Electromagnetism

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Lecture – 37
Electrostatics with conductors

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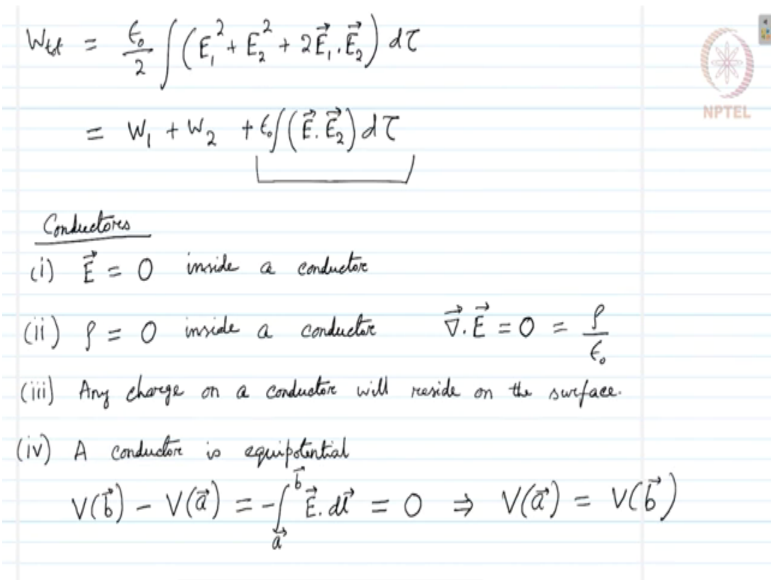

These were the comments on the energy for the electrostatic field. Now, let us consider the interesting case of conductors. Now, conductors have quite a few interesting properties. The first interesting property is that for an ideal conductor, the electric field inside a conductor is always 0. Why so, because if the electric field inside a conductor were non-zero the conductors have free charges to move around and that is why it is a conductor.

What would happen? The electric field will make those charges move in such a way that the electric field becomes 0 or the charges will move until the electric field inside the conductor becomes 0. So, this table condition for a good conductor is that the electric field inside a conductor will always be 0. Then comes an interesting statement. The volume charge density inside a conductor is also 0, why is that so?.

Because, we have no electric field inside a conductor therefore, the divergence of electric field inside a conductor has to be 0. And, if the divergence of the electric field has to be 0; that means, rho over epsilon naught has to be 0 and the only way that this could be violent this could happen is by rho going to 0, there is no other way.

Now, then comes the question, if we charge a conductor with some charge. If, we put some charge on a conductor, what is going to happen to that charge? Is it going to fly away from that conductor how that is not possible? So, if we put any charge on a conductor, it will reside on the surface of the conductor ok. Then, another statement another property that is a conductor is always equipotential.

What do we mean by this word equipotential? Equipotential means, everywhere on the conductor the electrostatic potential is going to be the same. Why so? If the potential was not same then there would be a current and there would be an electric field as well, consider point a and point b the potential at point b is expressed as V b the potential at point a is expressed as V a and the difference is nothing, but integration from point a to point b negative sign before that E dot dl.

This is how we have defined the potential difference. And, if the electric field inside a conductor has to be 0, then this potential difference also has to be 0, because the integration of 0 is going to give 0 0. Hence, that implies us that the potential at point a is equal to the potential at point b, there is no other choice and lastly the electric field outside the surface.

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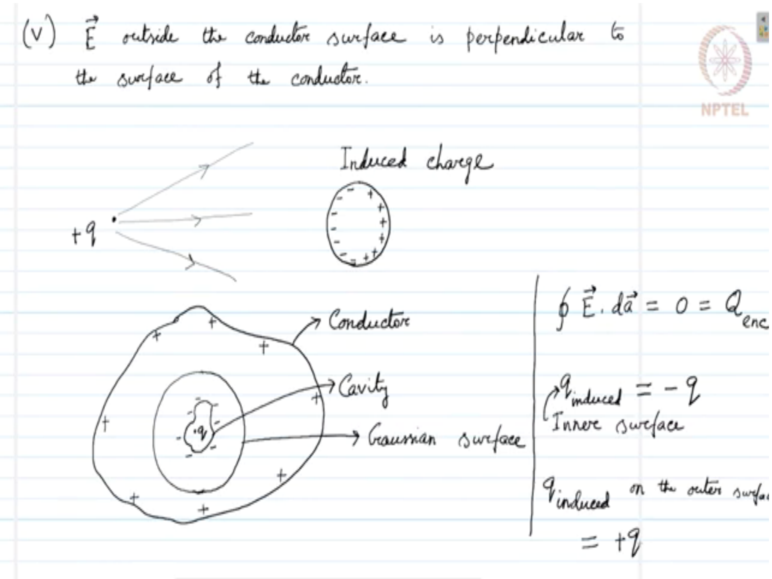

We can also make a conclusion about the electric field outside the surface it has to be perpendicular to the surface of the conductor. If, outside the surface the electric field has a tangential component, then that will make the charges move and that is not a stable condition. So, in a stable configuration the electric field outside the conductor, due to any surface charge of that conductor has to always be perpendicular to the surface of the conductor, that way it will find a stable situation.

