Electromagnetism
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Lecture – 26
Electric field lines, Flux, Gauss law

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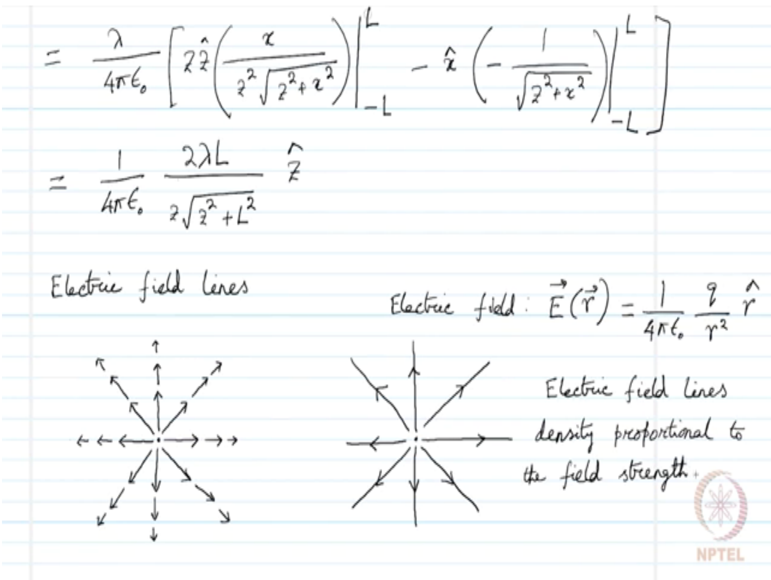

Now, we have got the answer. Let us now introduce the concept of electric field lines. What are electric field lines? If we consider a point charge single point charge of magnitude q then let us consider the single point charge of magnitude q located here. The electric field corresponding to the single point charge can be written as we have learned from Coulombs law E as a function of r would be given as if we consider the point charge is located at the origin, we can write it simply as 1 over 4 pi epsilon naught q over the position vector square.

Sorry the absolute value of the position vector squared and the direction of position vector. This will be the electric field provided the point charge is located at the origin.

Now, we can consider. So, we know that if we approach the charge then the electric field is stronger because it has 1 over r squared dependence inverse square law Coulombs law is an inverse square law. So, if we approach the charge the electric field is stronger and if we move away from the charge, then the electric field is weaker. So, in ordinary way of representing vectors we can represent it this way. We have a longer line here representing stronger field and then shorter line here even shorter line here with arrowhead that represents stronger field that becomes weaker progressively and that happens along every direction. So, let me try drawing in this direction longer than shorter than even shorter, longer shorter, even shorter, long, short, even short and also along these directions.

Like this, but when we talk about electric field lines the general the convention is that we will consider the point charge here and we will just draw the lines like this. These are called the field lines. So, we are drawing continuous lines along the direction of the electric field. So, we are drawing the lines at the positions where we have drawn the vectors earlier. The difference being that these are continuous line and the length of the line length of the arrow does not really represent the strength of the electric field rather the density of those lines that represent the electric field in case of electric field lines.

The density is proportional to the field strength. This is how we have we conventionally talk about the electric field lines.

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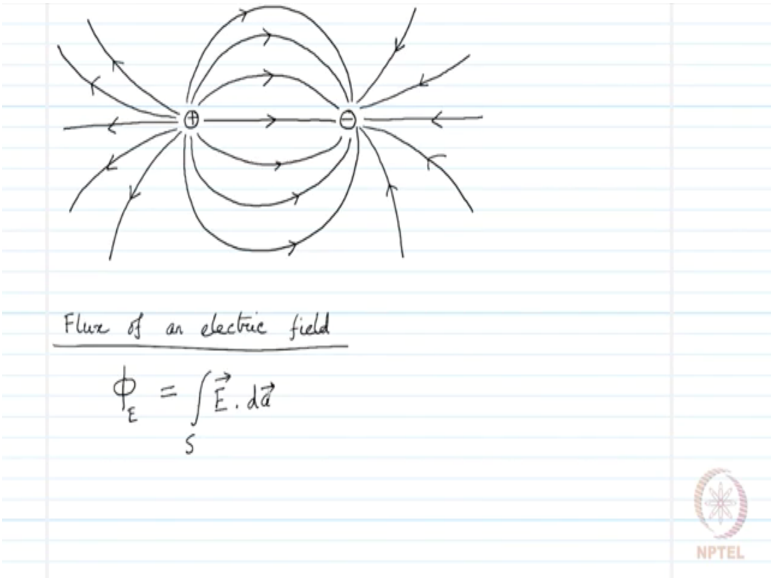

