Electromagnetism

Dr. Nirmal Ganguli
Department of Physics
Indian Institute of Science Education and Research, Bhopal

Lecture – 35
Work and energy of an assembly of point charges

Let us consider in case of electrostatic how we can represent Work and energy of this electrostatic system.

(Refer Slide Time: 00:41)

Work and energy

$$W = \int_{\vec{a}}^{\vec{b}} \vec{F}\cdot d\vec{l}$$

$$= -Q\int_{\vec{a}}^{\vec{b}} \vec{E}\cdot d\vec{l} = Q\left[V(\vec{b}) - V(\vec{a})\right]$$

$$V(\vec{b}) - V(\vec{a}) = \frac{W}{Q}$$

Work done to bring a charge Q from ∞ to $\vec{r}$

$$W = Q\left[V(\vec{r}) - \underbrace{V(\infty)}_{\to 0}\right]$$

$$W = Q\,V(\vec{r})$$

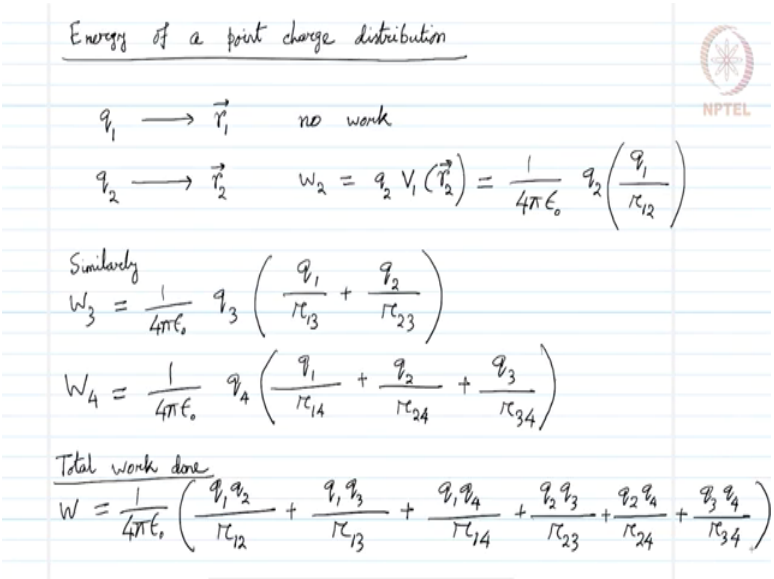
Work as we know is defined as W equals integration from point a to point b if we move something from point a to point b force times the line element and line integral over this. In the context of electrostatics if we consider a charge capital Q, move it from point a to point b under the influence of electric field E, then the work done can be written as minus Q E dot dl which is according to the definition of potential Q times V b minus V a. And clearly this answer is independent of the path that one takes from point a to point b.

So, V b minus V a equals work done over the amount of the point charge that has been carried and if one wants to bring a point charge Q from infinity and bring it to a position with position vector r and stick it there, how much work is done?

The amount of work done in this can process would be Q times V r minus V infinity. Just like what we have seen earlier and if we set this to 0, we find that the work done is nothing, but Q times the potential the electrostatic potential at the point r. Physically we can in in interpret the electrostatic potential as the amount of work done to bring unit charge from infinity to a point r. This is the physical interpretation of electrostatic potential.

Now let us consider the energy of a point charge of a charge distribution comprising few point charges.

(Refer Slide Time: 03:54)



So, we have seen that it if we bring some energy some point charge in an electrostatic potential, we have to work. But if initially there is no charge then if we bring one charge the first charge first point charge, we do not do any work. Starting from the second charge we start working. So, in order to bring the charge q 1 from infinity to the position r 1, no work is involved.

Now if we bring q 2 from infinity to the point r 2, then that involves some work. Let us call that W 2 that can be given as q 2 times V 1 r 2; V 1 is the potential due to the first point charge which is 1 over 4 pi epsilon naught q 2 times q 1 over r 1 2 this much. Similarly we can write down W 3 equals 1 over 4 pi epsilon naught q 3 times q 1 over r 1 3 plus q 2 over r 23; W 4 equals 1 over 4 pi epsilon naught q 4 times q 1 over r 1 4 plus q 2 over r 24 plus q 3 over r 34 and so on.

And so, with this distribution what is the total amount of work done? Total work done can be calculated for these four charges as 1 over 4 pi epsilon naught q 1 q 2 over r 1 2 plus q 1 q 3 over r 13 plus q 1 q 4 over r 14 plus q 2 q 3 over r 23 plus q 2 q 4 over r 2 4 plus q 3 q 4 over r 34.

This would be the amount of total work done in order to make this assembly of four charges. And if we can now generalize for n point charges the total amount of work done in a summation notation, we can write for n number of point charges.

(Refer Slide Time: 08:20)

For n point charges

Total work done $W = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\sum_{j>i}^{n}\frac{q_i q_j}{r_{ij}}$

$$W = \frac{1}{8\pi\epsilon_0}\sum_{i=1}^{n}\sum_{j\neq i}^{n}\frac{q_i q_j}{r_{ij}}$$

$$= \frac{1}{2}\sum_{i=1}^{n} q_i\left(\sum_{j\neq i}^{n}\frac{1}{4\pi\epsilon_0}\frac{q_j}{r_{ij}}\right)$$

$$\boxed{W = \frac{1}{2}\sum_{i=1}^{n} q_i V(\vec{r}_i)}$$

The total work done capital W that can be written as 1 over 4 pi epsilon naught sum over i equals 1 to n sum over j which is always greater than i and that also runs up to m q i q j over r ij. Now, we can remove this constraint that is j is greater than i by introducing a factor of half because if we remove this constraint, we will count each such term twice and if we count it twice, we can just we can adjust it by introducing a factor of half.

But still we will have to consider that j not j is not equal to i. So, we can write the work done as 1 over 8 pi epsilon naught including that factor half i equals 1 to n j naught equals i that runs up to n with j equals i, we will have r ij going to 0 and we will run into a problem.

So, we avoid that condition qi qj over r ij. This is the energy and that is nothing, but half sum over i equals 1 to n qi j naught equals i up to n 1 over 4 pi epsilon naught q i over r ij which suggests that this is equal to half sum over i equals q to n q i times this part this term in the parentheses, we recognize that this is the potential at the point r i due to the other charge distribution; sorry here this index must be j.

So, this is something we learn from this exercise that is the work done in a charge in a point charge distribution is if we bring in charge q i in the vicinity of an assembly of point charges, the work done is bringing its potential due to the previous charges multiplied with the amount of new charge and sum over all such new charges and half of that is the work done for bringing all those charges and assembling at one place.