Let us consider an interesting case with the conductors. Let us consider a point charge here of magnitude q and let us consider a conductor sphere here like this. So, if this point charge is a positive charge, then what will be the arrangement of charge on the conductor. So, this point charge will have an electric field and according to our convention the electric field lines will point like this and we have seen that conductor cannot allow any electric field inside it.

Therefore, what is going to happen with this? In order to negate this electric field it will have to develop some surface charge density, facing the positive charge it will be negative and away from the positive charge it will be positive. Because, the conductor has to be electrically neutral as well because it was before placing this point charge q the conductor was electrically neutral and this kind of charge is called induced charge. So, we will have induced charge on the conductor let us consider about this distribution of induced charge.

Let us consider a conductor of an arbitrary shape like this, this is a conductor. And, let us consider a cavity inside that conductor of some other arbitrary shape like this, this one is a cavity. And, let us consider a point charge placed here somewhere inside the cavity of magnitude q. Now, if this point charge is positive, then this part of the conductor will develop a negative induced charge and the outer surface of the conductor will develop a positive induced charge like this right.

Now, let us consider something interesting. Let us consider a spherical Gaussian surface here inside the conductor; this is the spherical Gaussian surface. We did not actually need any spherical symmetry any kind of Gaussian surface will be fine. Now on that Gaussian surface, if we try to calculate this E dot da over this closed surface, we can write this quantity equals 0, because inside the conductor the electric field goes to 0. If, the electric field goes to 0 this quantity is going to be 0 and if this quantity is going to be 0; that means, the total charge enclosed is also 0.

How is that possible? We have a point charge plus q; that means, the total induced charge that we have, inside this Gaussian surface; that means, the negative charge developed on this surface here, surface of the cavity that is going to be equal to minus q that is going to make the total enclosed charge 0. And, if that is the case; that means, this on the surface of the conductor we have minus q charge on the inner surface and since the conductor has to be electrically neutral, it has to develop positive q charge on the outer surface.

So, the induced charge in inner surface, this is on the inner surface is positive is negative q and q induced on the outer surface, that has to be plus q. Only that way we can satisfy the laws of electrostatics that we have developed so far.

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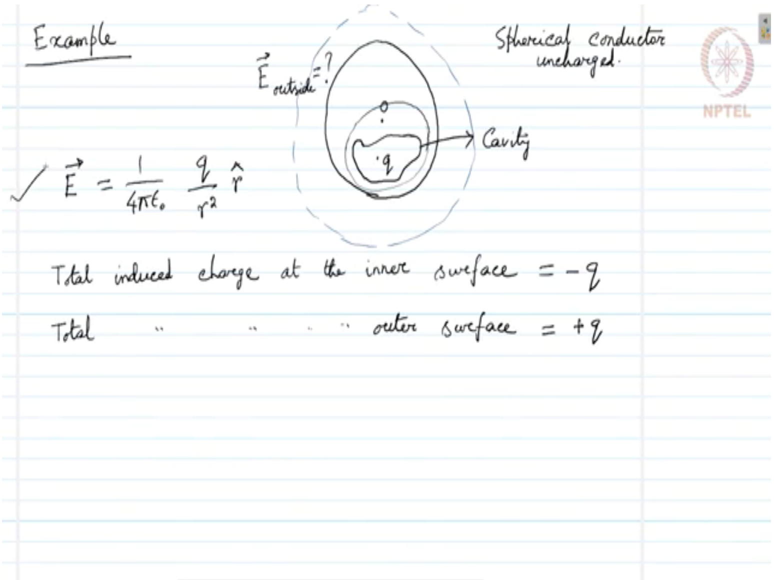

Let us consider an example related to this. Let us consider a spherical conductor that is not charged. And, here is the; here is the center of this spherical conductor, and we consider a cavity of an arbitrary shape somewhere here at an arbitrary location inside this spherical conductor, and that has a point charge q. So, this is our cavity.