Now if we consider two opposite point charges, let us consider a positive charge at this point and a negative charge at this point. How would the field lines look? The field lines it is along the direction of the electric field that means, it will go from the positive charge towards the negative charge. So, it will look somewhat like this away from that line it will be like this symmetrically above. It is like this and this will go on in the it will make many such lines connecting these two point charges like this and then when it cannot connect the two charges, it will just diverge something like this also in this direction, but in this direction the direction of this arrow would be towards the negative charge because that is the direction of the electric field.

This is how the field lines will look if we consider two opposite charges at a distance. Now let us introduce the notion of flux of an electric field. If we have a constant electric field then the flux is the field multiplied with the area through which the field is going, but if we have an electric field that is not constant over a given area then we cannot define it this way. We have to define it in an integral in an integral form like this phi E being the electric flux. It can be defined as integration over a surface on which we are interested in finding the flux E dot product with the area element.

Now, this in terms of field line is the measure of the number of field lines that are passing through this area S and this is proportional also to the charge enclosed in a volume.

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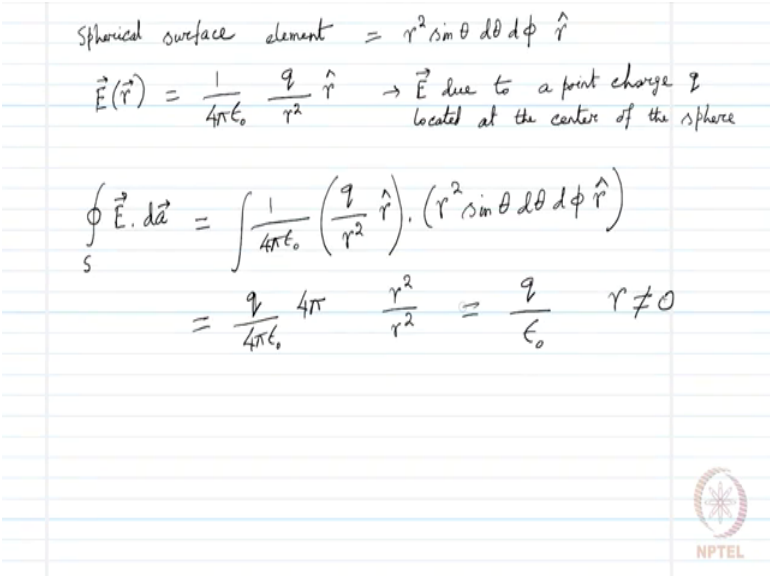

So, if we consider a spherical surface and the surface element would be given as r squared sin theta d theta d phi r cap. This would be the surface element, then we know that spherical surface encloses a volume and if we have an electric field at the origin of this at the center of this sphere sorry. If we have an electric field due to a point charge located at the center of the sphere then we can write the electric field this way E r is given as 1 over 4 pie epsilon naught. The magnitude of that point charge is q over r squared r cap.

So, this is the electric field due to a point charge q located at the center of this sphere which means it is located at the origin of our coordinate system. Now if we have this kind of an arrangement then if we try to find out the total flux of this electric field through the spherical surface we will perform a surface integral that surface is enclosing a volume that surface is a closed surface over E dot da and this way we will have integration over 1 over 4 pi epsilon naught q over r squared r cap.

Now, this quantity will have a dot product with the area element that is r squared sin theta d theta d phi r cap. Now because the direction of these two vectors are same we will have simply integration over. Ok, maybe I perform the integration over theta and phi already because we do not have any theta or phi dependence anywhere. So, it is the spherically symmetric expression for electric field. So, we can write down 1 over 4 pi epsilon naught which is a constant outside the integral multiplied with 4 pi outside the integral that will come from theta and phi integral and that is not a function of r then the integration we can also have q outside q is the amount of charge which is constant then we are left with r squared over r squared.

