Electromagnetism
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Lecture - 33
Calculation of electric potential from different approaches

Hello, earlier we have discussed about the electric potential. We have developed the concept of electric potential because we found that electric field was a rotational field that means the curl was 0. And because the curl was 0 we could represent it as the gradient of a scalar function b and that scalar function was called the electric potential. And the advantage of this method this procedure was that instead of worrying about three components of a vector, we could deal with only one component of a scalar that namely the electric potential.

And get store the exactly the same amount of information. And in order to be able to do that, we must be able to find out the electric potential right from the charge distribution and finding it out from the electric field is not that helpful in with this idea in mind. So, we have to find the electric potential right from a charge distribution not from the electric field. Let us try doing that.

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Electric potential of a localized charge distribution

$$\vec{E}\cdot\vec{dl} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,dr$$

point charge q at the origin

$$V(\vec{r}) = -\int_{\infty}^{r}\vec{E}\cdot\vec{dl} = -\frac{1}{4\pi\epsilon_0}\int_{\infty}^{r}\frac{q}{r'^2}\,dr'$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

In general $V(\vec{r}) = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{\mathscr{r}}$ $\qquad \vec{\mathscr{r}} = \vec{r} - \vec{r}'$

For an assembly of n charges

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i}$$

So, we will assume a charge distribution, just like when we calculated the electric field we assumed it. And we need a reference to be set for the integ.ration over the electric field to get the potential. So, the very earlier we calculated electric potential was that we considered E dot dl which is nothing but 1 over 4 pi epsilon naught q over r squared d r.

If we consider a point charge at the origin and with the setting the reference at infinity, we can write the potential due to this electric field as minus integral over infinity to r; r is a point of observation where we want to find the electric potential. Integration over E dot dl that will become minus 1 over 4 pi epsilon naught integration infinity to r q over r prime squared d r prime r prime is prime is just a dummy index.

So, that we can have the limit as r limit of the integration as r and that becomes 1 over 4 pi epsilon naught q over r this we have seen earlier. And that means, in general, we can represent the potential due to a point charge stored at any arbitrary point with position vector r prime as 1 over 4 pi epsilon naught q over curly r magnitude where this vector is r minus r prime. We have already defined it many times.

Now, if we consider an assembly of n number of charges, with each with magnitude q i we can write the potential as a function of position vector as 1 over 4 pi epsilon naught, sum over i equals one to n q i over r i; this is just the principle of superposition that the electric field obeys.

(Refer Slide Time: 05:48)

Continuous charge distribution

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{1}{\mathscr{r}}\,dq$$

For volume charge density

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\,d\tau'$$

Line charge density

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda(\vec{r}\,')}{\mathscr{r}}\,dl'$$

Surface charge density

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma(\vec{r}\,')}{\mathscr{r}}\,da'$$

Hold good if the reference is at ∞.

And if we consider a continuous charge distribution, with continuous charge distribution, we can write the potential as a function of r equals 1 over 4 pi epsilon naught, integration over 1 over curly r d q, where d q is the element of charge. For a volume charge density we can write the potential as 1 over 4 pi epsilon naught integration over rho which is a function of r prime over this r d tau prime.

And this is a volume integral and this is how we can find the potential directly from the charge distribution given without evaluating the electric field first. So, we are evaluating just the scalar in this case and not the vector not all three components of the vector. So, this if we can do it easily this would be a much more elegant way of dealing with electric field.

And if we consider instead of volume charge density, if we consider line charge density then we will have the potential given as 1 over 4 pi epsilon naught integral over lambda this is a function of r prime. That is the source coordinate system; over curly r dl prime if we consider a surface charge density. Then the potential would similarly be given as 1 over 4 pi epsilon naught integration over the surface charge density as a function of the prime coordinate the source coordinate system over curly r d a prime.

And these three expressions that we have written down here they hold good, if the reference is set as at infinity. These do not work otherwise, after developing this idea let us consider an example and find out how in practice we can find out the electric potential. Only from a charge distribution and not finding the electric field earlier.

