Electromagnetism
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Lecture – 28
Application of Gauss law on a flat 2D surface

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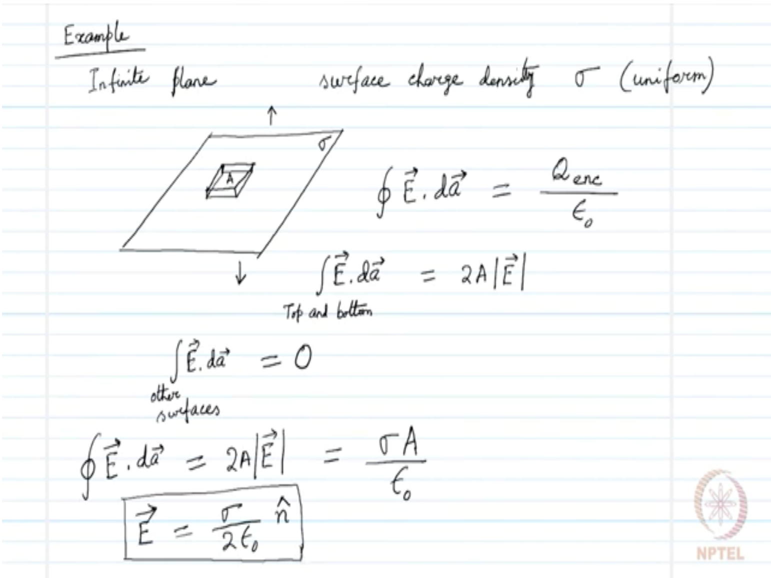

Let us consider another example. This time we consider an infinite plane, it has a surface charge density, uniform surface charge density sigma. And we are supposed to find out the electric field everywhere for this problem. So, this is our plane which extends to infinity in all direction and there is a surface charge sigma that is uniform on this plane extending to infinity.

So, if sigma is positive, we will have the electric field above this plane pointing upwards and below this plane pointing downwards, electric field will be pointing away from the positive charge density. Now, how to find out the magnitude of the electric field density? The direction we have already found out. We have to consider Gaussian surface that looks like a box.

Let us assume this kind of a surface with area A, this box is I have drawn it above this surface, similar box is there below this surface, so, this area is A here, we have similar thing below the surface as well. And if we now consider the Gauss theorem, so, we have a surface here where, the electric field is perpendicular to and the other surfaces the electric field is parallel to sorry, we have a surface here where the electric field is parallel to and the other surface is electric field is perpendicular to.

Let us write down the expression for E dot d a. So, if we consider this top surface of this box, on the top surface electric field is pointing upwards and the direction of this surface element is also upwards. So, E dot da will give the maximum value there and for the other surfaces side surfaces, that will E dot da will become 0. So, the Gauss law in integral form gives us E dot da is enclosed total enclosed charge over epsilon naught.

Now we have to find out E dot d a, E dot over the top and bottom surfaces gives us 2 A multiplied with magnitude of the electric field and on other surfaces because E and da are perpendicular on other surfaces, this quantity is 0; that tells us that E dot da over the surface enclosing this volume is 2 A times the magnitude of the electric field. And, how much charge have we enclosed?

The amount of charge enclosed is sigma uniform charge density multiplied with the area of the surface A and, if we have that then Gauss law will give us this and that means, we can write down the expression for electric field as sigma over 2 epsilon naught and we need to find out the direction of electric field that direction would be perpendicular to the plane carrying the surface charge density that is n cap.

Above the surface its above and below the surface its below, if sigma is positive. If sigma is negative, then that negative sign already tells you the direction of electric field. So, this expression is valid for an infinite plane carrying a charge density, uniform charge density sigma. So, here we have considered two examples. In one example, we have considered a cylindrical system with some charge distribution and there we have found that the electric field is a function of the distance from the cylinder even if we are.