So, after performing integration over theta and phi we are we get this kind of a quantity. So, r squared over r squared is nothing but 1 and 4 pi 4 pi cancels. So, we are left with the amount of charge over epsilon naught. We are left with only this; that means, we notice that the radius of sphere cancels out if the only if the radius is not 0 then it would not cancel out. Otherwise we have this expression for the total flux and that means if we have a charge enclosed within that enclosed by that surface then we have a finite flux and that flux is the total charge over epsilon naught and if we do not have any charge inside that surface inside that closed surface we will certainly not have any total flux through the surface.

Now, let us consider assembly of charges.

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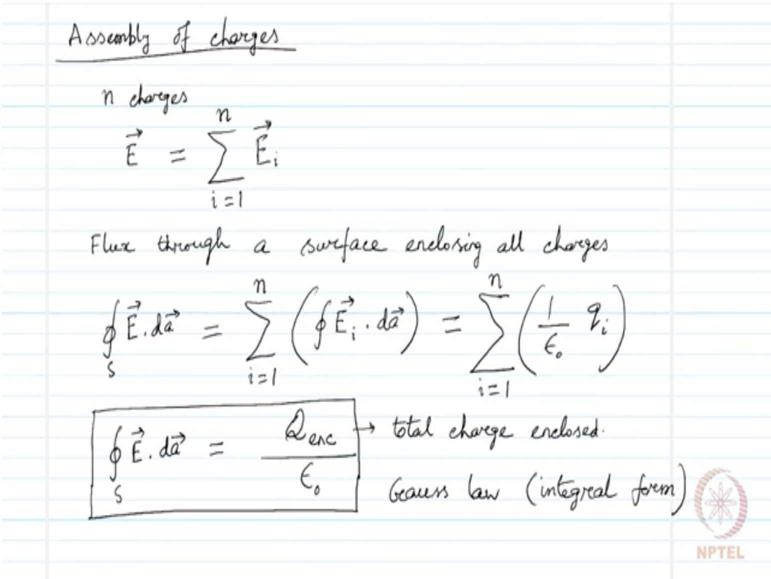

So, if we consider n charges then the electric field, the total electric field according to the principle of superposition can be written as sum over I equals 1 to n the vector sum over each over the electric field due to each charge. With this the flux through a surface that encloses all of these charges that can be given as cyclic surface integral over E dot da equals sum over i equals 1 to n cyclic surface integral over each electric field due to each individual charge dot da.

This is just the principle of superposition and that gives us very similar to what we have done earlier. Sum over i equals 1 to n 1 over epsilon naught q i; that means, the flux is proportional to the total charge enclosed by the surface if there are multiple charges and nothing else. That is all.

So, we can write looking at this expression the surface integral over E dot d a that is the flux through this closed surface is nothing, but the total charge enclosed that is capital q Q enc over the permittivity of free space epsilon naught. This is the total charge enclosed.

Now, this statement is very important. This is known as Gauss law of course, in integral form. Now if we use divergence theorem on this expression.

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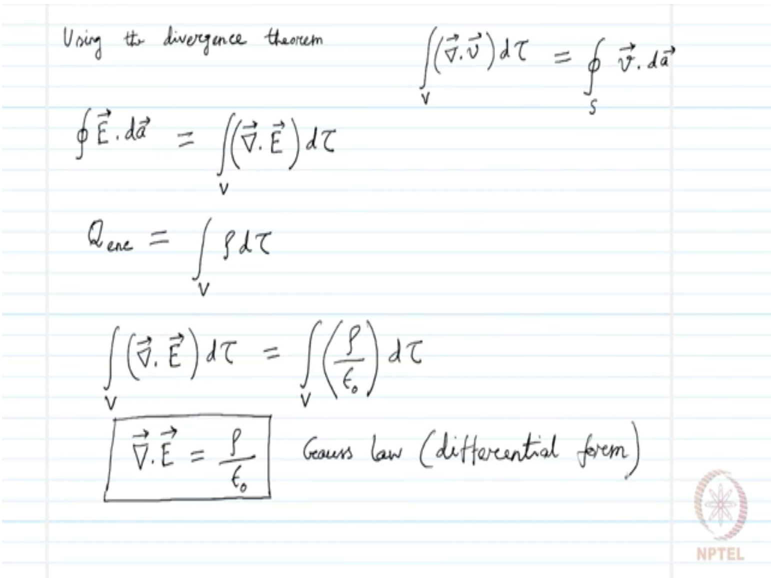

