Electromagnetism

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Lecture - 05
Divergence

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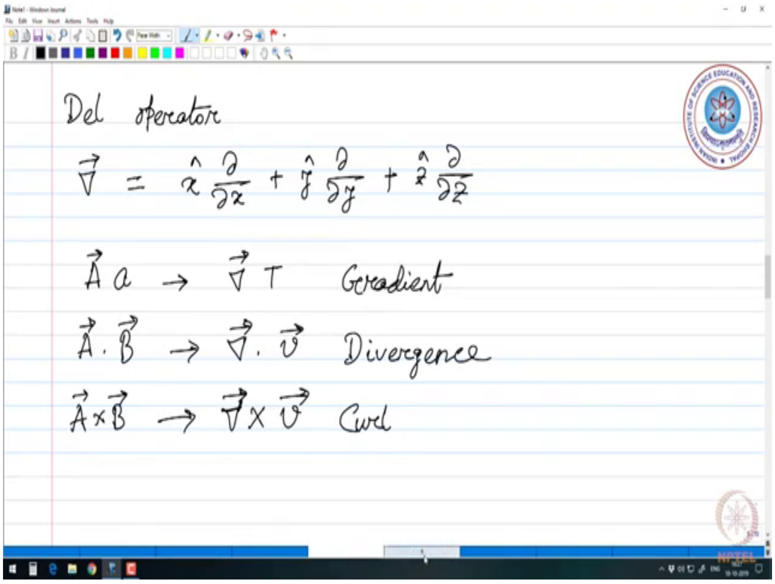

Just like earlier we had a vector multiplied by a scalar, it would be similar to this del operator acting on a scalar and this is known as the gradient. If we in analogy with the dot product write A dot B similar to that we can write del dot a vector quantity v; that means, this operator will operate on v. So, this kind of operation is known as Divergence and similar to cross product A cross B of two vectors we can write del cross v any vector quantity and this operation again it is not a cross product its known as curl now let us define the divergence first.

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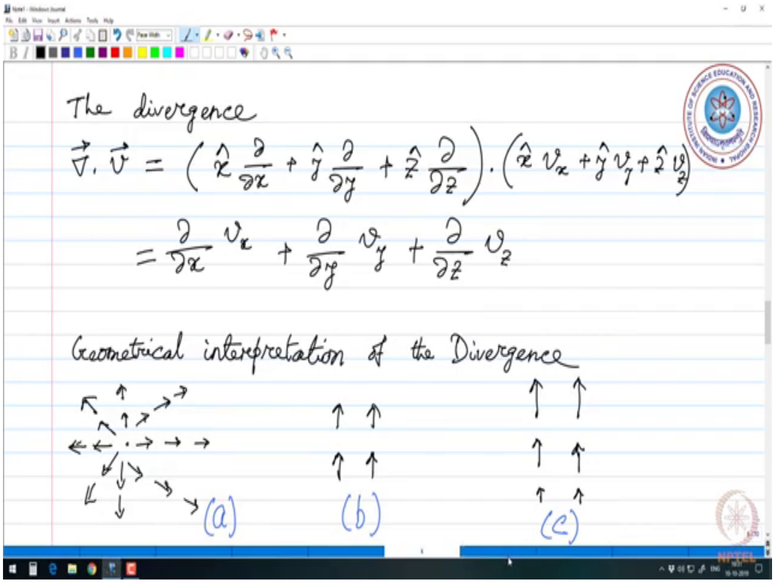

The divergence operator del dot v it operates quite similar to the notion of dot product. So, I am first writing the del operator x cap del del x plus y cap del del y plus z cap del del z and that will operate on x cap v x plus y cap v y plus z cap v z. And if we perform dot product we will get del del x of v x plus del del y of v y plus del del z of v z. So, this is exactly the operation; that means, we perform this divergence operation on a vector field that is v in this context, we have three components three Cartesian components of this vector v and after operating we get a scalar as the output. What does it geometrically mean?

So, geometrically divergence would mean if we consider a point and there is a vector field around it that goes somewhat like this. Here, I am showing a vector field that diverges from a point. In this kind of a situation we would say that at the point the vector field has a positive divergence; that means, the vector field is diverging from this point.

If we have the arrows pointing towards that point; that means, the vector field is converging to that point then the divergence of the vector field at that point would be negative and if we have a situation where the vector field does not change at a certain point, it does not diverge it neither diverges nor converges in that situation we can say that the divergence of the field there is 0, if we have this kind of a field the divergence would be 0.

