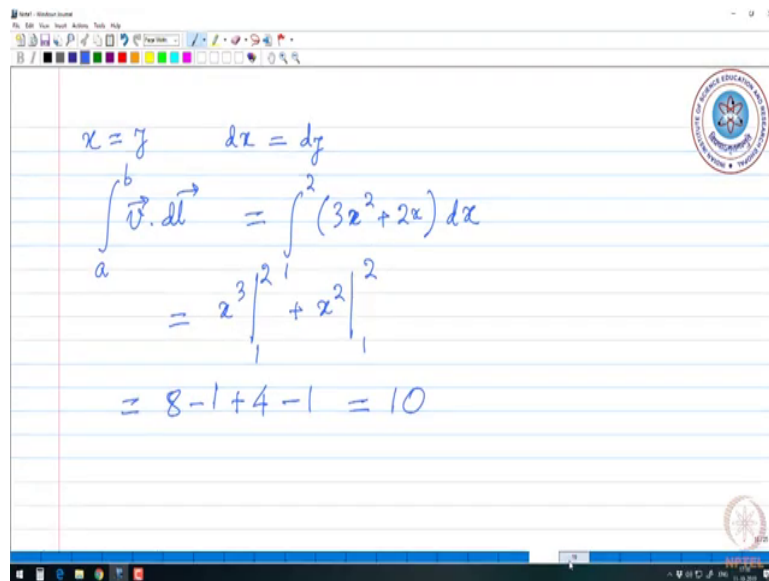


Electromagnetism
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Lecture - 10
Surface integral

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$$\begin{aligned}x &= y & dx &= dy \\ \int_a^b \vec{v} \cdot d\vec{l} &= \int_1^2 (3x^2 + 2x) dx \\ &= x^3 \Big|_1^2 + x^2 \Big|_1^2 \\ &= 8 - 1 + 4 - 1 = 10\end{aligned}$$

Let us learn what is Surface integral now.

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Surface integral

$\int_S \vec{v} \cdot d\vec{a}$ $\oint \vec{v} \cdot d\vec{a}$ closed surface enclosing a volume

Example $\vec{v} = 2xz \hat{x} + (x+2)y \hat{y} + y(z^3-3) \hat{z}$

$(i) d\vec{a} = dy dz \hat{x} \quad x=2$
 $\vec{v} \cdot d\vec{a} = 2xz dy dz = 4z dy dz$
 $\int \vec{v} \cdot d\vec{a} = 4 \int_0^2 dy \int_0^2 z dz = 16$

The diagram shows a 3D coordinate system with x, y, and z axes. A rectangular box is drawn with its bottom surface excluded. The top surface is labeled (v), the front face is (i), the right face is (ii), the back face is (iii), and the left face is (iv). The x-axis is labeled 'x', the y-axis is labeled 'y', and the z-axis is labeled 'z'.

So, surface integral for that we will need a surface and this vector is dotted with an area element a surface element da and we integrate over it and v the vector v must be a vector function, integral over surface, the surface may sometimes include a volume then this symbol for surface integral comes into effect $v \cdot da$ with this symbol it represents a closed surface integral that encloses a volume.

Let us understand what surface integral is by working out an example. So, we are given a vector function v equals $2xz \hat{x} + (x+2)y \hat{y} + yz^3 - 3z \hat{z}$. And there is a cubic box of arm length 2 and we are supposed to perform this surface integral over 5 sides of this box excluding the bottom one.

So, let us draw the picture first. We have this type of a box, the bottom surface is excluded for performing this integral and we will perform it over all the 5 surfaces. Let us name those

surfaces. The front surface is 1, the backward surface is 2, right hand side is 3 left hand side is 4, the top one is 5. And let us perform this integral over different surfaces for surface one the surface element da , area element is given as here what are the. So, this is x this is y and this is z

So, what are the variable on the first surface? We can see that x is a constant y and z they vary and this surface points along the x direction. So, this can be given as $dy dz \hat{x}$. With this; $\vec{v} \cdot d\vec{a}$ is given as $2xz dy dz$ that is $4z dy dz$. And if we perform the integral now integration over $\vec{v} \cdot d\vec{a}$, that will be 4 times integration over 0 to 2 dy because 2 is the arms length integration over 0 to 2 $z dz$ with this. So, we have put the value of x equals 2 here to get this expression ok.

Now, if we perform this integral; we will find that the value is 16 that is on the first surface.

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The image shows handwritten mathematical derivations for surface integrals. It is presented on a blue-lined background with a circular logo in the top right corner. The derivations are as follows:

(ii) $x = 0 \quad d\vec{a} = dy dz (-\hat{x})$
 $\vec{v} \cdot d\vec{a} = -2xy dy dz = 0$
 $\int \vec{v} \cdot d\vec{a} = 0$

(iii) $y = 2 \quad d\vec{a} = dx dz \hat{j}$
 $\vec{v} \cdot d\vec{a} = (x+2) dx dz$
 $\int \vec{v} \cdot d\vec{a} = \int_0^2 (x+2) dx \int_0^2 dz = 12$

Similarly we will perform integrals over all the other surfaces for surface 2. We will have x equals 0 and da would be, in the first surface we have forgot to mention that x equals 2 ok. We mention it here da would be $dy dz$ minus x cap, because this surface is pointing in the negative x direction the opposite direction we considered the outside of the surface to be the direction of the surface.

So, $v \cdot da$, this quantity becomes minus twice $x y dy dz$. And this is 0 because, we have x equals 0. So, integration over $v \cdot da$ in the second segment also clearly goes to 0. Let us consider the third surface now. In the third surface y equals 2 and da is given as $dx dy$, sorry y equals to so it is $dx dz$ y cap $v \cdot da$ can be obtained as x plus 2 $d x dz$.

Therefore the integral $v \cdot da$ on this surface is integration 0 to 2 x plus 2 dx multiplied by integration 0 to 2 dz . And if we work it out in to integrate the x part and multiply that with the integrated value of the z part we will get this quantity equals 12.

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The image shows a digital notepad with handwritten mathematical work. The work is organized into several steps:

- Step (iv): $y = 0$, $d\vec{a} = -dx dz \hat{y}$
 $\vec{v} \cdot d\vec{a} = -(x+2) dx dz$
 $\int \vec{v} \cdot d\vec{a} = -\int_0^2 (x+2) dx \int_0^2 dz = -12$
- Step (v): $z = 2$, $d\vec{a} = dx dy \hat{z}$
 $\vec{v} \cdot d\vec{a} = y(z^2 - 3) dx dy = y dx dy$
 $\int \vec{v} \cdot d\vec{a} = \int_0^2 dx \int_0^2 y dy = 4$
- Final result: $(i) + (ii) + (iii) + (iv) + (v) = 20$

The notepad also features a logo of the institution in the top right corner and a Windows taskbar at the bottom.

Component 4; here we have y equals 0 with that da becomes minus $dx dz$ \hat{y} cap. So, v dot da becomes minus x plus 2 $dx dz$.

So, integration over v dot da on this surface is minus integration over 0 to 2 x plus 2 dx times integration over 0 to 2 dz and this is minus 12 on surface 5; we will have z equals 2. So, da becomes $dx dy \hat{z}$ cap v dot da equals y times z square minus 3 $dx dy$ equals $y dx dy$. Because, z being 2, z square is 4 and z square minus 3 becomes 1. So, its $y dx dy$. Integration over v dot da gives us integration from 0 to 2 dx integration from 0 to 2 $y dy$ that is four. Now, if we find the total flux that is we add contribution from 1 2 3 4 and 5, we get that the total flux is 20.