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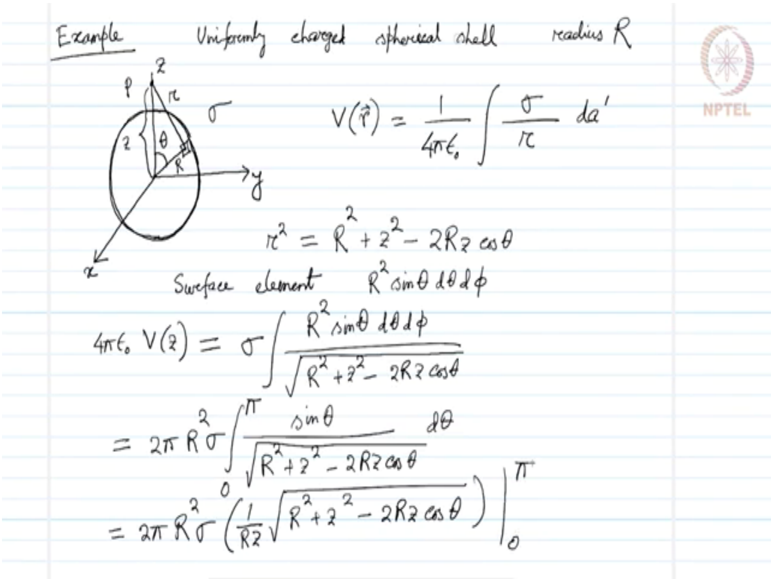

Let us consider a uniformly charged sphere, with radius capital R. And we want to find the potential due to this charge distribution. So, let us try drawing that sphere and because we have a sphere we will consider spherical coordinate system that would be the best given the symmetry of the system. So, these are the Cartesian axis in the spherical system. If we consider only a spherical shell here with surface charge distribution and the surface charge density is given by sigma that is uniform.

Then, we can consider a surface element here like this that is at a distance r from the origin. And if we consider a point p here on the z axis where we observe the potential, then this much length is given by z; this angle is theta. And from the point where we have the charge distribution to the point of observation this distance is curly R. If we have that then we can write the expression for the potential as 1 over 4 pi epsilon naught integration over sigma divided by curly r d a prime.

Now, we have considered the point of observation p on the z axis because we can orient the z axis in any direction. So, this is not a loss of generality in this case setting the point of observation on the z axis. We can write that curly r squared equals capital R that is radius of the sphere squared plus z squared minus 2 R z cosine of the angle theta this is the cosine law.

And the surface element can be given as capital R squared sin theta, d theta, d phi with this our 4 pi epsilon naught times the potential as a function of z only because we have considered the potential. We have considered our point of observation on z axis, this can be given as sigma comes outside the integral because we have considered uniform charge distribution surface charge distribution. R squared sin theta d theta d phi over R squared plus z squared minus 2 R z cosine of theta square root of this.

With this we can evaluate this and try to simplify this. We have performed the integral over phi because there is no other function of phi in this function in this expression. So, after performing that integral we got 2 pi outside and this R squared this is constant. So, it can come outside and only integration over theta is left the limit is from 0 to pi.

And in the numerator we have sin theta in the denominator, we have R squared plus z squared minus 2 R z cosine of theta square root of this quantity d theta. This is what we are supposed to integrate over and if we do that, we will have twice pi r squared sigma 1 over R z R squared plus z squared minus 2 R z cosine of theta square root of this entire quantity the limits are 0 to pi.

(Refer Slide Time: 16:10)

$$= \frac{2\pi R\sigma}{z}\left(\sqrt{R^2+z^2+2Rz} - \sqrt{R^2+z^2-2Rz}\right)$$

$$= \frac{2\pi R\sigma}{z}\left[\sqrt{(R+z)^2} - \sqrt{(R-z)^2}\right]$$

$$V(z) = \frac{R^2\sigma}{\epsilon_0 z} \quad \text{outside the sphere} \quad z > R$$

$$= \frac{R\sigma}{\epsilon_0} \quad \text{inside the sphere} \quad z < R$$

And that equals to 2 pi R sigma over z times R squared plus z squared plus 2 R z square root minus R squared plus z squared minus 2 R z square root, which is 2 pi R sigma over z R plus z square, square root minus R minus z squared square root. This simplifies to give us the expression for potential as a function of z as R squared sigma over epsilon naught z. And this is outside the sphere where z is greater than R.

And it becomes something else its R sigma over epsilon naught inside the sphere, where z is less than R. So, we can see that outside the sphere the potential is a function of z; that is distance from the center of the sphere on z axis because we have considered our point of observation on z axis. But inside the sphere because we have considered a spherical shell and charge distribution on the spherical surface only we have no charge inside the sphere.

So, there is no electric field inside the sphere only we have electric field outside. So, as soon as we come inside the sphere z becomes equals to R at the surface and R from the denominator and R square from the numerator that gives us only R at the numerator. And this becomes the constant potential everywhere inside the sphere.

So, it matches with our expectation but, we saw that it was not very easy to find out this potential right from the charge distribution. We have calculated the electric field from the charge distribution in a easier way than this. So, although finding the potential is in principle much more helpful for simple geometries, finding the potential from the charge distribution can be a bit tricky.