And let us consider another situation where we have the magnitude of the vector is increasing as we are increasing, as we are moving upward they can be this kind of a situation. So, let us call this the most leftmost case as situation a, the middle one as situation b and the rightmost one as situation c and let us see how we can calculate the divergences. So, for case a this is similar to the position vector.

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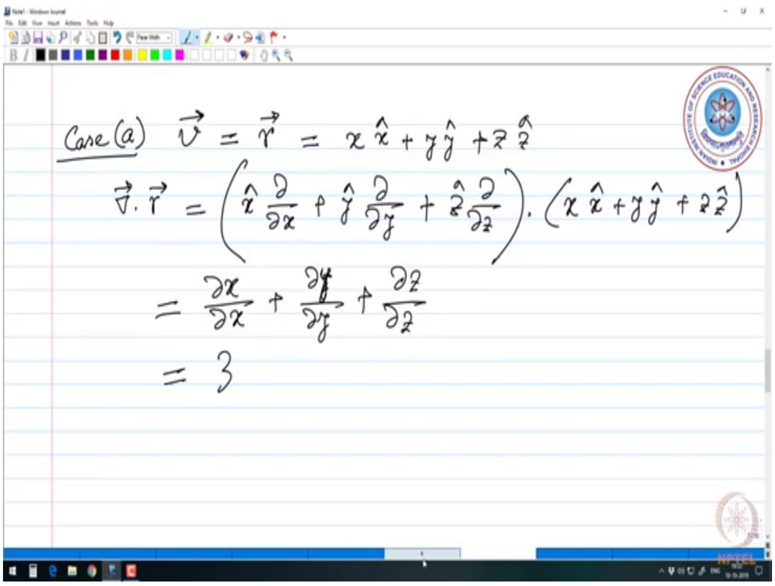

So, here the vector in question can be written as the position vector which is nothing, but x x cap plus y y cap plus z z cap in Cartesian coordinate system and if we calculate the divergence of it, we will see that this quantity would be x cap del del x plus y cap del del y plus z cap del del z dot x x cap plus y y cap plus z z cap and that will give us del x del x plus del y del y plus del z del z which is 3. We get one from each of these derivatives and finally, we get three as a result of it.

So, we can see that the divergence for this kind of a vector is positive. The situation b, case b in this picture is nothing, but its same the magnitude of this vector is constant and its pointing upward that is along the z direction.

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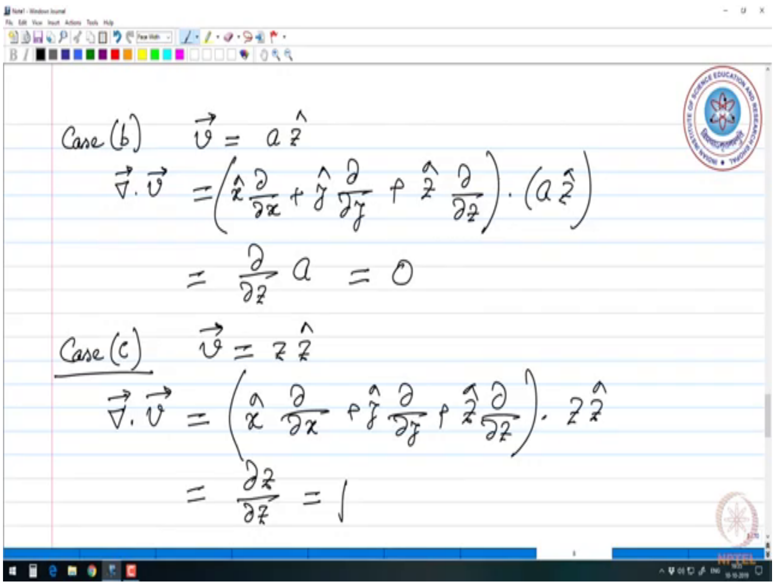

So, for case b we can write it in the following way, the vector field can be described by this equation any constant a multiplied by z cap. And now if we take the divergence of this quantity divergence v is given as x cap del del x plus y cap del del y plus z cap del del z a z cap. So, only the z component survives; that means, we have del del z of a constant which gives us 0. So, this vector neither diverges nor converges; let us come to case c, case c is where we have the magnitude of the vector is increasing as we move higher along the z direction.

So, this vector field can be represented as this v equals z z cap that is what we intend or any proportionality constant can also be there, but that does not matter that is not going to give any qualitative change in the picture. So, the divergence of this vector v can be written as x cap; so, the del operator first. This del operator operated on z z cap, only the z component will survive everything else will go to 0 as according to the rule of the dot product and we will be left with del z del z that is 1.

So, we have a positive divergence of this vector field at any point in the field that is something we can clearly see from here and this example also clarifies the physical meaning of divergence.