# Electromagnetism

**Dr. Nirmal Ganguli**
**Department of Physics**
**Indian Institute of Science Education and Research, Bhopal**

## Lecture – 12
## Fundamental theorems of vector calculus: The gradient theorem

Now, we are going to discuss about the Fundamental theorems of vector calculus; fundamental theorems for gradient divergence and curl.

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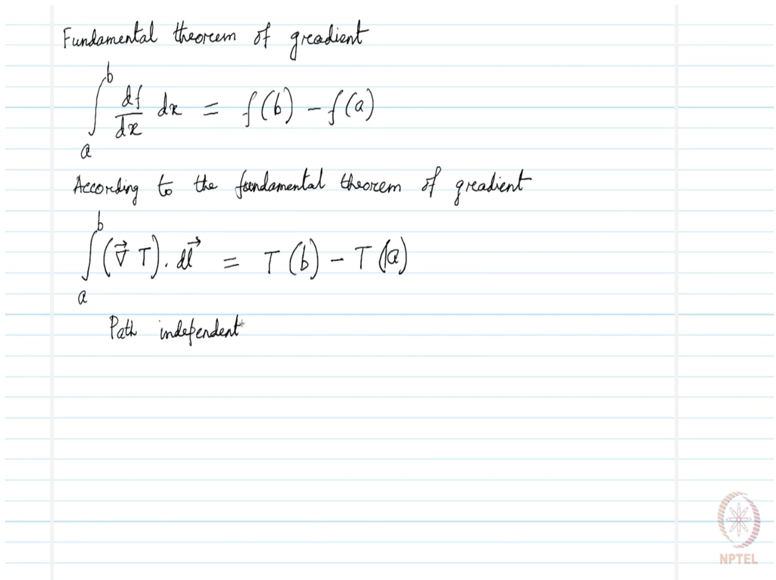


Let us start with the fundamental theorem of gradient. If we integrate over a range from a to b; the differentiated quantity that is the first order derivative df dx with respect x, we know we will get f b minus f a. The fundamental theorem of gradient says according to the fundamental theorem of gradient. Integration over the range a to b gradient of a scalar field T dot dl that is a line integral of it is T b minus T a; which tells us that the line integral of a gradient does not depend on the path, it only depends on the values of T at points, at the end points point a and point b.

It does not depend on which path we take to go from a to b. In other words it is path independent. We will not actually prove this theorem; rather we will see examples of it so, that we can work it out whenever necessary.

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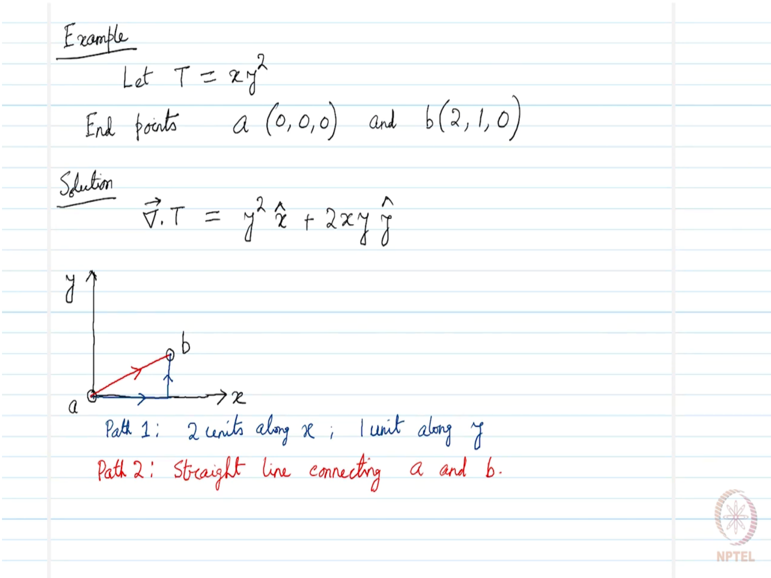


Let us consider an example, let us consider a function scalar field T equals x y squared and we shall check the fundamental theorem of gradient with two end points; a point the coordinate, coordinates are 0, 0, 0 and point b the coordinates are 2, 1, 0 in Cartesian coordinate system. So, we can first calculate the gradient of T.

The gradient of T is y squared, x cap plus 2 x y y cap; with this value for the gradient we are suppose to prove that in this Cartesian coordinate system this is the x axis, this is the y axis. We are given two points point a is the origin and point b has the coordinate 2, 1, 0; that is here because there is nothing in the z direction. I did not really plot anything, I did not plot the I did not show the z axis in this picture.

So, here we will consider two different paths; one path would be moving along the x axis for 2 units and then moving along the y axis for 1 unit this and this, this is path 1, path 1. Here we move along the x axis for 1, 2 units, followed by 1 unit along y axis. And we will consider the second path that connects point a and point b with a straight line like this. So, path 2 would be straight line connecting point a and point b; let us consider path 1 first and what it have for that.

