Physics through Computational Thinking
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Introduction to Non-dimesionalisation

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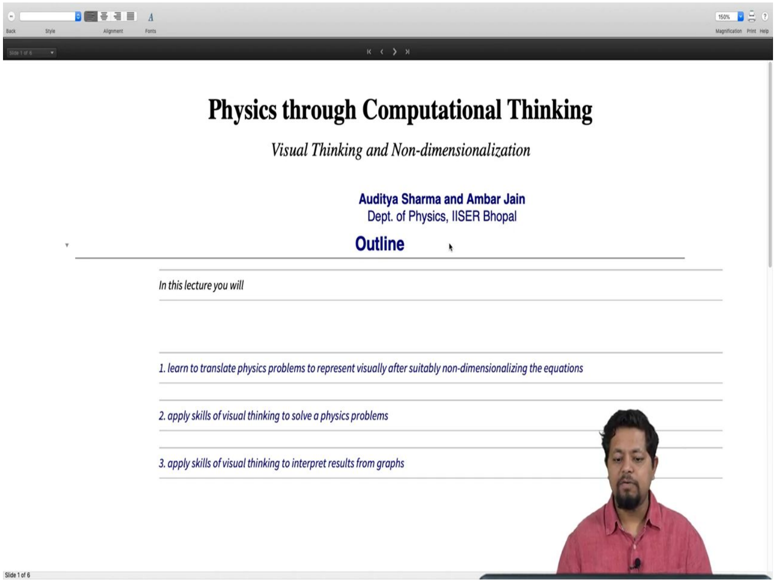

Hey guys, welcome back to Physics through Computational Thinking. In this module, we will talk about Visual Thinking and Non-dimensionalization. We will learn about how to translate a physics problem into a visual problem. We will learn about how to solve the problem visually. And then we will also learn about how to interpret results from the visual data that we have in front of them that will be the objective of today's session.

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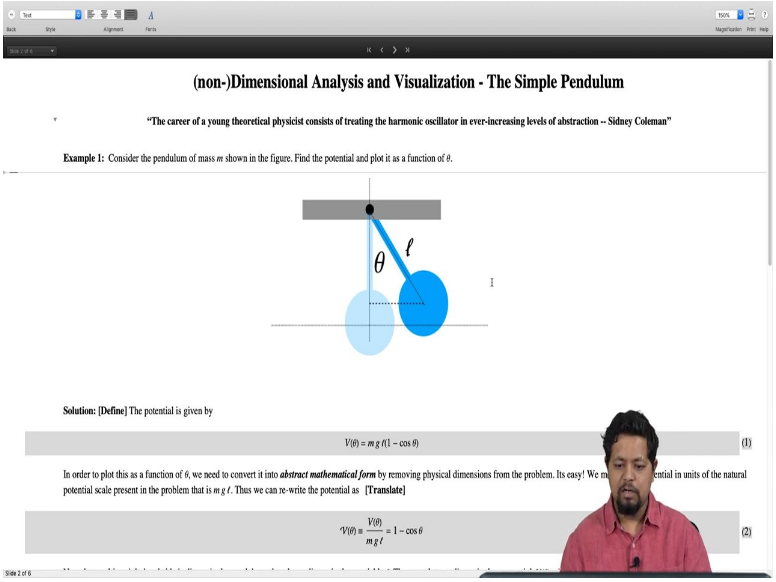

Let us go ahead and look at our first example. In this example, we will consider the simple pendulum, and we will talk about how to non-dimensionalize this and visualize this simple problem of the simple pendulum. Just to quote Sidney Coleman, who said, the career of a young theoretical physicist consists of treating the harmonic oscillator in ever increasing levels of abstraction.

What he means is that in our career of physics as a researcher, or as a student, we keep on coming back to the problem of simple harmonic motion again and again, and keep on solving it again and again in different ways, in different contexts. This is how the problem of a simple harmonic oscillator is so important. And even in this course, we will come back to it many, many times. So, let us look at one of the simplest systems and see how it is connected to simple harmonic motion.

Simple system that we are considering today is the simple pendulum. So, let us consider a simple pendulum over here of length l, and let us say the bob has mass m. And the problem we have to do is, find the potential and plot it as a function of θ. It is a very simple problem, we have to find the potential for this pendulum. And we have to plot this as a function of θ.

