calculating the force, you need to use the expression of the force on a current carrying wire due to magnetic field. You can now pause the video, try the solution and then restart the video to check the solution.

Force on the top and bottom part of the wire can be of the loop can be given as  $F_b$  that is bottom plus  $F_t$  that is top.  $I$  times integration over  $d\vec{l} \times \vec{B}$  over the bottom arm plus the same integral over the top arm. And  $L$  in the, if you look at this picture here then in the bottom arm, it would be along the negative  $z$ -direction and in the top arm it would be along the positive  $z$ -direction, the line element; considering that, please perform the integral and you will get the force  $F_b + F_t = -Ika^2\hat{z}$ .

Similarly, you can calculate the force on the wire from left side and right side.

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Problem 2: Solution (continued)

Similarly we can also calculate force on the wire left and right sides  
Forces on these two wires cancels hence contribution is zero

$$\vec{F} = -Ika^2 \hat{z}$$

0

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The image shows a video lecture slide. At the top left, a blue header contains the text 'Problem 2: Solution (continued)'. To the right of the header is the NPTEL logo. The main content area contains the text 'Similarly we can also calculate force on the wire left and right sides' followed by 'Forces on these two wires cancels hence contribution is zero'. Below this text is the equation  $\vec{F} = -Ika^2 \hat{z}$  and a circled zero. At the bottom of the slide, there is a navigation bar with the text 'Nirmal Ganguli (IISER Bhopal) Electromagnetism swayam NPTEL'. A small inset video of the presenter is visible in the bottom right corner.

But if you calculate the force on the left side and right side, there would not actually be any resultant force that force will cancel each other. You can find out from the situation what we have. Therefore, the total force on this current carrying loop that will be  $F$  equals minus  $Ika$  squared and that is along  $z$  cap direction; that means, the total force would be along the negative  $z$ -direction, ok. That is about this tutorial.