Electromagnetism
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Lecture - 04
Vector differential calculus: Gradient

Hello everybody. In the last discussion we have learned vector algebra and now it is about Vector calculus. So, we will start with differential calculus; obviously, and before going to vector calculus we would first see how ordinary derivatives are calculated with a scalar function.

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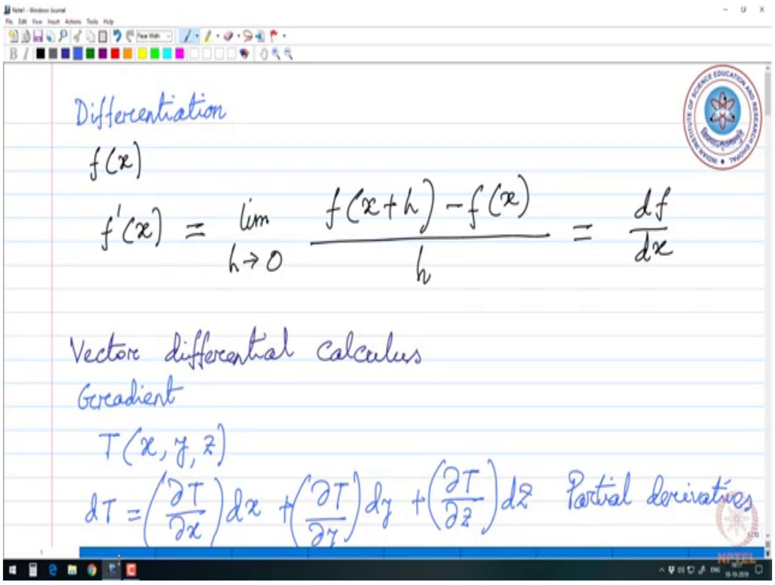

So, let us consider differentiation first, if we have a function f x then its derivative first order derivative with respect to x that is f prime x is defined as limit h tends to 0 f x plus h minus f x over h. We are familiar with this this definition and this in the shorthand notation is written as df dx.

Geometrically we interpret this derivative as the slope of the function. Now, if we are interested in vector differential calculus then the simplest operation would be gradient of a scalar field that becomes vector. How do we define gradient? Let us consider we have a scalar field T and that is a function of x, y and z in 3 dimensional Cartesian coordinate system. So, we can write dT as del T del x dx plus del T del y dy plus del T del z dz in the form of partial derivatives..

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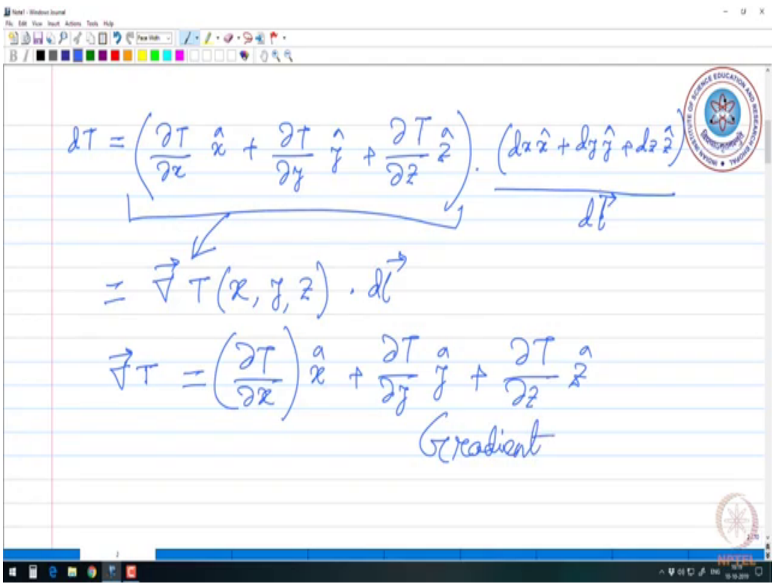

Now, if we make a grid we want to take the gradient of this then the gradient can be so, dT can be written in the form del T del x x cap plus del T del y y cap plus del T del z z cap. This quantity dotted with dx x cap plus dy y cap plus dz z cap. And we know that this right hand side is nothing, but the differential line element; that means, this quantity becomes the gradient of the scalar field T that is a function of x, y and z dotted with the line element dl.

Now, this gradient of T is nothing, but this quantity here. So, here we define gradient of T written in this way nabla followed by T is del T del x x cap plus del T del y y cap plus del T del z z cap. Geometrically gradient means the direction in which the scalar field is changing. So, gradient of a scalar field is a vector quantity that has a magnitude and a direction, it is very important to note.

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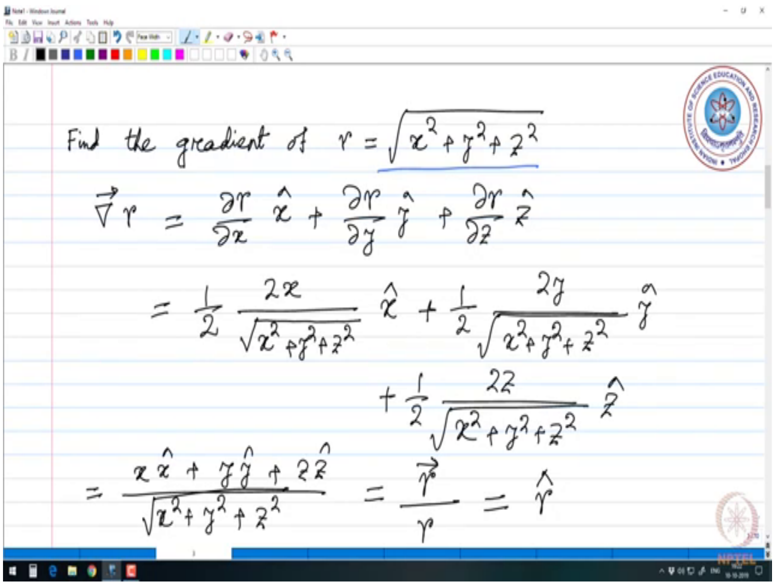