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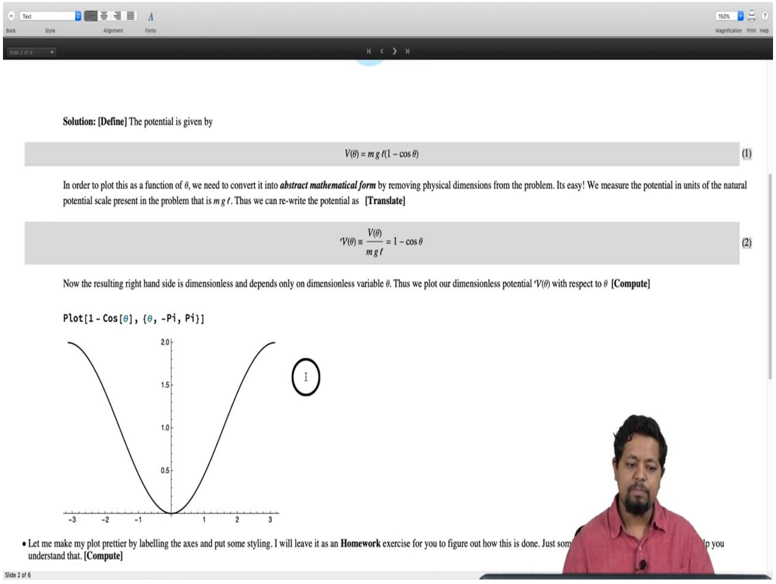

So, we have to plot the potential of this pendulum. What we are going to do is the simple problem and we will break it into four steps: define, translate, compute and interpret. This very simple problem will lay down those four steps for us. So, pay attention on how we go about to do that.

So, the first step is to define. The problem is, we have to plot the potential. So the potential is simply given by $mgl(1-\cos\theta)$. So, first we have to lay down a coordinate system. We are considering this line as a line of 0 potential, the angle between the vertical position of the pendulum and any arbitrary position of the pendulum we are taking that as θ.

Now, if this is θ and this is the 0 potential, we have to find the height by which the bob is lifted up from this line and that height is given by, this is the length l minus the projection of this l onto this which is $l\cos\theta$, so $l - l\cos\theta$, therefore, my potential becomes $mgl(1-\cos\theta)$.

Now that we have got this potential, I want to plot it. But one of the hurdles that I get in plotting is what do I do for mgl? Should I take some values for the mass m, put in the value of $g = 9.8 m/s^2$ and take some length? How should I go about plotting this potential? How should we deal with mgl?

What we are going to do with that is we will non-dimensionalize this problem, that is we will translate this physics problem into a math problem by removing all the dimensions that are coming from physics. We will take them out and turn in some simple mathematical equation or mathematical problem. We will analyze that and we will plot that.

So, this is the second step of translate. We want to bring this physical potential into an abstract mathematical form, to do that we will non-dimensionalize. So, in this problem, the angle theta is already dimensionless. It is measured in radians, which is a dimensionless unit. The potential however has units of energy, which is measured in units of mgl, and that is why the potential is $mgl(1-\cos\theta)$.

So, in order to non-dimensionalize, we will define a new potential, $V(\theta)$, which is equal to $V(\theta)/mgl$. And we will, when we divide by mgl we get, $\mathcal{V}(\theta)=1-\cos\theta$. This is now a dimensionless potential, and we can actually go ahead and plot it.

So, let us go ahead and just plot it. You have done this before. And that brings up to this step of computation or visualization, so this is the third third stage of the problem, this is where we compute. So, my computation here is I am going to make a plot using the plot function. You have seen this before, you have done this before. And the plot is just the plot of $1-\cos\theta$ in the limits of $-\pi$ to π.

