Electromagnetism
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Lecture - 50
Electric Polarization and Bound Charges

So, if we have a polarized object, there will be certain electric field emerging due to that polarized object. Let us try to find that out what is the electric field that emerges and, how is it a function of spa and how it emerges as a function of that electric polarization.

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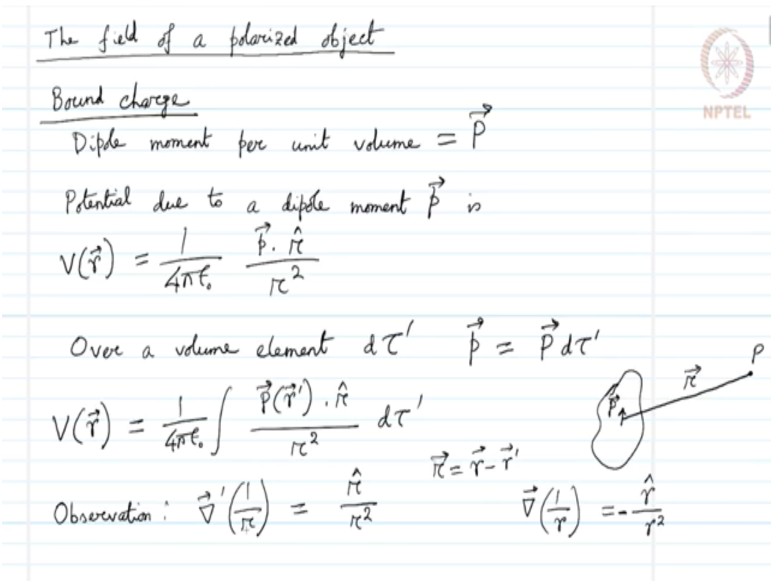

Let us introduce the concept of bound charges. We have dipole moment per unit volume given as capital P that is the polarization. And the potential due to a dipole moment small p, can be written as, V r equals 1 over 4 pi epsilon naught p dot, r cap r squared p dot r cap over r squared.

So, this is the potential due to a dipole with dipole moment small p, now that we have the dipole moment per unit volume. So, over a volume element d tau prime what will be the dipole moment? That would be polarization vector multiplied with d tau d tau prime. Therefore, the potential due to this polarized object can be written as 1 over 4 pi epsilon naught.

Now, we have to integrate over t as a function of r prime dot r cap over r squared, d tau prime. And this is what we have to do; that means, we are considering in this material here where we have a dipole moment a polarization P. And we are considering ourselves somewhere here at the point of observation and this distance is this r vector curly r vector.

We have considered something like this, now we have to integrate, this quantity. How do we do that? We make an interesting observation that del prime of 1 over curly r gradient of 1 over curly r is r cap over r squared we have calculated. We have earlier seen that, the gradient of 1 over r is nothing, but r cap over r squared with a negative sign.

Now, here because we are taking this gradient over the primed coordinate and our curly r vector is nothing, but r minus r prime. So, over the primed co-ordinate this minus sign will not be there will be a positive sign and this is how we can represent, del prime of 1 over r. And if we can represent this vector curly r cap over curly r squared as a gradient, that is going to make some make a this work very interesting.

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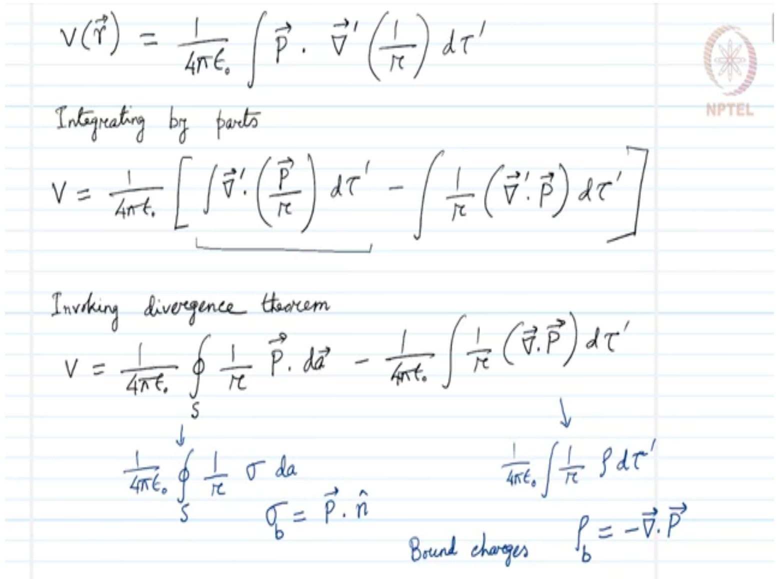

