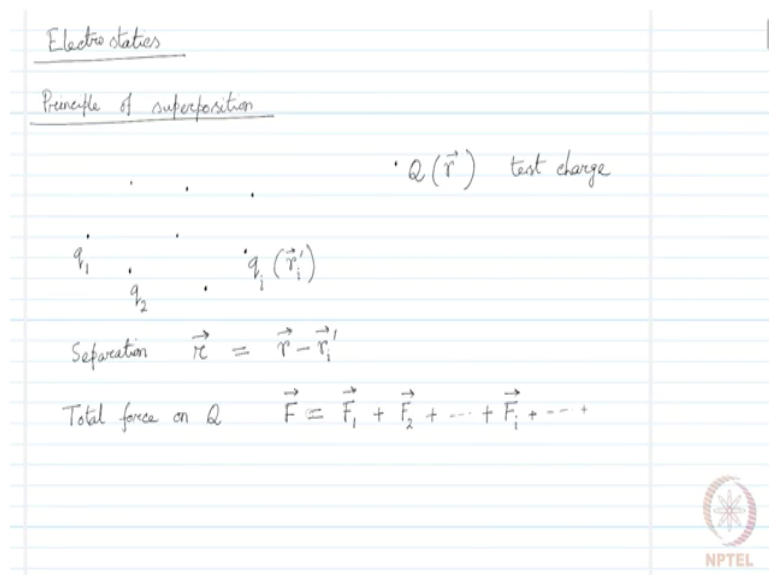


Electromagnetism
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Lecture – 23
Introduction to Electrostatics

After getting introduced to the mathematical background that is required for this course Electromagnetism let us move on to Electrostatics. So, we will start the electrostatics with the principle of superposition.

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Let us consider a situation where we have few charges distributed over a region, we consider few point charges distributed like this, where this one is say q_1 , this is q_2 and so on, say this is q_i i th charge at r_i prime position, r_i prime is the position vector for this i th charge. And

let us consider a test charge here marked as capital Q at location position vector r this is the test charge.

Now, the distance between the test charge and the point charge at ith position, the separation can be given as curly r vector equals r minus r prime r i prime. So, if we now consider the total force on the test charge capital Q due to this charge distribution that we have we can write the total force on capital Q as the force due to the first charge plus the force due to the second charge plus the force due to any ith charge plus so on.

So, the force the amount of total force on Q would be the vector sum due to the force from all charges, this is the principle of superposition. And this is very important in the context of electrostatics, we will find its use later on. One must remember that it is the force that we are summing and no other quantity.

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Coulomb's Law

point charge q test charge Q
distance \vec{r}

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \hat{r}$$

↳ permittivity of free space


Electric field

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \dots$$

$$= \frac{1}{4\pi\epsilon_0} \left(\frac{q_1 Q}{r_1^2} + \frac{q_2 Q}{r_2^2} + \dots \right)$$

$$= \frac{Q}{4\pi\epsilon_0} \left(\frac{q_1}{r_1^2} + \frac{q_2}{r_2^2} + \dots \right) = Q \vec{E}$$

↳ electric field



Now, let us move on to finding an expression for the electric field, the force in due to two charges the electrostatic force due to two charges and the expression was given by coulombs. So, it is known as coulombs law. If we have a point charge small q and a test charge that is also a point charge capital Q at a distance curly r vector, then the force acting on the test charge capital Q due to the point charge small q will be $\frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2}$ along the \hat{r} direction.

We will explain why what is epsilon naught later epsilon naught is known as the permittivity of free space. We will explain it later let us now introduce the concept of electric field we have seen that the total force can be expressed as the superposition of F_1 plus F_2 plus so on which is nothing but $\frac{1}{4\pi\epsilon_0} \frac{q_1Q}{r_1^2} + \frac{q_2Q}{r_2^2} + \dots$. Taking the capital Q out, we can write it as $Q \left(\frac{q_1}{r_1^2} + \frac{q_2}{r_2^2} + \dots \right)$, or we can write this as capital Q times the electric field E . That means, if we bring a test charge q in this region it will experience an electric field, and due to that electric field the force on it would be the amount of charge times the electric field.

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$$\vec{E}(\vec{r}) = \sum_{i=1}^n \frac{q_i}{r_i^2} \hat{r}_i \quad \vec{r}_i = \vec{r} - \vec{r}'_i$$

Example

$$E_z = 2 \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \cos\theta$$

$$r = \sqrt{z^2 + \left(\frac{d}{2}\right)^2}$$

$$\cos\theta = \frac{z}{r}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2qz}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^{3/2}} \hat{z}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2q}{z^2} \hat{z} \quad (z \gg d)$$

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That means we can define the electric field E that is a function of a position vector as if there are n charges, then $q_i / r_i^2 \hat{r}_i$. Now, here we must remember that the coordinate of a charge is represented by the position vector \vec{r}'_i , and the at a point that is represented by the position vector \vec{r} , we bring in the test charge. So, the distance between these two points are given as $\vec{r} - \vec{r}'_i$ that is the distance from the position vector \vec{r} to that position vector of the test charge \vec{r}'_i .

So, this is known as \vec{r}_i . It is like here is our test charge q_i , its position vector is given as \vec{r}'_i this is a point charge here we have our test charge Q and its position vector is \vec{r} . So, the vector connecting these two would be given as $\vec{r} - \vec{r}'_i$ vector, and this is denoted as \vec{r}_i with respect to any arbitrary origin let us say located here. So, this much is \vec{r}'_i and this much is \vec{r} .

So, we have found the expression for electric field. Now, let us consider an example. Let us consider a charge distribution with two point charges. We have one point charge at the left of value magnitude q , at the right we have another with the same magnitude, and the distance between them is d let us say. We want to find out the electric field at this point which is z distance above the by the line bisecting sorry it is above the bisecting point of the line connecting these two charges Q .

So, if we have this kind of an arrangement, we can see that the electric field due to the left hand side charge would point in this direction, the electric field due to right hand side charge will point in this direction, and the resultant direction of the electric field would be along z -direction. If we consider this angle to be θ , then we can write down that the z component of the electric field from one charge q is $\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \cos\theta$, where this is r this length. And for if we consider both the charges, we will have to multiply it with 2, and r will be given as $\sqrt{z^2 + \frac{d^2}{4}}$ square root of this, cosine of θ is given as $\frac{z}{r}$.

With this we can write the resultant electric field as $\frac{1}{4\pi\epsilon_0} \frac{2qz}{(z^2 + \frac{d^2}{4})^{3/2}}$ \hat{z} . Now, if we consider the limit where z is much greater than d , then in this limit we will have the electric field expressed as $\frac{1}{4\pi\epsilon_0} \frac{2q}{z^2}$ along \hat{z} direction that is valid only within this limit.