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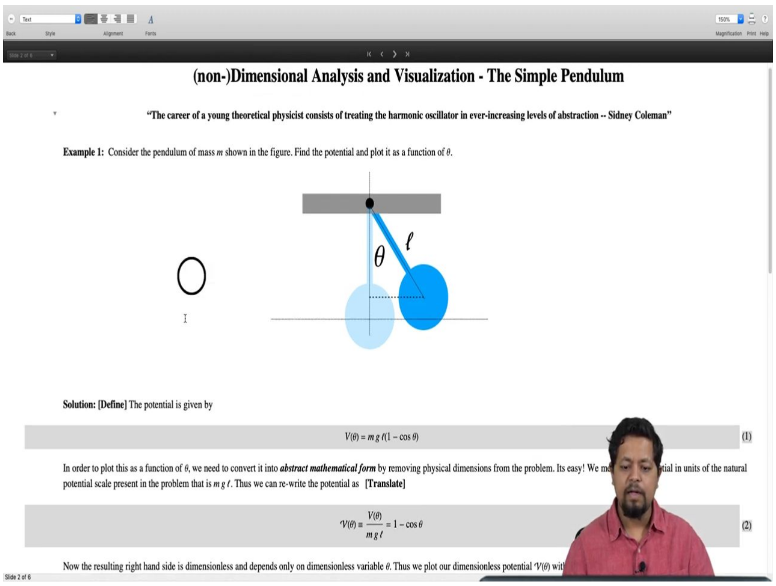

Let us go back to this problem here. $\theta=0$ means vertical position, $\theta=-\pi/2$ means horizontal position with the bob being over here, $\theta=\pi/2$ means bob being over here,

$\theta = \pi$ means bob being vertically upwards. So, we are plotting all the way from $\theta = -\pi$ to $+\pi$.

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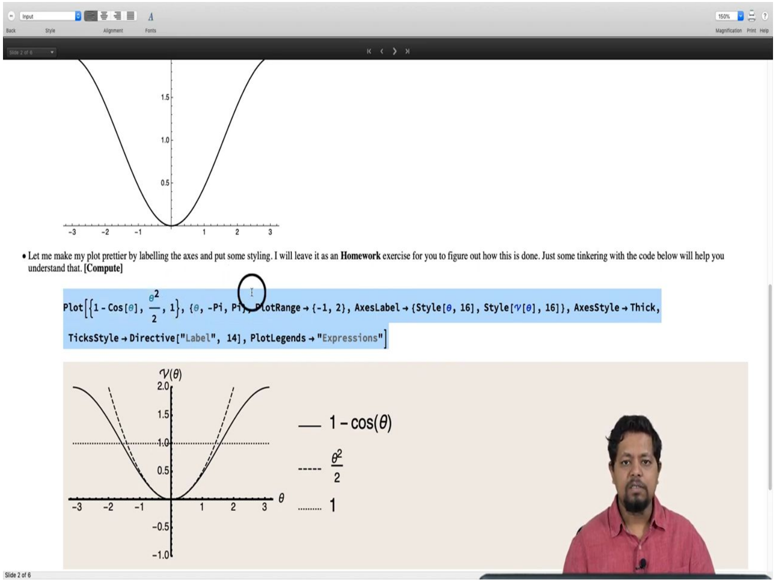

So, here is my plot. Now, let us go ahead and make this plot a little bit more prettier by adding various labels, etc. And to do that, I will execute this command over here. When I do that, I get the plot below. Now, leave it as a homework exercise to figure out what are all these various options I have added, little bit of tinkering with this one, one line of code, you will be able to understand some of the options that I have used here that you have not seen before.

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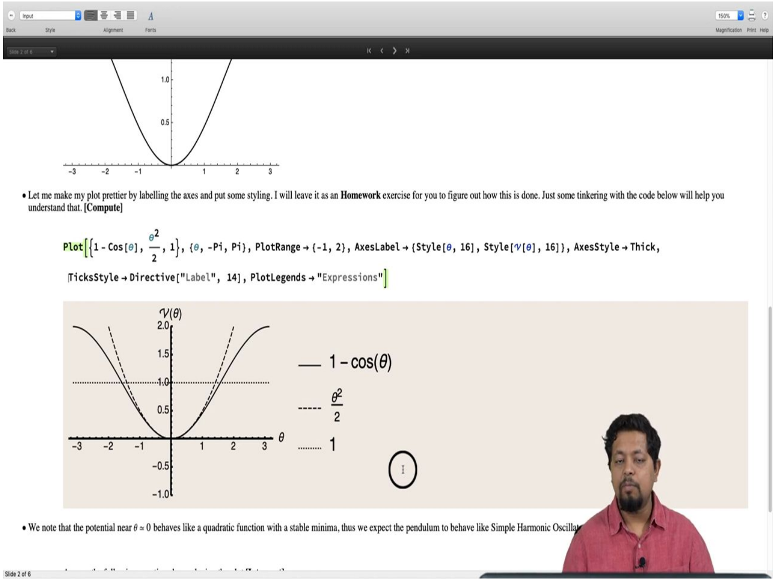

So, when I make use of, when I plot it this way, my picture gets labeled, it starts looking prettier. And now I can use it to understand and analyze my results. So, here is my solid line, solid line corresponds to $1-\cos\theta$, I am going to approximate that with a quadratic potential, the quadratic potential is ½ θ^2.

