Electromagnetism
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Lecture – 16
Generic curvilinear coordinates systems: Unit vectors and components

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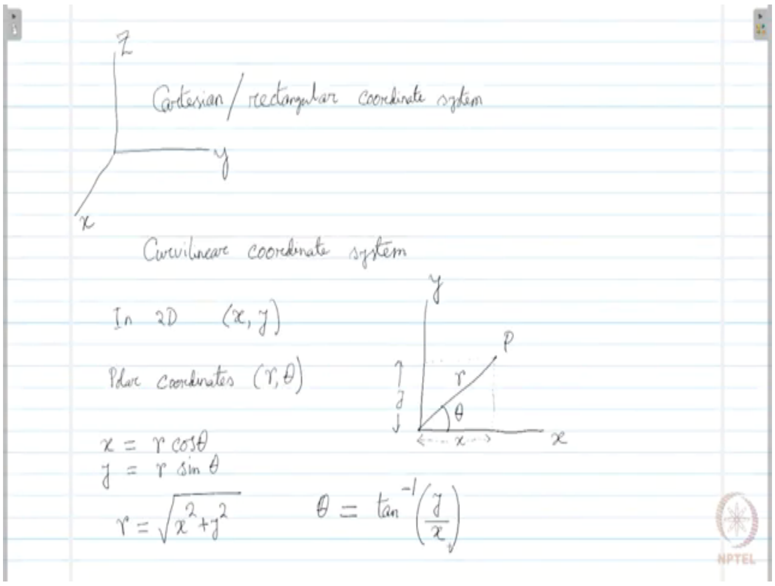

Now, this is the simplest possible curvilinear coordinate system and this is in 2 dimension, but mostly we will deal with 3 dimensional systems. So, we need to go for curvilinear coordinate systems and we need to introduce it in a general fashion.

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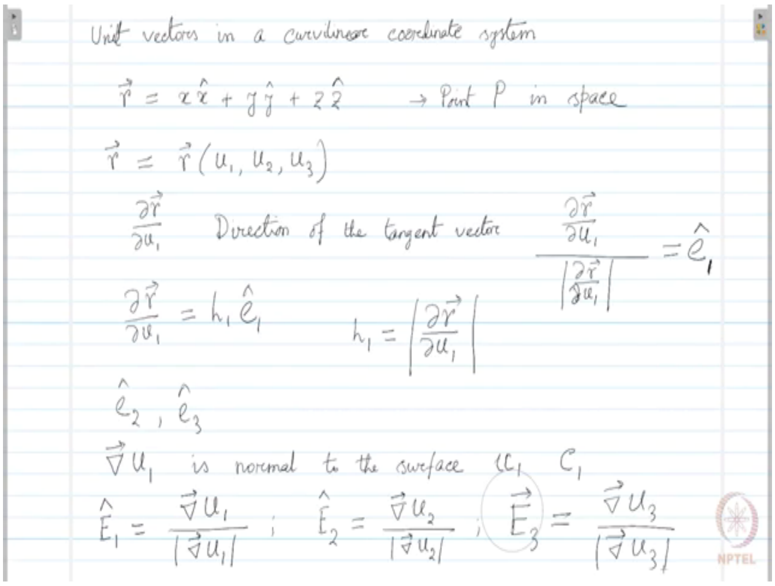

So, we need to first understand how to find out unit vectors in a curvilinear coordinate system. So, if we consider in 3 dimension a vector given by r equals x x cap plus y y cap plus z z cap, then and if it represents a point P in space then we know x cap y cap and z cap these are the unit vectors, along xy and z direction. Now, if we consider that the same vector r is represented in a curvilinear coordinate system as a function of u 1, u 2 and u 3 three coordinates.

And we can consider a tangent vector to the u 1 curve at point P for which u 2 and u 3 are constant; that means, we are talking about del r del u 1 this would be a tangent vector in the u 1 coordinate on the curve represented by r and the direction of this tangent vector, can be given as del r del u 1 over the magnitude of del r del u 1 like this.

And this can be called the first unit vector e 1 and; that means, if we have h being h 1 being the first coordinate then del r del u 1 can be written as h 1 e 1 unit vector; that means, h 1 is given as the absolute value of del r del u 1 this.

