Electromagnetism
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Lecture – 18
Special curvilinear coordinate systems: Cylindrical and spherical

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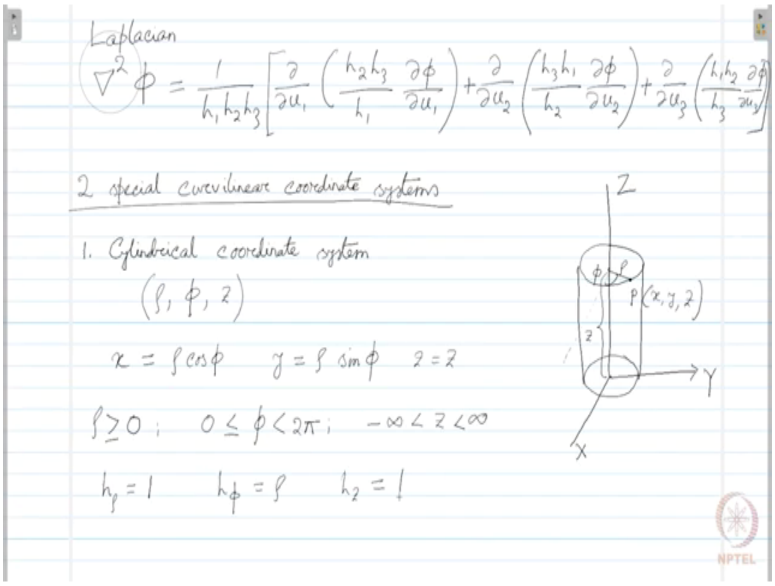

Now, let us consider special; special curvilinear coordinate system 2 special system. The first one is cylindrical coordinate system; that means, u 1 u 2 and u 3 will form a cylinder. How? Let us, draw a cylinder here like this makes a cylinder and the origin of our conventional Cartesian six coordinate system we consider here, this happens to be the X axis this is the Y axis and this is the Z axis.

In this notation, if we consider a point P here; this is the position of the point P. Now, the distance of this point P is given distance of this point P from the Z axis is given by rho. And, the angle that it makes with X axis, so I try to draw a parallel of X axis here the angle that it makes with this is phi and the Z value that is this much is given by z. So, we have coordinates rho, phi, and z; we also have r theta and phi coordinates. Sorry, we also have X, Y and Z coordinates of this point P.

Now, if we just compare with our discussion of polar coordinate systems we can write down that x will be given by rho cosine of phi, y will be given by rho sin of phi, and z equals simply z nothing else. And there are interesting facts to note that rho will always be greater than or equal to 0, there cannot be any negative value for rho in this system; in cylindrical coordinate system. The range of phi is 0 less than equal to phi less than 2 pi anything beyond 2 pi is already covered within this range anything less than 0 is already covered within this range.

And z has the limit just like the Cartesian coordinate system, minus infinity less than z less than plus infinity. And in terms of the h components h rho equals 1 in this case h phi equals rho and h z equals 1, these are the coefficients to the Cartesian coordinates; that converts it to the Cartesian coordinate system.

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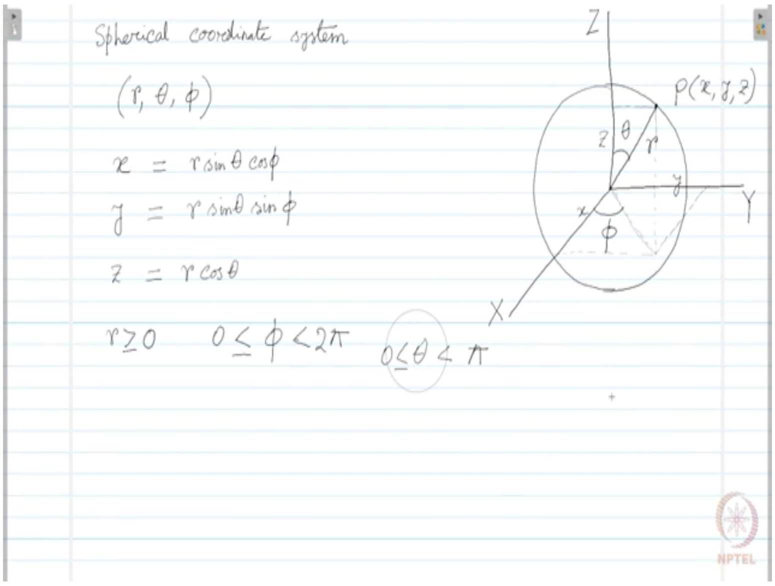

Now, let us consider the other one spherical coordinate system. AC [FL] In case of spherical coordinate system let us try to draw a sphere here and the center of the sphere happens to be the origin of the Cartesian coordinate system this is the X axis of the Cartesian coordinate system this is the Y axis and this is the Z axis.