Let us see how that does; that means, the expression for the potential becomes 1 over 4 pi epsilon naught, integration polarization vector, dot product gradient over the primed coordinate system; 1 over curly r d tau prime, and now if we integrate it by parts. So, we have a part P and a part this gradient function. We will get the potential equal to 1 over 4 pi epsilon naught integration del prime dot P over curly r divergence of this quantity d tau prime minus integration. 1 over curly r del prime dot P d tau prime. This is what we will obtain by performing integration by parts. And now we can bring in the divergence theorem over the first part that is this part.

So, let us invoke the divergence theorem. We can write them the potential as 1 over 4 pi epsilon naught integration closed integration over the surface binding this volume. Surface enclosing this polarization distribution 1 over r P dot da, we can do that and the next term remains as it is 1 over 4 pi epsilon naught volume integral over 1 over r del dot p divergence of P d tau prime that is what we get.

Here the first term looks like the potential due to a surface charge distribution and the second term looks like, the potential due to a volume charge distribution the first term is integration over a surface and the second term is the integration over a volume. And if you look at this 1 over pi epsilon not 1 over r dependence of the potential times whatever is there and the surface element.

In case of the first term and volume element in case of second term whatever is remaining that is like the charge distribution term; that means, this 1 is analogous with this term is analogous with 1 over 4 pi epsilon naught integration over a surface 1 over r sigma da. It is analogous with this and this term is analogous with 1 over 4 pi epsilon naught integration 1 over r rho d tau prime.

Now, comparing these two terms we can write that sigma equals P dot n cap n cap is the perpendicular direction to this surface element that is perpendicular to the surface. And rho equals the negative divergence of the polarization vector that is what we find. And because these are not real charges these are not external charges, these are the charges that these are something that behaves like charge due to the polarization and it cannot come out of this material. So, we call it bound charges this is sigma b this is rho b.

Bound charges are just fictitious charges that is developed, that is that helps us interpret the consequence of electric polarization in a material in a dielectric material.

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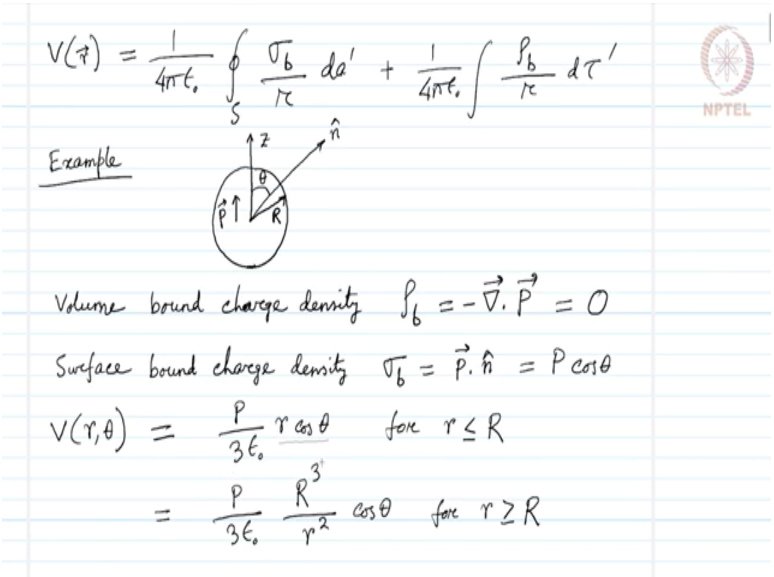

And if we write this way, then in terms of these bound charges the potential can be expressed as a function of the position vector. 1 over 4 pi epsilon naught closed surface integral over sigma b over r d a prime plus 1 over 4 pi epsilon naught integration rho b over 4 d tau prime. This is the expression of potential in terms of the bound charges that we have just depth we have just developed the concept of. Let us consider an example of example for finding the bound charges in a system.