Similarly, we can find e 2 and e 3 using the tangents from u 2 and u 3 curves at point P; and then we have all the unit vectors. Now, we can consider that we can consider the gradient of u 1 this is a vector at P that is normal to the surface u 1 equal c 1.

So, this is the gradient of u 1 this vector is normal to the surface u 1 equals c 1. Then a unit vector along this direction can be given as capital E 1 unit vector is the gradient of u 1 over the absolute value of the gradient of u 1 like this. Similarly, E 2 and E 3; I mean capital E 2 and capital E 3 can be written as gradient of u 2 over the absolute value of gradient of u 2.

And E 3 equals gradient of u 3 over the absolute value of the gradient of u 3, we can write it this way. So, these capital E 1 capital E 2 capital E 3; these are normal's these are unit vectors normal to the surface while small e 1 small e 2 and small e 3 these are unit vectors tangential to the surface.

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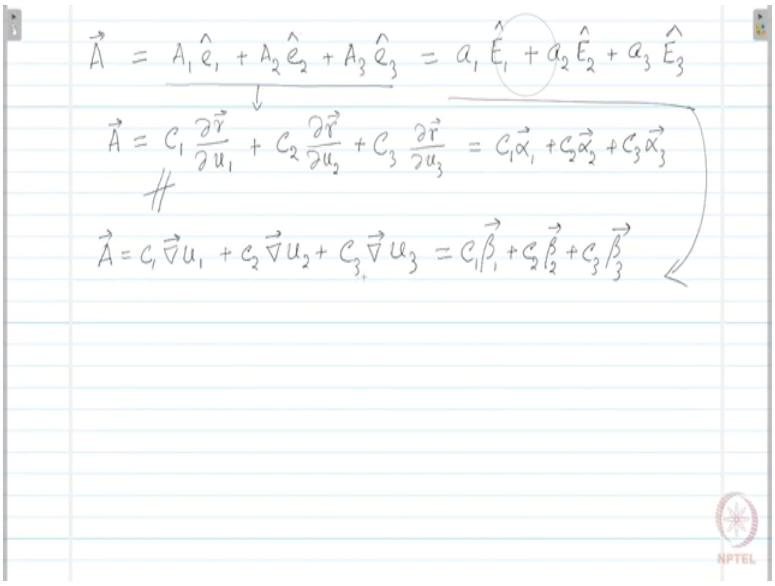

If we want to represent a vector, given by a vector a can be represented in terms of the unit vector small e 1 small e 2 small a 3 or capital E 1 capital E 2 capital E 3 in terms of small e 1 small e 2 small e 3; it would be A 1 e 1 plus A 2 e 2 plus A 3 e 3.

Similarly, in terms of capital E 1 E 2 E 3 we are using small a 1 a 2 a 3 and capital E 1 E 2 E 3. I will in a minute explain what that means, we can represent it this way where small capital A 1 capital A 2 capital A 3 or small a 1 small a 2 small a 3; a 3. These are the components of a in each system and how we have found out the system one was from the partial derivative of the position vector with respect to u 1 u 2 u 3.

And the other one was from the absolute value of the gradient of the unit vector. Sorry, absolute value of the gradient of u 1 u 2 u 3 that way we found these components we can write it in any of the ways. In other words, we can represent A as in the first notation here, C

1 del r del u 1 plus C 2 del r del u 2 plus C 3 del r del u 3 that is C 1 alpha 1 vector plus C 2 alpha 2 vector plus C 3 alpha 3 vector.

And in so, this is this notation here and the other notation would be like we are writing it here, A is given as C 1 of course, these this C 1 and this C 1 are not equal; gradient of u 1 plus C 2 gradient of u 2 plus C 3 gradient of u 3. That is equal to C 1 beta 1 plus C 2 beta 2 plus C 3 beta 3.

So, we have found out how unit vectors are calculated and so, these capital C 1 C 2 C 3 that we have obtained in this notation the first notation are called the contra variant components, while the small C 1 C 2 C 3 that we have found in the second approach is called the covariant components.