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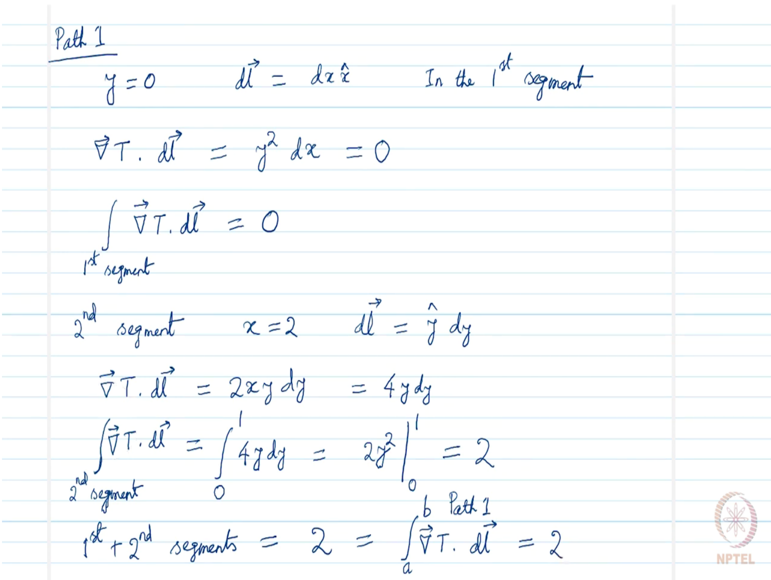

For path 1 for the first segment we have y equals 0; and dl is entirely given by dx x cap in the first segment. We are suppose to write down gradient of T dot dl and this becomes y square dx; because the gradient of T is nothing, but y square x cap plus 2 xy y cap. And here dl is along x cap direction no contribution from y component will come only the x component contribution will come and that is y square dx the value of y is 0, so y square dx is 0.

With this if we integrate over the first segment in path 1; gradient of T dot dl we will find it to be 0. Now, let us consider the second segment. In the second segment we have x equals 2 and dl now we are moving along y direction. So, dl equals y cap dy gradient of T dot dl in this direction would only have the y component 2 x y dy; because there is no x component of dl. So, for after the dot product x component will go away. And since the value of x is 2, we can write this as 4 y dy.

Now, if we integrate it over the second segment, gradient of T dot dl we will get the range of integral is here dy y changes from 0 to 1, the argument is 4 y dy and that gives us 2 y squared over the range 0 to 1 which is nothing but 2. So, from the first segment we got 0, from the second segment we got 2, the sum is 2; that means integral from a to b over path 1 gradient of T dot dl this is 2.

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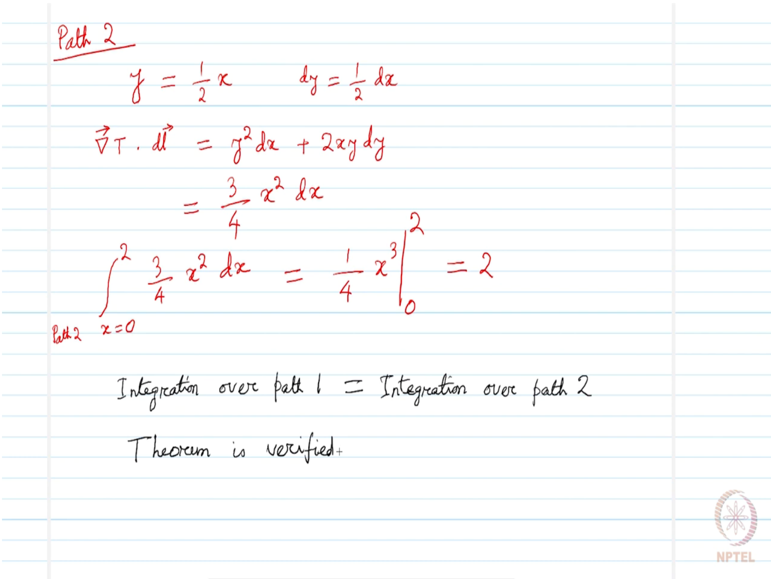


Now, let is consider path 2. What do we have in path 2 is, the value of because we are moving over a straight line and the end points coordinate is x equals to y equals 1; the starting point is 0, 0. So, y equals half of x, then dy equals half of dx that is clear. Gradient of T dot dl that would become y square dx plus 2 x y dy. And if we now convert everything in terms of dx, just by the relationship we have found above it would become 3 over 4 x squared dx.

Now, we are suppose to integrate over x in path 2; x equals 0 to 2 is the range, we will integrate three over four x squared dx. And as a result we will get 1 over 4 x cubed 0 to 2 is the range equals 2. So, we can clearly see that the integration over path 1 equals to the integration over path 2.

So, the theorem the fundamental theorem for gradient is verified.