So, let us try to find the electric field produced by uniformly polarized sphere of radius R; we have a sphere here radius R. And this is the z axis, this is uniformly polarized with P polarization and uniform polarization does not mean radial; it means the polarization is along one direction and we assume it to be along the z direction. And let us consider any point somewhere here. So, the direction of the surface is along this direction n cap the angle that makes with the z axis is theta, let us assume that.

So, what would be the volume charge density? Volume bound charge density that is given by rho b equals the negative divergence of the polarization, but our polarization is uniform; that means, its constant and its divergence is going to be 0. Therefore, there is no bound volume charge density. And what is the surface bound charge density? That can be expressed as sigma b equals P dot n cap n cap we have marked, this is the direction of n cap; that means, that would be the polarization times cosine of theta where theta is this usual spherical coordinate theta.

And then potential due to such a surface charge density would become in as a function of r and theta it would be again symmetric along azimuthal direction phi. So, it will not be a function of phi and if we perform the integral inside and outside the sphere we would find that the polarization would be P over 3 epsilon naught r cosine sorry. The potential would be 3 over sorry p over 3 epsilon naught r cosine of theta for our point of observation within the sphere. And this would be P over 3 epsilon naught, capital R cube that is radius of the sphere cubed over small r squared that is distance squared cosine of theta for r greater than equal to capital R.

Now, since r cos theta equals z the field inside the sphere becomes uniform inside the sphere we have r cos theta here and if we put z for that, then we will if we take the gradient of this potential we will get a constant field along the z direction. So, the electric field inside becomes uniform let us work out what I am talking about let us consider this expression.

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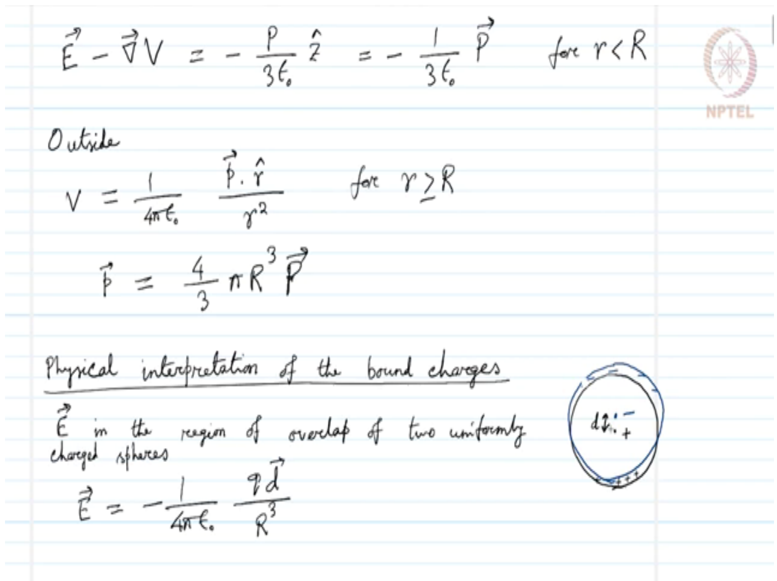

So, electric field can be given as the negative gradient of the potential which is minus P over 3 epsilon naught z cap; because r cos theta equal to z and this is minus 1 over 3 epsilon times the P vector. This is for any point inside the sphere what happens outside? Outside the sphere the potential is identical to that of a perfect dipole at the origin. Let us see how the potential for a perfect dipole at the origin is 1 over 4 pi epsilon naught the dipole moment dot r cap over r squared.

