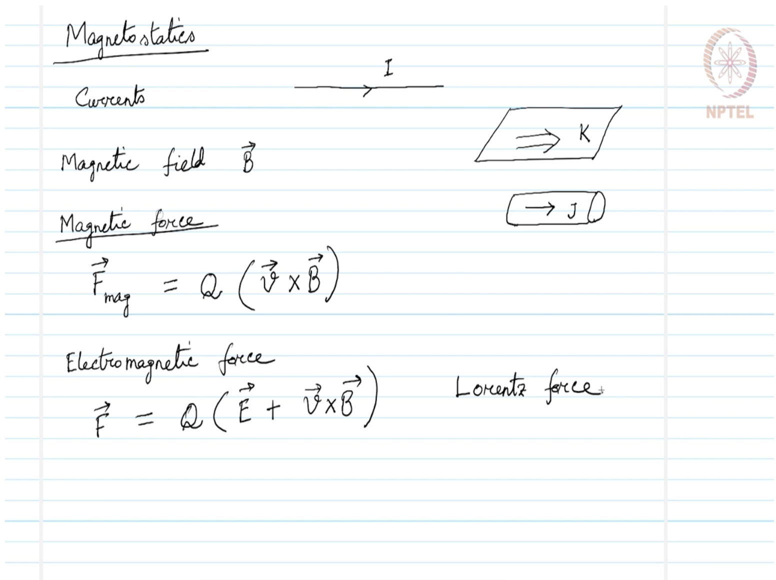
Electromagnetism
Dr. Nirmal Ganguli
Department of Physics
Indian Institute of Science Education and Research, Bhopal

Lecture – 55
Motion of a charged particle in electromagnetic field

(Refer Slide Time: 00:38)



Now, let us move on to magneto statics. In magneto statics before developing the concept of magnetic field let us understand what develops a magnetic field just like electric field was developed by charges magnetic fields are developed by currents. So, what are currents? That is very important to understand.

Current is nothing, but the flow of charges along one direction and that gives us a current. Now, let us consider different kind of situations for example, if we have 1 dimensional geometry and charge carriers flow in this direction we get a line current, if we have a surface like this and the charge carriers move in this direction we get a surface current.

So, line current is expressed as I surface current is expressed as K and if we consider a volume any arbitrary volume we are considering a cylindrical shape and if a volume current flows through this that is indicated as J that is that is the kind of a volume current that we can understand.

Now, we can actually express the total current from volume current and surface current where by integrating it over the cross section in case of a volume current and over the line perpendicular to the flowing charges for a surface current.

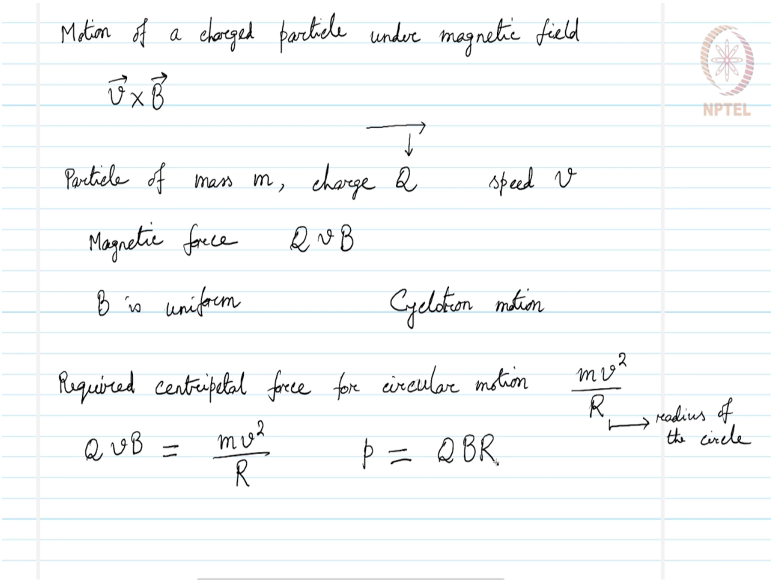
Now, let us consider the magnetic field. We will discuss about the origin of magnetic field later now we are just assuming that there exists a magnetic field B and what happens to the charge carriers, what happens to a charged particle under the influence of a magnetic field that is what we are interested in finding out now.

