

Electromagnetism
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Lecture - 38
Capacitors

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Capacitors

+Q on one conductor
-Q on the other conductor

Potential difference
 $V = V_+ - V_- = - \int_{(-)}^{(+)} \vec{E} \cdot d\vec{l}$

$C \equiv \frac{Q}{V}$ $V \equiv CQ$
↳ Capacitance unit → Farad

Now, we will discuss about Capacitors. Suppose we have two conductors and we put for one conductor we charge one conductor with positive charge and the other conductor with negative charge. So, we put plus Q charge on one conductor and minus Q charge on the other conductor in that case, this kind of a system can have some electric potential and it can hold some energy this kind of system is called a capacitor.

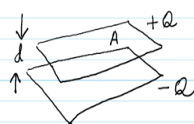
Now, it develops a potential difference between the plus Q and minus Q charged conductors. So, the potential difference if we call that V, then we can write it as the difference between the potential of the positively charged conductor and the negatively charged conductor which is nothing but minus integral from the negatively charged conductor to the positively charged conductor electric field dot the line element.

And since electric field is proportional to the charge, positive Q and negative Q and the potential difference is also proportional to the charge then there would be proportionality constant namely C the constant equivalent to Q over the potential difference, now C is called the capacitance. We had V the potential difference proportional to Q with the help of proportionality constant we can write V equals C Q, now C is called the capacitance and its unit is farad.

Farad is defined in the way that if we put plus 1 coulomb and minus 1 coulomb charge on two conductors and if the potential difference between them is 1 volt then the capacitance is 1 farad.

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Example Find the capacitance

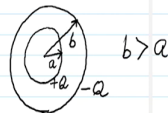


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$$E = \frac{Q}{A \epsilon_0} \quad V = \frac{Q}{A \epsilon_0} d$$

$$\text{Capacitance } C = \frac{Q}{V} = \frac{A \epsilon_0}{d}$$

Example $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$



Potential difference

$$V = -\int_b^a \vec{E} \cdot d\vec{r} = -\frac{Q}{4\pi\epsilon_0} \int_b^a \frac{1}{r^2} dr = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right)$$

Let us consider an example of it. Let us consider two parallel plates, because that we are already familiar with one plate like this the other plate is below this one, looks like this. Let us consider the separation between them is d and A is the area of each plate. Now, the problem asks to find the capacitance for this arrangement. In order to solve this problem, we need to assume that we put plus Q charge on one plate for example, the top plate and minus Q charge on the other plate for example, the bottom plate.

And if the area of these plates are much larger compared to the separation between them, we can for every practical purpose assume that these two plates are of infinite extent. And if we assume that then we can use the result for electric field that we have obtained in the case of two; in the case of two parallel infinitely extended plates conductors.

So, the electric field the magnitude of the electric field can be expressed as Q over $A \epsilon_0$ and the potential will be given as Q over $A \epsilon_0 d$ this becomes like this; because we have the electric field constant in between these two parallel plates and the charge is uniformly distributed.

So, having the electric field constant we in order to obtain the potential we can just multiply it with the distance d between them and that will give us the difference in potential. If we have this then the capacitance can be given as capacitance C equals Q over V and the expression will come to be $A \epsilon_0$ over the separation between the place plates.


Let us consider another example, let us consider two concentric spherical shells one like this, the other one like this. There is a common center my drawing is not that good forgive me for that, and the radii of these two shells are a and b respectively, clearly b is greater than a . In this case, and we put plus Q charge on the inner shell minus Q charge on the outer shell and try to find the capacitance of this arrangement, how do we do that?.

If we have this kind of an arrangement then electric field inside the smaller sphere is 0, electric field outside the bigger sphere is also 0. And there is an electric field in between these two spheres and what is the amount? From Gauss law we can find that the amount of electric field in between these two concentric spheres that will come only because of the charge on the smaller sphere.

It will have no contribution from the charge on the bigger sphere. If we have that, we can write down the electric field in between these two concentric spherical shells as 1 over $4 \pi \epsilon_0$ Q over r^2 assuming origin at the center of the spheres and therefore, the potential difference can be expressed as V equals minus integration from b to a we are considering this integral because we can consider that the potential outside the bigger sphere is 0, there is no electric field. So, the potential at infinity and just outside the bigger sphere will be same we have no electric field anywhere and when it enters in between then there is the. There is an electric field.

So, the potential changes and we are calculating the potential difference from the big surface of the bigger sphere to the surface of the smaller sphere. And that is expressed as, $E \cdot dl$ integration over that minus sign of it and this can be expressed as minus Q over $4\pi\epsilon_0$ naught integration b to a 1 over r squared dr which essentially becomes Q over $4\pi\epsilon_0$ naught 1 over a minus 1 over b . In this limit if we have this as the expression for the potential difference.

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$$C = \frac{Q}{V} = 4\pi\epsilon_0 \frac{ab}{b-a}$$


Then the capacitance would be Q over V that equals $4\pi\epsilon_0$ naught a times b over b minus a this would be the capacitance for this arrangement.