Let us brush up what is divergence theorem. The divergence theorem tells us that the volume integral of the divergence of a vector field v this is equal to the surface integral over the closed surface enclosing this volume V the surface integral of v dot da. This is the statement of the divergence theorem.

So, we have here over a closed surface the surface integral of E dot d a that can be written as a volume integral over the divergence of the electric field times the volume element. And we know that the total charge enclosed is nothing, but the volume integral of the volume charge density. Now, so the right hand sides of these two expressions by comparing these 2 expressions and looking at the right hand sides we can write from the integral form of Gauss law the volume integral of the divergence of the electric field equals the volume integral of rho over epsilon naught. And if this has to be true for any given volume then the quantities in the left of the parentheses of the left hand side and the quantity in the parentheses of the right hand side they must be equal.

That means we can write down divergence of the electric field can be given as the volume charge density over the free space permittivity epsilon naught. This is the Gauss law; the same Gauss law this time in differential form. Can we directly calculate the divergence of an electric field? Let us try doing that.

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Direct calculation of the divergence

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_{\text{all space}} \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\,\rho(\vec{r}\,')\,d\tau' \qquad \vec{\mathscr{r}} = \vec{r} - \vec{r}\,'$$

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{4\pi\epsilon_0}\int \vec{\nabla}\cdot\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)\rho(\vec{r}\,')\,d\tau'$$

$$\vec{\nabla}\cdot\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) = 4\pi\,\delta(\vec{\mathscr{r}})$$

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{4\pi\epsilon_0}\int 4\pi\,\delta(\vec{\mathscr{r}})\,\rho(\vec{r}\,')\,d\tau' \qquad \vec{r} = \vec{r}\,' \Rightarrow \vec{\mathscr{r}} = 0$$

$$= \frac{1}{4\pi\epsilon_0}\,4\pi\,\rho(\vec{r}) = \frac{\rho(\vec{r})}{\epsilon_0}$$

If we have a volume charge density rho as a function of r prime then the electric field at a position vector r for that can be given as 1 over 4 pi epsilon naught integration over r cap upon r squared the volume charge density rho r prime d tau prime where primed coordinates are the coordinates of the source charge reference frame.

And we have to perform this integral over all space and according to our notation curly r is nothing, but position vector in unprimed minus position vector in primed coordinate system. So, if we try to calculate the divergence of electric field directly then we will write it as 1 over 4 pi epsilon naught integration divergence of the vector r cap over r squared times rho r prime d tau prime ok. So, we have already calculated earlier the divergence of r cap over r squared.

So, let us write that down the divergence of r cap over r squared that we have learned from vector algebra is nothing, but 4 pi dirac delta function of r vector. If we have that then the divergence of electric field can be expressed as 1 over 4 pi epsilon naught integration over 4 pi dirac delta of r rho r prime d tau prime.

Now, when does this argument r go to 0 when there is no distance between r and r prime. So, when r equals r prime we have currently r equals 0. So, at that point we will have this integral giving some value. So, if we consider it in the form Dirac delta multiplied with a function integrated over all space then we will have the value of the function at the point when the argument of the dirac delta is 0.

So, this will become nothing, but 1 over 4 pie epsilon naught. We will have 4 pi here coming out and the function that is rho r prime when r prime equals r this will have the value. So, we will get rho r coming out here this is the expression that. So, 4 pi from denominator and numerator cancels and we are left with rho over r sorry rho as a function of r over epsilon naught. So, we can directly calculate the divergence of the electric field without looking at the flux and the integral theorem what we have done. So, we can directly arrive at the differential form of Gauss law.