Now, we want to find out what is the electric field outside the sphere? This is our question let us try to solve this problem, we have point charge q here at somewhere in the cavity. So, the electric field due to this point charge q can be written as 1 over 4 pi epsilon naught q over r squared r cap. We have total induced charged at the inner surface, here on this surface, that is exactly equal and opposite to the point charge there.

And, the total induced charge plus q on the total induced charge out on the outer surface equals plus q; that means, what is the effective situation? We can effectively consider that inside the sphere there is no charge only plus q charge is distributed on the outer surface of the sphere, if we look at the sphere from outside.

If, we consider a Gaussian surface somewhere here, that is like this, then it will see no charge enclosed within this Gaussian surface. So, it will find the electric field equals 0 and electric field must be 0, there because it is inside a conductor. So, everything is consistent if we consider a Gaussian surface outside here somewhere, then it will only see an induced charge distribution on the surface of the sphere, because everything else got everything else compensated each other. Therefore, that induced charge distribution that would also be uniform because it is a spherical surface.

Due to that uniform charge distribution, we will have an electric field that is similar to this point charge located at the center of the sphere. So, this is going to be the expression for the electric field for this system, that is pretty interesting, no matter where you keep the cavity and where you keep that point charge inside that cavity, the electric field is going to be the same in case of a conductor. Now, let us consider the surface charge distribution on a conductor and the force on it.

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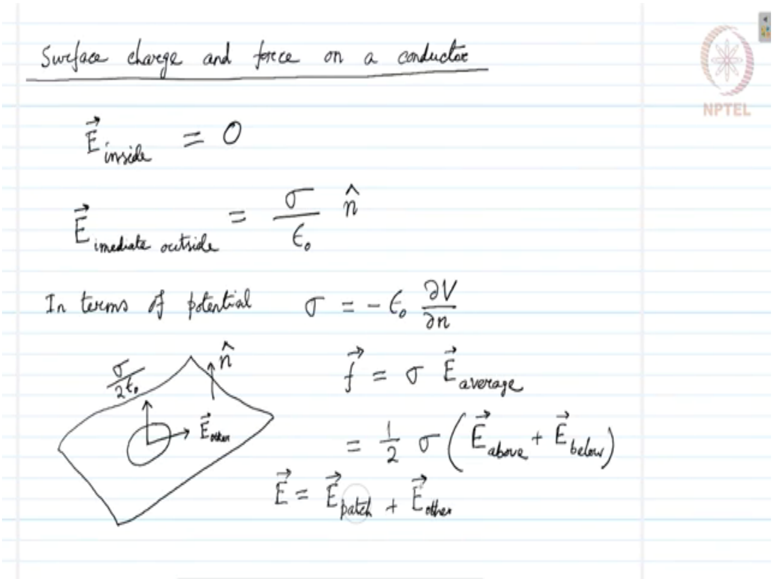

Because, the field inside a conductor is always 0, the boundary conditions would require that field immediately outside the conductor is so let us write down the electric field inside that is always 0. So, the electric field immediate outside, the boundary condition will require that this becomes sigma over epsilon naught n cap it has to be perpendicular to the surface, because if there was any parallel component to the surface of this electric field, that needed to be continuous across the surface.

And, it cannot there cannot be any electric field inside the conductor therefore, no question of existence of a parallel component of the electric field immediately outside the conductor, that is ruled out.

And, with this consideration in terms of potential we can write sigma the surface charge density becomes minus epsilon naught del V del n where del V l n del n is the normal derivative we have earlier found out for this is now; that means, if we know the potential or the electric field, we can calculate the surface charge distribution. Let us consider a picture like this, here we have a surface of a conductor, this is the normal to the conductor surface that is n cap and we considered this region.

Where we have perpendicular component of the electric field, due to the surface charge distribution and due to the surface charge distribution this electric field would be as we have found out earlier a sigma over twice epsilon naught, we are not considering this conductor to be of infinite extent.

But, immediately above the conductor we can still assume that the distance from our point of observation from of from the conductor is much less compared to the extent of the conductor. So, it would behave like an infinity and this would be the perpendicular component of the electric field. And, let us consider some other component of the electric field here.

And, in the presence of an electric field the surface charge will experience a force. If, there is another component of the electric field existing. If, we write the force as f, then f can be written as sigma times the average of the electric field.

And, that can be essentially written as half sigma times the electric field above plus the electric field below. Why do we average this? Because, if you consider this patch on this surface that we have drawn here and the patch cannot apply electric field on itself. So, the patch will experience electric field from the rest of this surface. And, that will bring in some parallel component of the electric field on this patch itself although for the entire surface there is no parallel component of the electric field.