So, if you approximate cosine theta as $1-\theta^2/2$ for small angles, $1-\cos\theta$ becomes ½ θ^2 and that is my dashed line. So, the dashed line is the quadratic potential. That corresponds to ½ θ^2. And you see that it matches very well for this small angle, which is where this expansion is supposed to hold. And I have also plotted a dotted line in this picture, which is the line corresponding to $\mathcal{V}(\theta)=1$, this is just for reference.

Now, we do notice here that the potential very well agrees with the quadratic potential over here and that is why for simple pendulum we often make an assumption that this angles are small and we treat a simple pendulum as a simple harmonic oscillator. Simple pendulum is not really a simple harmonic oscillator because potential is not ½ θ^2 but $\cos\theta$, for small angles it behaves like a simple harmonic oscillator.

And which is what we use when we allow the pendulum to oscillate, measure its time period and use the time period to obtain the gravitation constant by using time period, or $\omega=\sqrt{g/l}$.

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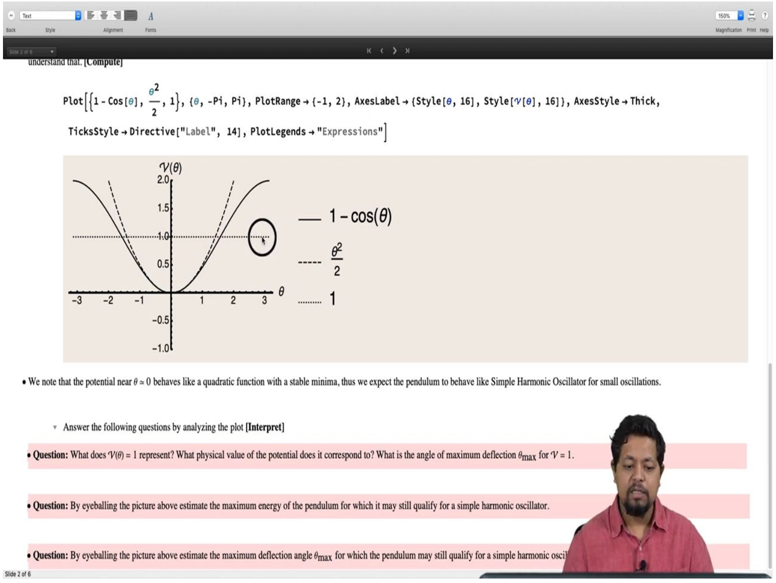

Now, let us come to the interpret stage and for the interpret stage I have got three questions for you, we will look at these three questions and we will analyze or we will find the answer of these three questions using this visual data or this visual representation that we have created.

So, the fourth stage of interpret, my first question to use, what does $\mathcal{V}(\theta) = 1$ represent? What physical value of the potential does it correspond to? And what is the angle of maximum deflection $\theta_{\max}$ for $\mathcal{V}(\theta) = 1$. So, first question is, what is $\mathcal{V}(\theta) = 1$? What is its physical value and what value of potential does this correspond to?

Second question is, by eyeballing the picture above, estimate the maximum energy of the pendulum for which it may still qualify for simple harmonic oscillator? So, we do agree that for small angle this is a simple harmonic oscillator, it behaves like a simple harmonic oscillator.

But about what angles does it work? How far it can go? Is it $\theta = 30^\circ, 60^\circ, 90^\circ$, for how big a theta should be that I can believe the the behavior of the simple pendulum is going to be like a simple harmonic oscillator? And I want you to do this just by eyeballing the picture. So, this is where we want to look at the visual data, visual representation and interpret how well a system is approximated by the approximation we are making here, which is the small angle approximation.

Third question is, by eyeballing the picture above, estimate the maximum deflection angle $\theta_{\max}$, for which the pendulum may still qualify for simple harmonic oscillator? I am sorry, the second question was estimate the maximum energy of the pendulum for which it may still qualify for simple harmonic oscillator? And the third question is, find the maximum angle theta max for which the pendulum may still qualify for simple harmonic oscillator? So, if you want to pause the video at this moment, you can go ahead and pause the video. Think about these questions. And when you play back, I will discuss the answers with you.

