Electromagnetism

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Lecture - 10
Surface integral

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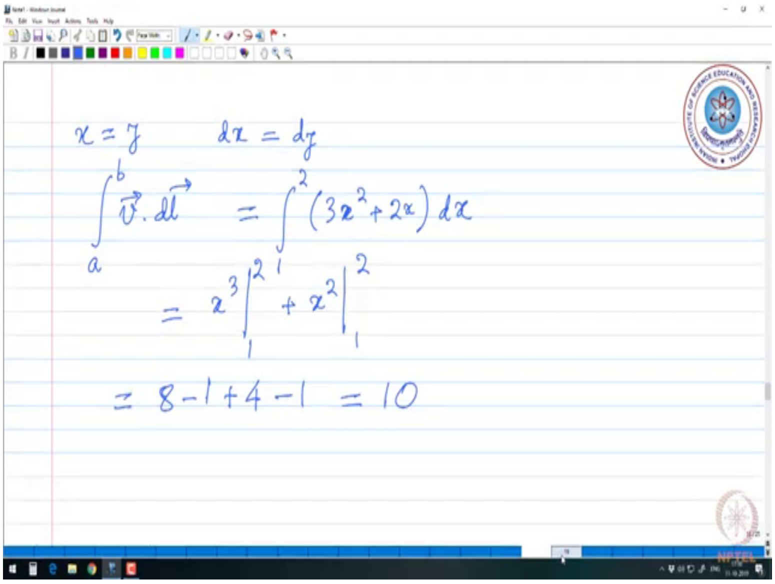

Let us learn what is Surface integral now.

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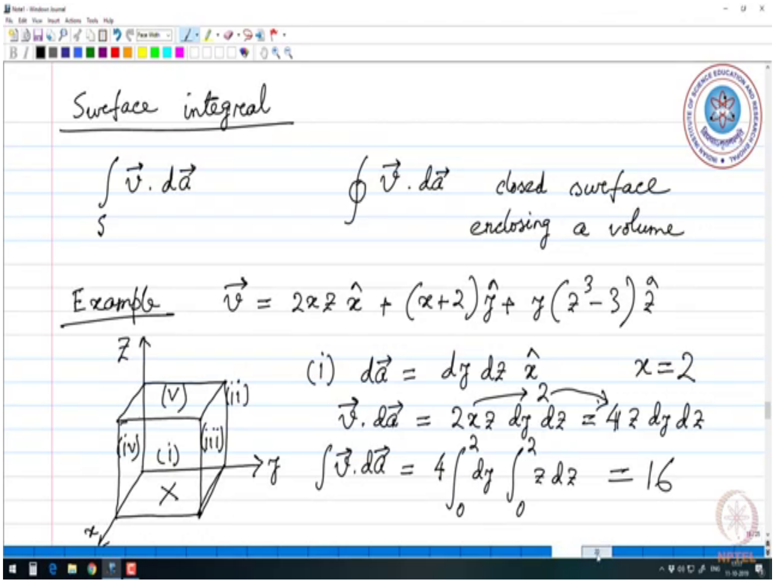

So, surface integral for that we will need a surface and this vector is dotted with an area element a surface element da and we integrate over it and v the vector v must be a vector function, integral over surface, the surface may sometimes include a volume then this symbol for surface integral comes into effect v dot da with this symbol it represents a closed surface integral that encloses a volume.

Let us understand what surface integral is by working out an example. So, we are given a vector function v equals 2 x z x cap plus x plus 2 y cap plus y z cube minus 3 z cap. And there is a cubic box of arm length 2 and we are supposed to perform this surface integral over 5 sides of this box excluding the bottom one.

So, let us draw the picture first. We have this type of a box, the bottom surface is excluded for performing this integral and we will perform it over all the 5 surfaces. Let us name those surfaces. The front surface is 1, the backward surface is 2, right hand side is 3 left hand side is 4, the top one is 5. And let us perform this integral over different surfaces for surface one the surface element da, area element is given as here what are the. So, this is x this is y and this is z

So, what are the variable on the first surface? We can see that x is a constant y and z they vary and this surface points along the x direction. So, this can be given as dy dz x cap. With this; v dot da is given as 2 xz dy dz that is 4 z dy dz. And if we perform the integral now integration over v dot da, that will be 4 times integration over 0 to 2 dy because 2 is the arms length integration over 0 to 2 z dz with this. So, we have put the value of x equals 2 here to get this expression ok.

Now, if we perform this integral; we will find that the value is 16 that is on the first surface.