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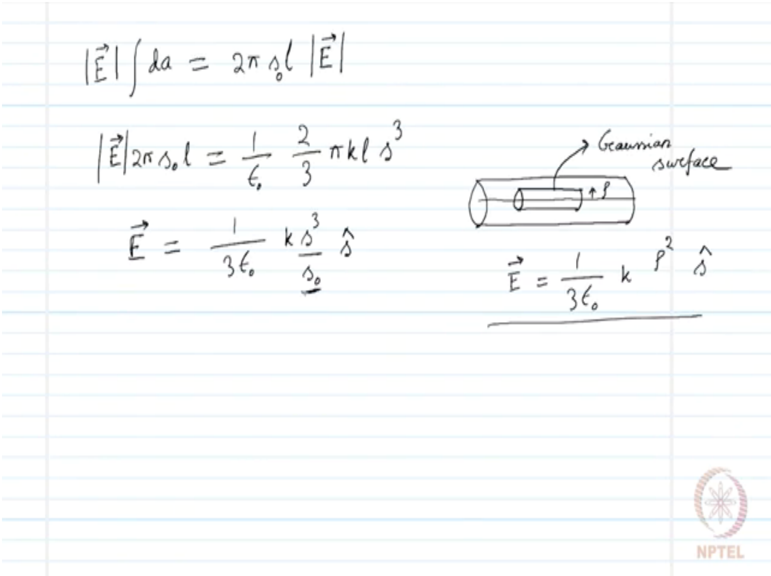

So, the distance from the cylinder comes in here if we are outside the cylinder. We have seen that it is a function of the distance from the cylinder. If we consider a surface charge density of infinite extent, we see that the electric field is not a function of the distance from the surface, it is same, no matter how far you go from the surface its uniform, that is something interesting. Please, think about why it happens so. And this example intrigues us to look at something even more interesting.

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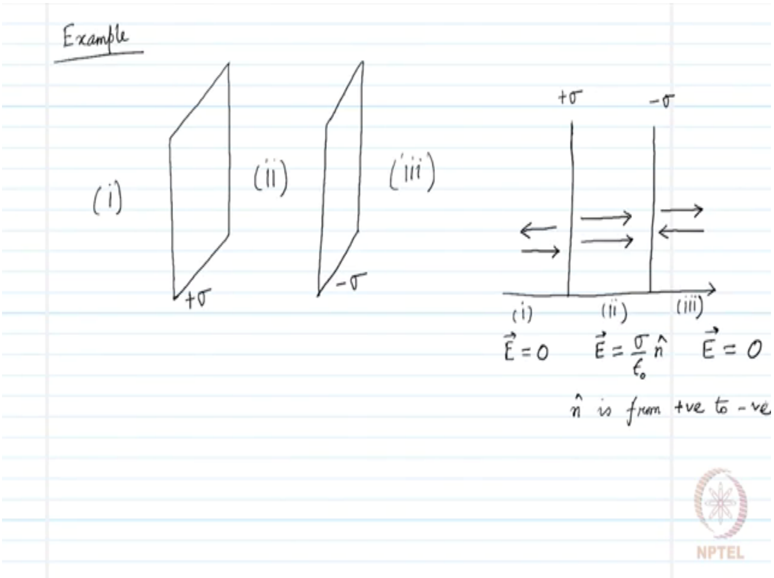

Let us consider another example. We have two parallel planes having uniform charge density both planes are of infinite extent. Although my drawing is not that good, the idea is that these two planes are parallel of infinite extent and the left hand side one carries a uniform charge density plus sigma the right hand side one carries a uniform charge density minus sigma. We have three regions, in region i that is in the left to the left hand plane region ii that is in between two planes and region iii that is right to the right side plane, we are supposed to find the electric fields.

How do we find the electric fields? Let us consider. So, if they are separated along x axis, let us draw x axis here and lines here for simplicity. This becomes our region i, this is region ii and this is region iii. For positive charge on this lines that is this plane, we will have electric field along this direction right, this has plus sigma charge density and this is minus sigma charge density.

So, for minus sigma charge density we will have electric field in region i along this direction and because the electric field magnitude for this uniform charge distribution over infinite surface infinite plane, it does not depend on the distance from the plane; these two electric fields are equal and opposite, so, they will cancel each other.

Similarly, here in this region for the positive one the positive sigma plane we will have electric field in this direction, for the negative sigma plane we will have electric field in this direction and they will cancel each other exactly, because the magnitude of the electric field does not depend on the distance from the plane. And in between we will have for the positive plane electric field along this direction, for negative plane electric field will be along this direction the same direction, so that gets reinforced.

So, in region i we have electric field equals 0, in region iii we have electric field equals 0 and in region ii we have electric field equals sigma over epsilon naught n cap, where n cap is from positive to negative planes. So, this is the principle of parallel plate capacitors.