Alright, let us go to the answer of the first question, what does $\mathcal{V}(\theta) = 1$ represent? $\mathcal{V}(\theta) = 1$, is this horizontal dotted line, it corresponds to energy of the system or energy of the simple pendulum. As the pendulum oscillates, its angle θ changes as the angle θ changes, its potential energy changes according to this solid curve.

And its kinetic energy changes according to the difference between this dotted line and the solid curve, which means, the difference between the dotted lines and solid curves, that is the height above the solid curve up to this dotted line. This is the measure of the kinetic energy that differences the measure of the kinetic energy.

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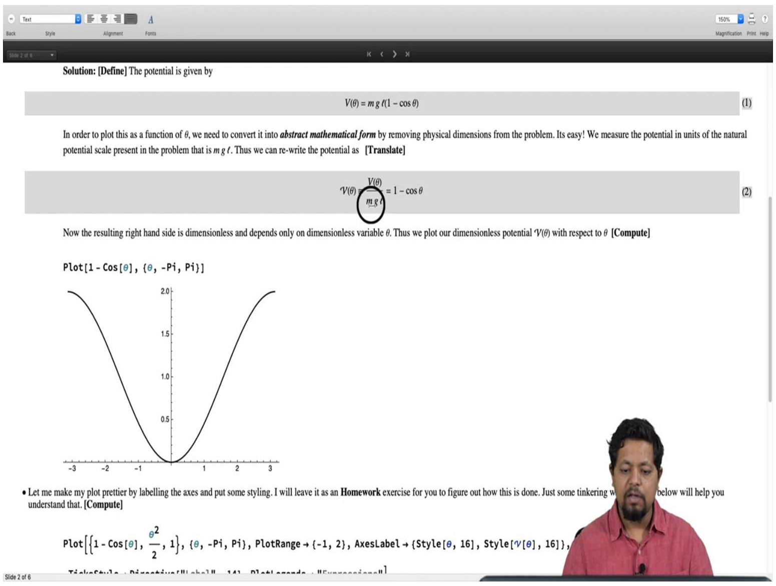

So, this represents the total energy of my system. And how much energy is that, that corresponds to because $\mathcal{V}(\theta) = 1$ and V is measured in units of mgl, this is how we non-dimensionalized V, $\mathcal{V}(\theta) = 1$ corresponds to $V(\theta) = mgl$ or energy equal mgl.

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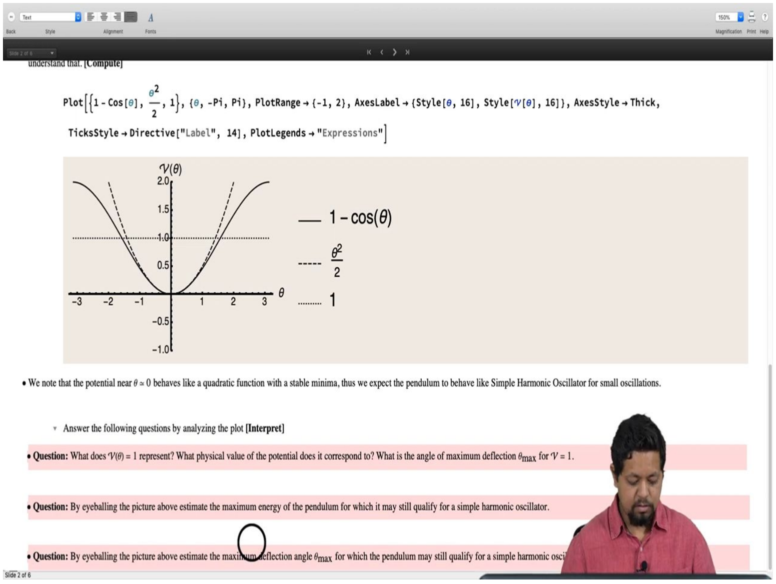

So, this dotted line corresponds to my system having an energy of mgl that means kinetic energy plus potential energy is equal to mgl, the potential energy is represented by the solid curve, the difference between the dotted line and the solid curve is the kinetic energy. So, when I start from here my kinetic energy slowly increases and then decreases.

At this point, my kinetic energy becomes maximum, potential becomes minimum, and again the kinetic energy starts to decrease and at this point it becomes 0 and this is the turning point for the pendulum the pendulum turns, comes back and starts going in that direction. So, the pendulum oscillates between these two values of the angle theta from.