So, magnetic field exerts a force. If we write magnetic force as F mag of course, this is a vector this magnetic field exerts a force on moving charged particles, if the amount of charge is Q and it moves with a velocity v then the magnetic force on the charged particle at a magnetic field B would be given as Q times v cross B.

So, similarly we can write down the electromagnetic force from our earlier discussion about electric field and this discussion about magnetic field. So, the total electromagnetic force would become Q times electric field plus v cross the magnetic field and this is also known as the Lorentz force; the total electromagnetic force is also known as the Lorentz force.

Now, how does a charged particle move under the influence of an electromagnetic field, electric as well as magnetic field? Let us find that out.

(Refer Slide Time: 04:56)

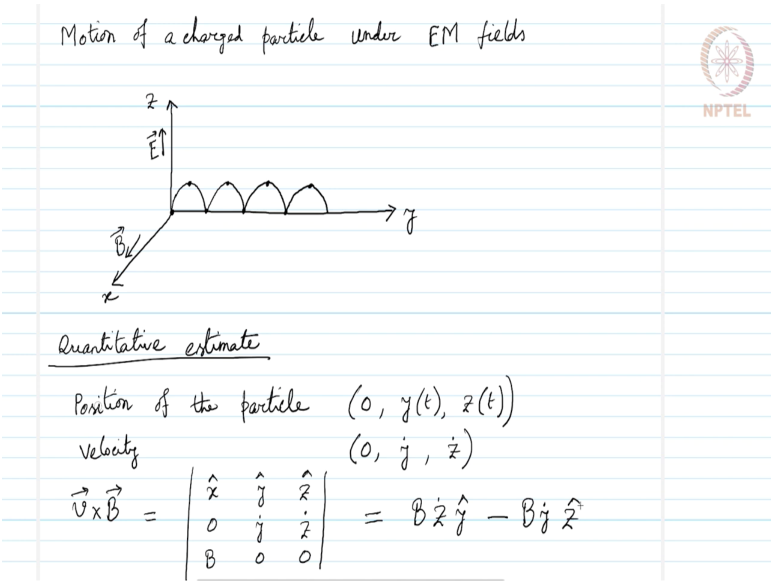


If we consider only magnetic field that will make the situation simpler. So, let us start with that. So, magnetic field as we have seen it its direction is v cross B. So, magnetic field does not act along the velocity of the particle, it also does not act along the direction of the magnetic field, its perpendicular to both. And a particle if a particle is say moving in this direction and the magnetic there is a magnetic field that exerts a force along this direction, then that force will act like a centripetal force and it will curve the motion of this particle.

For particle of mass m and charge Q it moves with speed v, the magnetic force its magnitude would be Q v B and that will be the that will be it provided the velocity is perpendicular to the magnetic field and if we consider B to be a uniform magnetic field, then this arrangement would give. So, the magnetic field will provide a centripetal acceleration, centripetal force and that way we will realize a cyclotron motion. So, the charged particle would move along a circle and it will keep on doing that.

Now, for circular motion we need an amount of centripetal force in order to maintain that motion over that circle. So, the required amount of centripetal force is for circular motion m v squared over R where capital R is the radius of the circle. So, for this kind of a trajectory we must have Q v B being equal to m v squared over R and; that means, the momentum of that particle would be Q B R, momentum is expressed as p here. Now, if we also include the electric force into consideration.

(Refer Slide Time: 09:04)



So, this time we are considering motion of a charged particle under electromagnetic force under the influence of electromagnetic fields. So, as a simple example let us consider that the magnetic field points along x direction and electric field points along the z direction.