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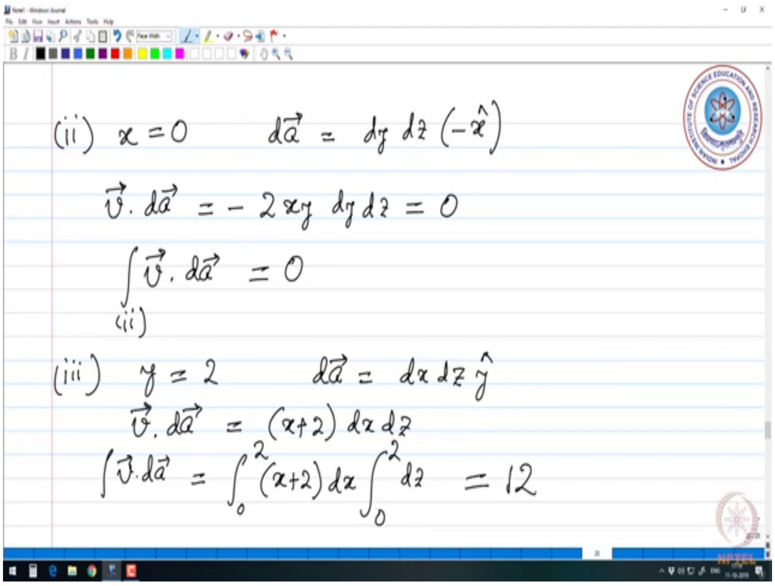

Similarly we will perform integrals over all the other surfaces for surface 2. We will have x equals 0 and da would be, in the first surface we have forgot to mention that x equals 2 ok. We mention it here da would be dy dz minus x cap, because this surface is pointing in the negative x direction the opposite direction we considered the outside of the surface to be the direction of the surface.

So, v dot da, this quantity becomes minus twice x y dy dz. And this is 0 because, we have x equals 0. So, integration over v dot da in the second segment also clearly goes to 0. Let us consider the third surface now. In the third surface y equals 2 and da is given as dx dy, sorry y equals to so it is dx dz y cap v dot da can be obtained as x plus 2 d x d z.

Therefore the integral v dot da on this surface is integration 0 to 2 x plus 2 dx multiplied by integration 0 to 2 dz. And if we work it out in to integrate the x part and multiply that with the integrated value of the z part we will get this quantity equals 12.

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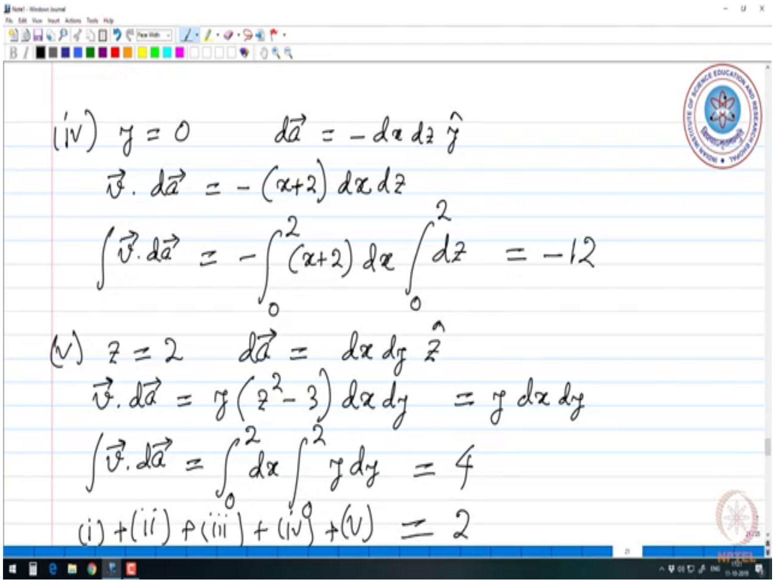

Component 4; here we have y equals 0 with that da becomes minus dx dz y cap. So, v dot da becomes minus x plus 2 dx dz.

So, integration over v dot da on this surface is minus integration over 0 to 2 x plus 2 dx times integration over 0 to 2 dz and this is minus 12 on surface 5; we will have z equals 2. So, da becomes dx dy z cap v dot da equals y times z square minus 3 dx dy equals y dx dy. Because, z being 2, z square is 4 and z square minus 3 becomes 1. So, its y dx dy. Integration over v dot da gives us integration from 0 to 2 d x integration from 0 to 2 y dy that is four. Now, if we find the total flux that is we add contribution from 1 2 3 4 and 5, we get that the total flux is 20.