So, if we can if we do not consider the electric field due to this patch, then this patch will experience some electric field in having parallel component on this acting on this patch. If, we consider that kind of a situation. So, then the electric field becomes electric field on the patch, electric field due to the patch plus electric field due to other part of the conductor, that is going to be the total electric field.

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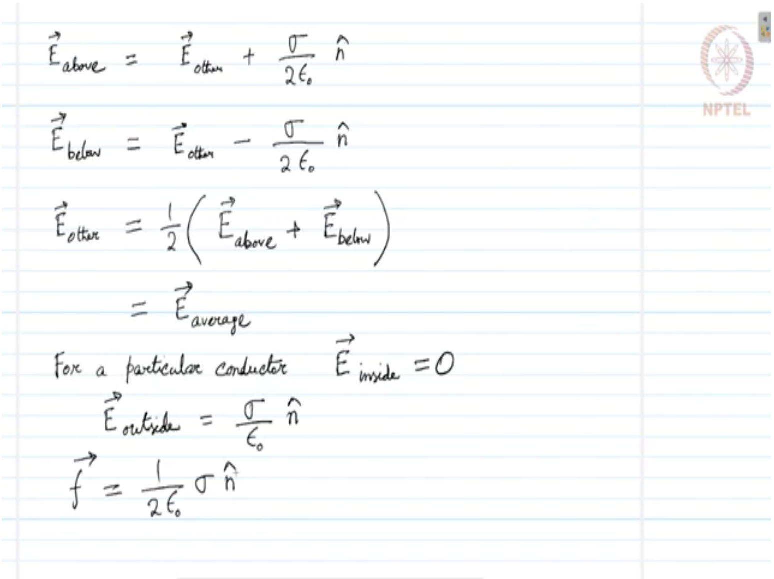

And, with this consideration we can write that the electric field above is electric field due to other part of the conductor surface excluding the patch plus sigma over 2 epsilon naught n cap. This is the perpendicular component of the electric field and electric field below, that can be expressed as electric field due to other parts of the conductor surface excluding the patch, that is the parallel component minus sigma over 2 epsilon naught n cap that is the perpendicular component of the electric field below.

So, the electric field parallel component due to the other parts of the conductor surface excluding the patch is going to be averaged over the electric field above and the electric field below. This is the other part of the electric field; that means the parallel component of the electric field that is acting on the patch of our consideration and that is nothing, but the electric field average.

So, this has a parallel component as well. And, if we consider a particular conductor with electric field inside to be 0; that means, below the surface the electric field is 0, we have only electric field above the surface, let us write that as electric field outside the surface. And, that is going to be sigma over epsilon naught n cap we can find this using Gauss law, then the force on the conductor is going to be 1 over 2 epsilon naught sigma n cap.

This is going to be the force due to force on the charge on the surface, force on the surface charge distribution, due to the conductors on the electric field due to this charge distribution itself. So, this is the amount of force the conductor is going to be going to experience.

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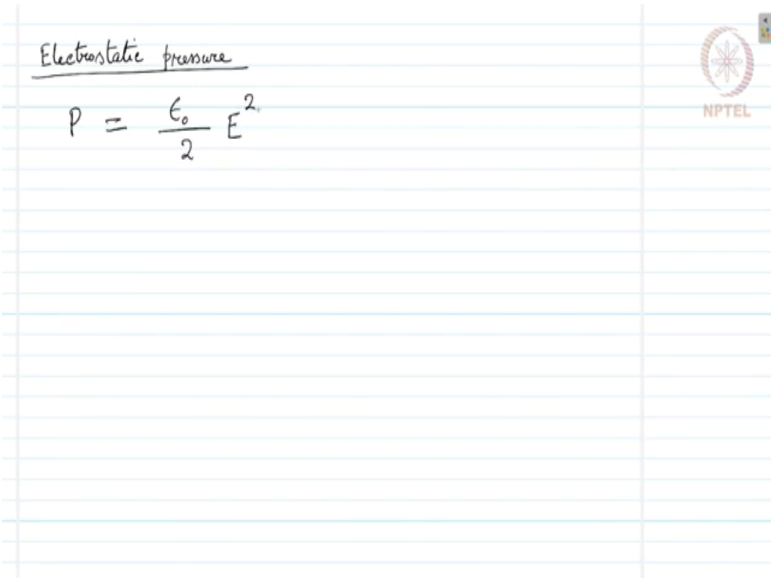

Now, this force is going to develop an electrostatic pressure on the conductor. So, the pressure can be expressed as sigma naught over 2 E squared. This is the electrostatic pressure that the electric field will exert on the conductor itself and the electric field came from the conductor itself, due to the distribution of surface charge.