Let us draw the coordinate system this is the z direction, y direction and x direction. We have a particle with positive charge and its released from the origin at rest, electric field is along this direction, magnetic field is along this direction, we have a positively charged particle at the origin and its released from there at rest our job is to find out what is going to happen to this particle.

Qualitatively we can think that initially the particle is at rest. So, the magnetic force is 0 on this, the electric force will accelerate the particle in the z direction. So, initially the motion of the particle will be along the z direction and once the electric field accelerates it, it will pick up a speed and the magnetic force will start to apply on it. And what will the magnetic field do? It pulls the particle in the y direction.

And when the; when the particle starts moving once again the electric force under the influence of electric force it slows down reducing the magnetic force on it and it this thing gets repeated. So, initially the particle moves along this direction and when it moves along that direction that is z direction, then we get a magnetic field magnetic force applicable on it, the direction of the magnetic force would be z cross s. So, that is y direction and that bends the particle like this.

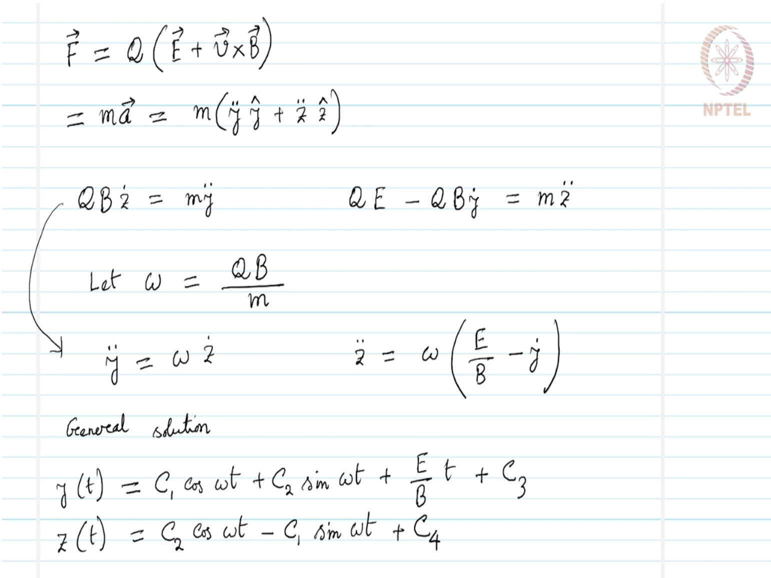
Now, the velocity of the particle is at this point along y direction and when that happens we will have a force on the particle magnetic force on the particle that is y cross x that is minus z direction. So, that way the particle will come back here and it will be immobile and there will be no magnetic force on it again electric force will pull it up here, magnetic force will pull it down there and this thing will be repeated this kind of a motion will be there of the particle.

So, after doing this qualitative estimate let us try to write down the equations of motion and try to make a quantitative estimate of whatever we have discussed so far. So, there is no force in the x direction to begin with, the position of the particle can be described at any time by. So, there is no force along x direction any time what, so ever throughout this motion.

So, we can describe the position of the particle at any time t as 0 there is no displacement along x direction, y that is a function of t and z that is a function of t. And the velocity can be described as there is no velocity along x direction, y dot velocity along y direction that is the time derivative of y and z dot velocity along z direction that the times derivative of z.

So, we have to calculate v cross B to find out the magnetic force and that is x cap, y cap, z cap, 0 y dot z dot B 0 0 this determinant and that is B z dot y cap minus B y dot z cap.

(Refer Slide Time: 14:44)



Now, we can write down the equation of motion as using the second law of motion force equals the charge times the electric field plus velocity cross the magnetic field. That means, mass times the acceleration equals this quantity is also equal to mass times the acceleration that is mass times y double dot in y direction plus z double dot in z direction.

Now, if we have this then treating the components separately we can write down we have already found the expression of v cross B, now treating the components separately we can write down Q times B times z dot that equals m y double dot and Q times the electric field that is along z direction minus Q B y dot this quantity equals m z double dot.