Now, let us consider a point on the spherical surface point P somewhere here this is our point p now the distance of this point P from the origin that is given as r. So, we found one coordinate r and now we can project this point onto the x y plane say we project it here. So, the x component is this much the y component becomes this much we can see.

Now, if we draw a line from the center to this point on the x y plane then that makes an angle with the X axis that is given as phi and this line marked with r that makes an angle with Z axis that marked with theta. So, we have coordinates r, theta and phi these three coordinates are there. What are the relations of these coordinates with the Cartesian coordinate system? So, P has xyz coordinate as well what is x in our picture? X is this much, y is this much, and z is this much.

So, x as you can see from this picture we have projected r on to x y planes. So, that gives us r sin theta; r sin theta is this line and then we project this line onto the x axis. So, the angle is phi cosine of phi this is x. Similarly, y for y we need r sin theta just because it is projected onto x y plane and the projection of this line onto Y axis will bring in sin phi and the z coordinate is not z; it is something else here we have r and if you project onto Z axis that will be r cosine of theta.

So, this is the relationship and it is also interesting to note the limits on it r can never be negative the range of phi is from 0 to 2 pi, and the range of theta it starts from 0 and it ends at pi; not 2 pi. Because if we increase the value of theta 2 pi; that means, it will r will actually trace the negative Z axis and by tracing 2 pi angle using phi we can actually trace the entire sphere entire spherical space that is accessible to us and varying the value of r we can access the whole space. So, we do not need any larger value for theta in this case.

So, after introducing the 2 special curvilinear coordinate systems that we are going to use a lot in the course electromagnetism let us consider few examples of these coordinate systems.

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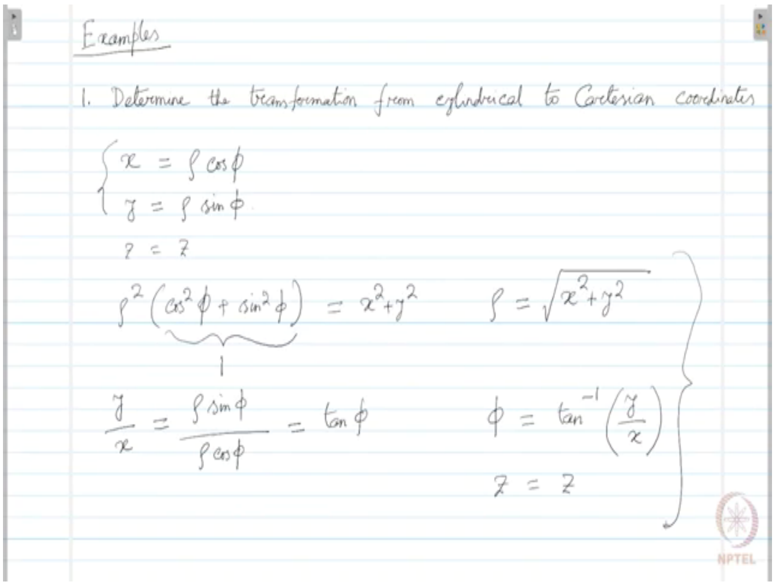

Let us consider the first example where we asked to determine the transformation from cylindrical to rectangular coordinates. How do we do this? We know that; we know the other transformation that is we know that x is given by rho cosine of phi, y is given by rho sin of phi and z equals z.

So, if we want to find the reverse transformation then we can note that rho squared. We are actually squaring and adding these two rho squared cosine square of phi plus sin square of phi this is x square plus y square. In other words, so cosine square phi plus sin square phi gives us 1; in other words, we have rho equals x square plus y squared square root of this.