Let us consider one example of calculating a gradient, let us try finding the gradient of the position vector not really vector the magnitude of the position vector r in 3 dimensional Cartesian coordinate system. As we know the definition of gradient is this is given as del r del x x cap plus del r del y y cap plus del r del z z cap. And, with the definition of r that is given here we can clearly see that the derivative of r with respect to x will bring us half in front.

And, because there is x square we will have 2 x here and it will the power of r would be minus half; that means, 1 over x square plus y square plus z square square root of this quantity x cap. Similarly, for derivative with respect to y it will be half 2 y over x square plus y square plus z square square root of that y cap and for z it would be half 2 z over the same thing which is the half and two these things will cancel. And, we will be left with x x cap plus y y cap plus z z cap over x squared plus y squared plus z squared square root of this quantity.

So, the numerator is nothing, but the r vector over the denominator the magnitude of r; that means, the gradient of the scalar of the scalar magnitude of the position vector gives us the direction of r, that is unit vector along the position vector r. Let us consider another example.

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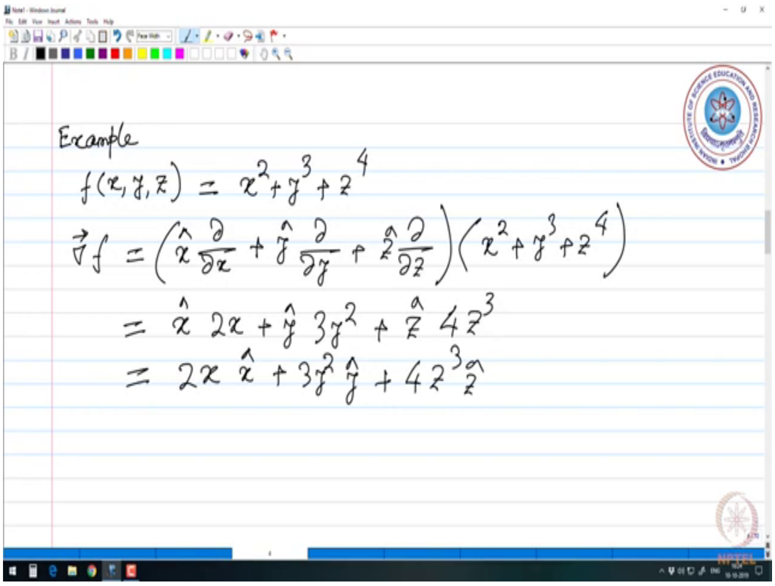

Let us consider a scalar field, if that is a function of all three position coordinates x, y and z is given as x squared plus y cubed plus z power 4. Now, if we calculate the gradient of this scalar field we will have we can write it as x cap del del x plus y cap del del y plus z cap del del z. This is the gradient operator and this operator is operated on the scalar field that is x square plus y cubed plus z power 4. So, we will get x cap 2 x plus y cap 3 y squared plus z cap 4 z cubed which is rearranging it properly twice x x cap plus 3 y squared y cap plus 4 z cubed z cap.

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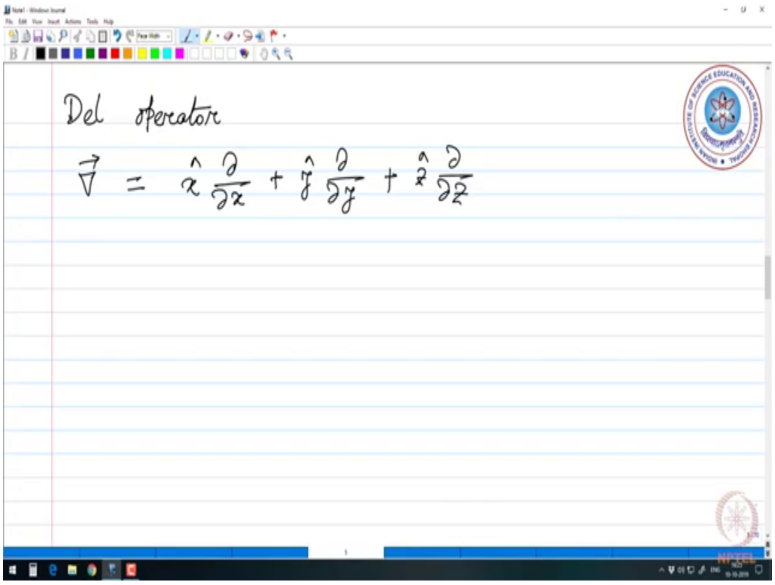

So, we can see that we have developed a del operator out of this. The way in the previous example we have written the gradient of operator was this operator was equal to x cap del del x plus y cap del del y plus z cap del del z. Although this is a differential operator, it looks pretty much like a vector and just because it looks like a vector we can define its product with a scalar, its product with another vectors that will look like a dot product or a cross product.

But, we have to remember that it is not really a dot product and cross product of vectors rather it is a differential operator, we have to be careful about that.