And if we compare with the equation written here, we can write down that in this case the dipole moment vector p that would be given as 4 over 3 pi R cubed times the polarization P. Now let us try to physically interpret what the bound charges are. If we consider a material for example, if we consider a sphere like this and we have applied an electric field.

So, the charge centres have moved somehow earlier for this black sphere. Let us say we had a charge centre here. And now under the influence of an electric field the positive charge centre is still there the nucleus is still there, but the negative charge centre has shifted the electron cloud has shifted a bit as a result the negative charge centre. Let us assume have shifted here. So, we have the positive charge centre here and the negative charge centre here.

So, the distance between these two charge centres if that is given as d. Then that develops the polarization within the material. And in this polarization what we have we have in this part of the material some positive charge excess of positive charge. And in this part of the material we have some excess of negative charge that kind of a situation happens. And actually these charges that we have drawn at the extremes of this material bound charges are similar to something like this; because of the shift of the charge centre you can see that the charges inside the material cannot cancel each other properly as it used to happen earlier.

Now, this difference in the charges that is the bound charge that we have seen so, far in terms of mathematics; physically we can interpret that bound charge in this way. The electric field in the region of overlap between two uniformly charged sphere that we have done we have considered here two uniformly charged sphere they are shifted a bit.

In that case the electric field can be written as minus 1 over 4 pi epsilon naught q times the d vector over R cubed where R is the radius of the spheres capital R is the radius of the spheres and q is the total charge of the positive sphere and also same. So, minus q is the total charge of the negative sphere d is the vector that we have drawn here the separation between the charge centres. And in terms of polarization we can write down that the polarization vector p, is q d that is 4 over 3 pi R cubed times the polarization vector.

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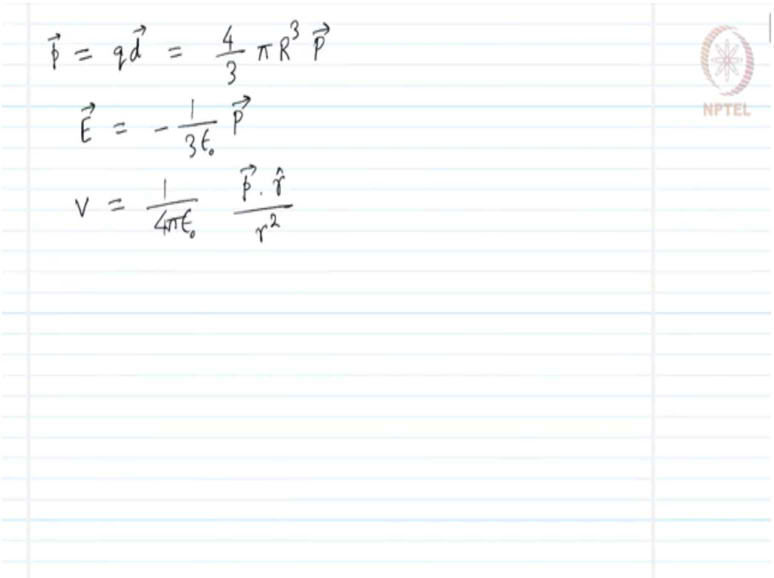

So, the electric field becomes minus 1 over 3 epsilon naught the polarization vector; that means, we have considered here two spheres; one black sphere with all positive charge in it one blue sphere with all negative charge in it and they are superimposing on each other. With this situation we have seen that there is a difference in the charge centre and that develops a polarization in the material and a dipole moment in the material.

And that exactly brings in the kind of electric field that we have seen earlier in case of surface bound surface charges, uniform polarization in the material. And the corresponding potential outside the sphere would be 1 over 4 pi epsilon naught p dot r cap over r squared. So, d is very small and its some fraction of the atomic radius.

If we consider that the atomic charges the electron cloud of the atom is getting shifted and in along if we think along that line. So, we can physically interpret the polarization by considering two equally and oppositely charged spheres with slight difference in overlap between each other. That is how we can physically understand what these bounds bound charges are. So, bound surface charge we have seen using this analogy.