Now, if we try to find out y over x. So, we divide this equation by this equation here and that gives us rho sin phi over rho cosine phi which is nothing but tan phi. Then we can clearly understand that phi is nothing, but tan inverse of y over x and z is z. So, we have obtained the reverse transformation; that means, we have determined the transformation from cylindrical coordinate to Cartesian coordinate.

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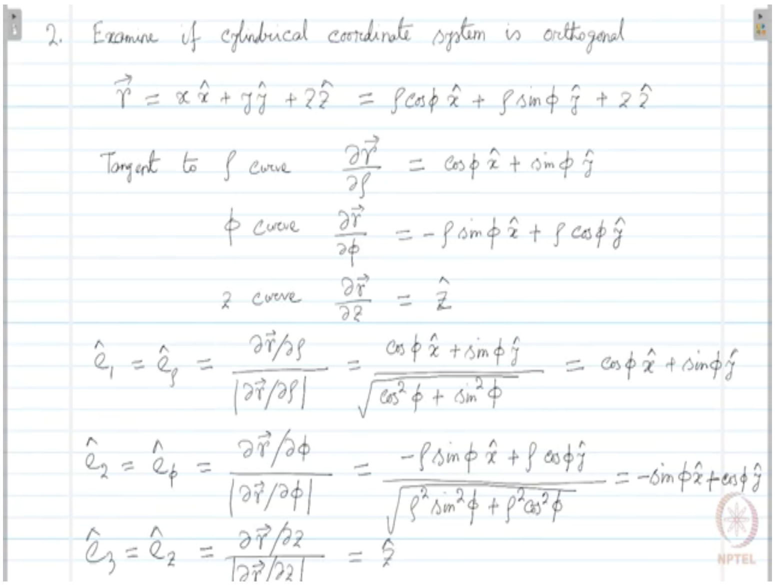

In the next example, we examine where; whether cylindrical coordinate system is orthogonal or not. How do we do that? We considered a position vector in Cartesian coordinate system the position vector r is given as x x cap plus y y cap plus z z cap and that means, in terms of cylindrical coordinate axis we can write it as rho cosine phi x cap plus rho sin phi y cap plus z z cap this way. Now, the tangent vectors to rho phi and z curves that can be given as tangent to rho curve would be del r del rho for phi curve it would be del r del phi and for z curve, it would be del r del z.

So, let us evaluate these quantities del r del rho is cosine phi x cap plus sin phi y cap. Del r del phi equals minus rho sin phi x cap plus rho cosine phi y cap, and the del r del z is nothing but;

the unit vector along Z axis. So, we have the unit vectors e 1 e 2 e 3 that is e rho e phi e z we can determine these unit vectors. So, e 1 that is nothing but e rho becomes del r del rho over the absolute value of del r del rho. So, this can be given as cosine phi x cap plus sin phi y cap over cosine square phi plus sin square phi square root of this which is.

So, the denominator becomes 1 only the numerator remains cosine phi x cap plus sin phi y cap. Similarly, e 2 that is e phi can be given as del r del phi over the absolute value of del r del phi that is minus rho sin phi x cap plus rho cosine phi y cap over rho squared sin squared phi plus rho squared cosine squared phi square root of this. Which is again we can see that the numerator becomes rho squared times one.

So, rho squared square. So, this becomes rho squared square root that is rho and rho cancels from the numerator the denominator becomes only rho, it cancels from the numerator and we are left with minus sin phi x cap plus cosine phi y cap. E 3 is e z that is trivial, can be written like this, but it is we know that nothing but z cap. Now, if we have these we have to now check for the dot products between different unit vectors and see whether it satisfies the condition for being orthogonal curvilinear coordinate system.

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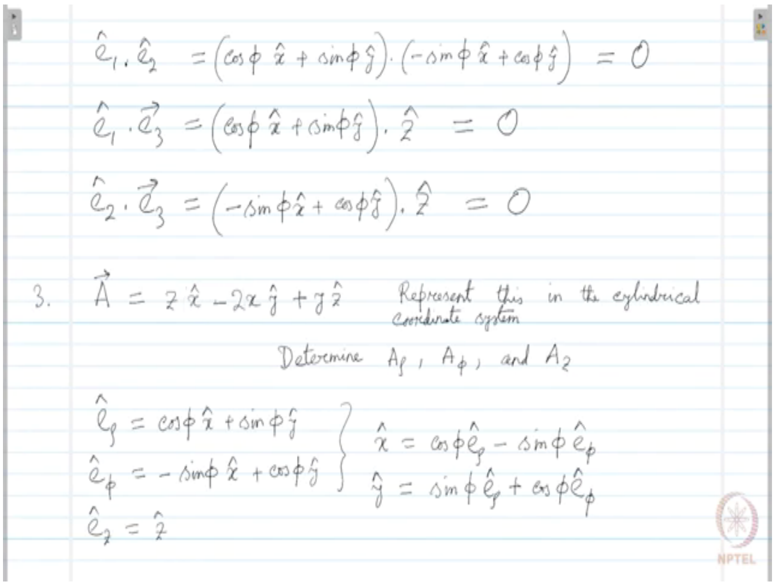

