**Electromagnetism**
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**Lecture – 41**
**The method of images**

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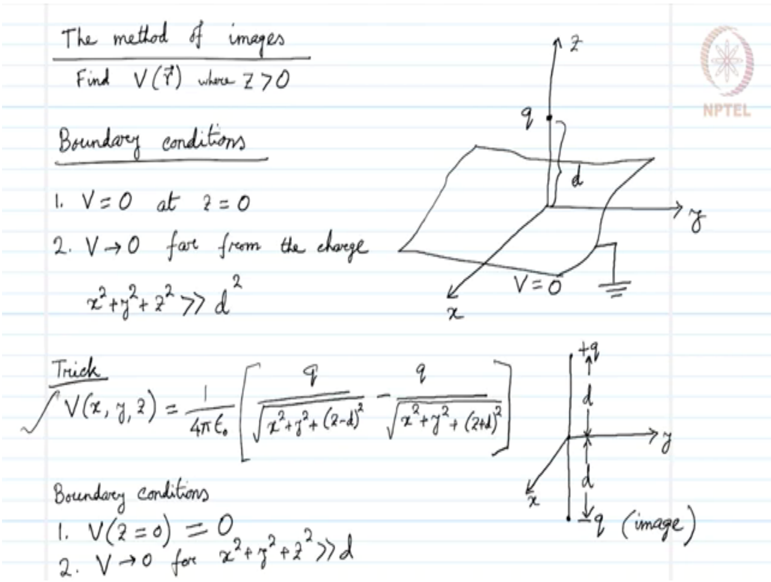


The interesting thing is The method of images. What is it? Suppose at a suppose, we have a point charge q here, this is a point charge q and we have an infinite conducting plane like this. This is x axis y axis and z axis and this distance is d. If we have this kind of a situation and if we consider that this infinite conducting plane is grounded. If this kind of a situation happens, grounded means the potential on this plane equals 0, on this conductor plane. If we have this kind of a situation, then we want to find out the potential above this plane. So, the problem asks us to find V at any point r where the z component of r, that is z greater than 0.

So, we are interested in the upper half of this infinite plane from mathematical point of view, it is about solving the Poisson equation, Laplace equation would not help; because in our region of interest, we have a source charge we have a point charge q here at this point. And using the subject to the boundary condition, we can find something very interesting about this kind of a problem.

So, what we, if we write down the boundary condition for this problem; we see that the potential equals 0 at z equals 0. Because, we have an infinite conducting plane that is grounded and the second boundary condition is that the potential tends to 0 far from the charge. That means, if we have x square plus y square plus z square, that is much greater than d squared then, the potential tends to 0; that means, at infinity the potential will go to 0. Now if we consider the first uniqueness theorem. Actually, we are considering Poisson equation in this case, so, it is about the corollary of the first uniqueness theorem; let us see what it is.

So, that guarantees that there is only one function that meets the requirement; that means, the solution to the potential above this plane is unique from the first the corollary of the first uniqueness theorem. If we can make a clever guess or find a trick for a valid potential satisfying the boundary condition; that is all we need to solve this problem. So, what kind of trick can we do? Let us work some trick out. Let us forget about the actual problem and make the trick, we are going to study a completely different situation and this situation is we have a point charge here at a distance d from the origin. We have another point charge below at a distance d from the origin.

So, here is plus q charge, here is the origin of the coordinate system and here is an image charge minus q. This distance is d, this distance is also d. Let us consider this kind of a this kind of an arrangement. If we consider this kind of an arrangement; what are we going to have? Do we have both boundary conditions satisfied? The potential on xy plane is going to be 0, because of the arrangement of plus q and minus q point charges the way they are at a separation d from the origin on the same line that is the z axis.

So, the potential on x and y sorry xy plane is going to be 0 and far away from these charges, that is when we move to nearly infinity; we will still have 0 potential. So, both our boundary conditions are satisfied with this pseudo arrangement instead of the original problem. But here this minus q charge that we have considered this charge does not exist, this is called the image charge.

And if we now find the potential due to this arrangement, then we can write that V x y z equals 1 over 4 pi epsilon naught plus q over x square plus y square plus z minus d squared square root of this minus, because of the negative image charge, x squared plus y squared plus this time z plus d squared square root. This is the expression for the potential. And we have boundary conditions, boundary conditions are V at z equals 0 that is on the xy plane that is 0 and the second boundary condition is V tends to 0 for x square plus y square plus z square much greater than d. Everything is satisfied here. And the only charge in our region of interest that is z greater than 0 above this xy plane is the plus q charge, there is no other charge in there.

Thus, all conditions of the problem are satisfied and so, whatever potential we have found we have calculated here is going to be the potential for z greater than 0. So, this is the correct potential that is what we claim and that satisfies all the boundary conditions that are at hand interesting.