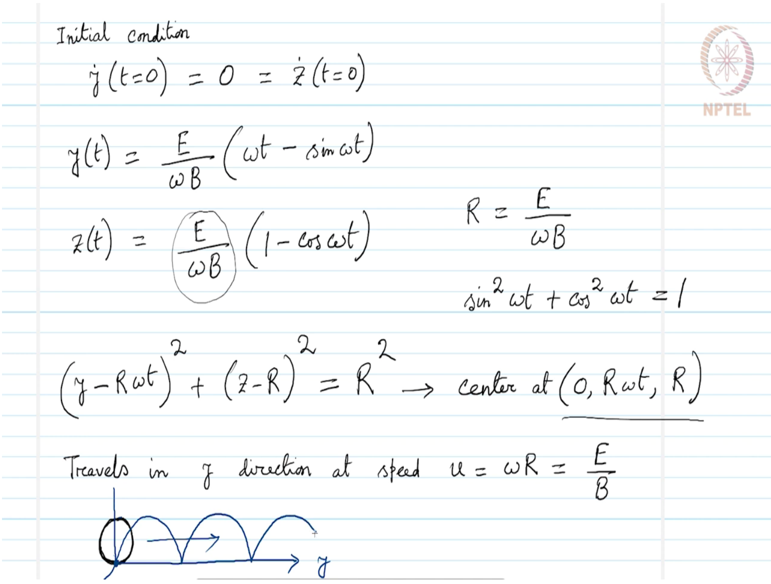
That means we have a second order differential equation, second order in time and it these are coupled. So, y double dot has z dot in it and z double dot has y dot in it. Now, let us consider a frequency omega for this motion that equals Q B over m, if that happens to be the frequency that happens to the let us not call it frequency right now let us just denote it as omega, then these two equations we can write as y double dot equals omega times z dot and z double dot equals omega times E over B minus y dot ok.

After getting this we can differentiate the first equation and put the value of z double dot from the second equation. So, we get the third order time derivative with respect to y and second order time derivative for z comes here and if we put the value of z double dot from the second equation, we get one equation without cross terms; that means, only in terms of y and.

So, in terms of y triple dot and y dot we will one equation and if we solve that equation the general solution for that equation would be y as the function of time can be given as C 1 cosine of omega t plus C 2 sin of omega t plus E over B times t plus C 3 which is the constant of the integral. And z t z as a function of time can be expressed as C 2 cosine of omega t minus C 1 sin of omega t plus C 4 that is also a constant of the integration.

With these two equations using the initial condition, initially what we had? We had the particle at rest at the origin.

(Refer Slide Time: 19:11)



So, the initial condition can be written down as y dot at t equals 0 is 0 there is no velocity along any direction. So, z dot at t equals 0 is also 0 and the particle is at the origin. So, y and z those are also 0 at t equals 0. Now, if we apply this initial condition on the general solutions for the differential equations, we will find the specific solution to this problem y t equals E over omega B omega t minus sin omega t and z t would be given as E over omega B one minus cosine of omega t.

Now, let us put this quantity E over omega B as R let us consider this and let us also use the identity that sin squared omega t plus cosine squared omega t this will be 1 using these we can write down that y minus R omega t squared plus z minus R squared equals R squared. This is the equation of a circle of radius R whose centre is at 0, R omega t R, circle of radius capital R with centre at this position.

And this travels in y direction at a constant speed. How much speed? u equals omega r; that means, E over B it travels along the y direction at a constant speed. So, what is the trajectory of the particle, how are we finding this? If we consider a circle like this and if there be a charge there be a particle like this here ok.

Let us not start from there let us start from here if this was the origin, then this is the y direction we are rolling this circle this way. So, with we are rolling in this circle along this direction; that means, the motion of this point will look like something like this and that is what we qualitatively expected and after working out this problem we found exactly the same solution to this problem.