Electromagnetism
Dr. Nirmal Ganguli
Department of Physics
Indian Institute of Science Education and Research, Bhopal

Lecture – 42
Induced charge

If that is the case, then let us see in the original problem what is the situation. What kind of induced charges do we have?

(Refer Slide Time: 00:42)

Induced charges in the original problem

Surface charge density $\sigma = -\epsilon_0 \frac{\partial V}{\partial n}$

$\frac{\partial V}{\partial n} = (\vec{\nabla} V) \cdot \hat{n}$ → normal derivative

$$\sigma = -\epsilon_0 \frac{\partial V}{\partial z}\Big|_{z=0}$$

$$\frac{\partial V}{\partial z} = \frac{1}{4\pi\epsilon_0}\left\{\frac{-q(z-d)}{\left[x^2+y^2+(z-d)^2\right]^{3/2}} + \frac{q(z+d)}{\left[x^2+y^2+(z+d)^2\right]^{3/2}}\right\}$$

$$\sigma(x,y) = \frac{-qd}{2\pi\left(x^2+y^2+d^2\right)^{3/2}}$$

We have the expression for the potential for z greater than 0 positive z values. We have already found it. And we can calculate the surface charge density from the expression of the potential. Surface charge density sigma can be expressed as minus epsilon naught times the normal derivative of the potential on that surface. So del V del n just to recap, is the normal derivative the gradient of V dot n cap for this surface. n cap is the unit vector pointing outward of the surface.

So, if we now calculate sigma that is minus epsilon naught del V del z at z equals 0. Because, z is the direction normal to the plane that we have considered, the infinite metallic plane infinite conducting plane. And we have the expression for the potential V. So, del V del z can be given as 1 over 4 pi epsilon naught minus q z minus d over x square plus y square plus z minus d squared.

This quantity power 3 over 2 plus q times z plus d over x squared plus y squared plus z plus d squared. This quantity over 3 over 2. This quantity power 3 over 2. Now this is del V del z and we want to find del V del z at z equals 0. So, we will z 0 here for z and 0 here for z also at the numerator and we will be left with.

So, if we now calculate the expression for sigma that is the surface charge distribution that will be minus q d over twice pi x square plus y square plus d squared power 3 by 2. This will be the surface charge distribution and as expected for plus q point charge in the z x at a distance d along the z axis we have an induced charge on the grounded metallic sheet of infinite extent of a negative sign.

And this charge distribution as you can see if we have larger values of x and y this induced charge amount of the induced charge reduces. And on the other side of this metallic plane conducting plane, the induced charge will go to the ground because we have grounded the this conducting plane.

So, by this charge distribution induced charge the distribution that we have found out the metallic conducting plane will make its will maintain itself at 0 potential. So, let us try to find out the total induced charge for this system. For total induced charge we will have to integrate over the entire plane.

(Refer Slide Time: 05:26)

Induced charges in the original problem

Surface charge density $\sigma = -\epsilon_0 \dfrac{\partial V}{\partial n}$

$$\frac{\partial V}{\partial n} = (\vec{\nabla} V)\cdot \hat{n} \rightarrow \text{normal derivative}$$

$$\sigma = -\epsilon_0 \left.\frac{\partial V}{\partial z}\right|_{z=0}$$

$$\frac{\partial V}{\partial z} = \frac{1}{4\pi\epsilon_0}\left\{ \frac{-q(z-d)}{\left[x^2+y^2+(z-d)^2\right]^{3/2}} + \frac{q(z+d)}{\left[x^2+y^2+(z+d)^2\right]^{3/2}} \right\}$$

$$\sigma(x,y) = \frac{-qd}{2\pi\left(x^2+y^2+d^2\right)^{3/2}}$$

So, the total charge will be integration over sigma da. And let us move to polar coordinate system now. The coordinates are r and phi. So, we have to represent sigma in terms of that which is just a function of r its symmetric in phi. So, this will be minus q d over 2 pi r squared plus d squared over 3 over 2, x square plus y squared equals r square in case of two dimension that is simple.

So, if we integrate it we will get Q over phi the range of integration is 0 to 2 pi over r its 0 to infinity. We have the integral minus q d over 2 pi r squared plus d squared power 3 over 2 r d r d phi this is the surface element. And this turns out to be q d over r squared plus d squared square root of this quantity. The limits are from 0 to infinity and that equals minus q. That means, the amount of the total induced charge is the negative of the point charge amounts that we have at a distance d above this plane which is consistent with all our expectations.