First, let us try e 1 dot e 2 that is cosine phi x cap plus sin phi y cap dot minus sin phi x cap plus cosine phi y cap this turns out to be 0. e 1 dot e 3 equals cosine phi x cap plus sin phi y cap dot z cap which is obviously 0 and e 2 dot e 3 is given by minus sin phi x cap plus cosine phi y cap dot again z cap and this is also; obviously, 0. Therefore, the coordinate system that is the cylindrical coordinate system that we have considered and we have worked out so far; turns out to be an orthogonal coordinate system orthogonal curvilinear coordinate system.

Now, let us consider yet another example in cylindrical coordinate system itself. Let us consider a vector A given by z x cap minus 2 x y cap plus y z cap. So, this is represented in Cartesian coordinate system and now we want to represent this in the cylindrical coordinate system. What does it mean? That means, we will have to determine A rho A phi and A z. How do we do that first we will have to find out the unit vectors e rho, e phi, and e z and we have already found that out in the previous problem; previous example.

So, let us just write that that down e rho is cosine phi x cap plus sin phi y cap e phi is minus sin phi x cap plus cosine phi y cap and e z is z cap. Now, if we compare the first and second unit vectors we can write down by comparing these two that x cap is; cosine phi e rho unit vector minus sin phi e phi unit vector. Similarly, y cap equals sin phi e rho unit vector plus cosine phi e phi unit vector.

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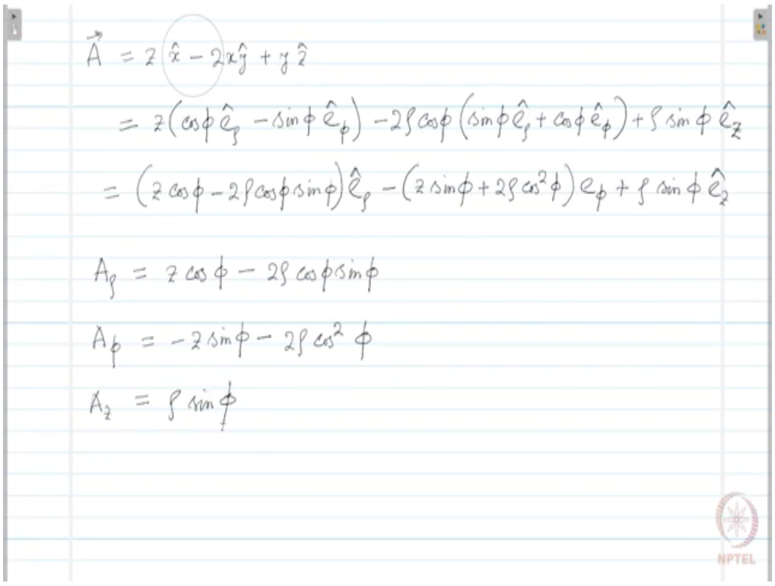

Once we have these we can represent the vector A as, z times cosine phi e rho minus sin phi e phi cap. E phi unit vector minus 2 rho cosine phi becomes common sin phi e rho unit vector plus cosine phi e phi unit vector plus rho sin phi e z unit vector that is nothing else, but z cap.

Rearranging these we can write that it is z cosine phi minus 2 rho cosine phi sin phi e rho unit vector minus z sin phi plus 2 rho cosine squared phi e phi unit vector plus rho sin phi is ez unit vector. And that means, we have actually found out the components A rho is given as z cosine of phi minus 2 rho cosine of phi times sin of phi. A phi is given as minus z sin phi minus 2 rho cosine squared phi and A z is rho sin phi.