So, this is my $\theta_{\max}$ for $\mathcal{V}(\theta) = 1$, it oscillates between these two points as it oscillates between these two points, energy increases, reaches a maximum, energy decreases, pendulum comes to a stop, it turns changes its direction, energy increases back again and so on and so forth.

So, what is this $\theta_{\max}$ for $\mathcal{V}(\theta) = 1$, well this is an intersection point. So you can go ahead and solve for that, you have to substitute $1 - \cos\theta = 1$, which leaves $\cos\theta = 0$, and that gives you $\theta = \pi/2$. That is for the first question. Now, the second question, I want you to eyeball the picture and estimate the maximum energy of the pendulum for which it can still qualify for a simple harmonic oscillator.

Clearly the dotted line over here, which corresponds to $\mathcal{V}(\theta) = 1$ or energy equal to mgl. You see the dashed potential and the solid potential do not agree with each other. That means this is where the approximation is definitely failing. If it has to be considered as a simple harmonic oscillator, I should come down over here, down to this point. Around this region, my system can qualify as up to this region, my system can qualify as a simple harmonic oscillator.

So, the value over here maybe, to be on the safer side, somewhere around here, so maybe at $\theta = 0.8$. So, by eyeballing I can at least say that at $\theta = 0.8$. And the corresponding energy equal to 0.4. My system can qualify as a simple harmonic oscillator. This is just by eyeballing. So, these numbers are approximate. But let us go ahead and check out if this approximation is valid for small angles?

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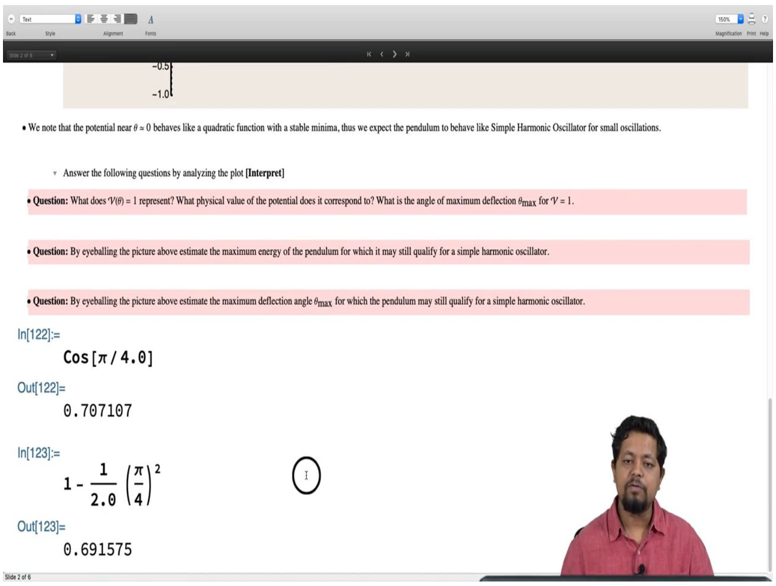

So, to do that, what I will do is, I will evaluate cosine of let us estimate what is 0.8 corresponds to. So, if $\pi/2$ is about 3.14 by 2 is 1.5. This is what ½ of that so this is pi by 4. So, our estimate is that at theta equal to $\pi/4$ I can still treat the system as a simple harmonic oscillator.

So, let us, let me go ahead and put $\pi/4$ as a $\cos(\pi/4)$ and we will evaluate its value. It is 0.707. And our estimation is, for this $\cos\theta$ is $1-\theta^2/2$. So, I will go ahead and check what is $1-\theta^2/2$, so in this case, θ is $\pi/4$ and I want to square it up, multiply by half and evaluate it, that is 0.69, it is very close to 0.707.

So, our estimation from eyeballing was correct that for $\theta = \pi/4$ the system will still can be approximated as a simple harmonic oscillator. And that is the reason why when you do an experiment with a simple pendulum even with wide angles, angles as much as 30°, 40°. You can get acceleration due to gravity very accurately.

If you have never done this experiment before, go ahead and take a pendulum. Make a pendulum out of some objects that you can find in your office or home, oscillate the pendulum, count the time period for 10 oscillations, divide by 10. Do this, repeat this many times, take an average of many of these values. That is your one time period, using the time period, calculate what is the acceleration due to gravity and you will find that you will get a pretty accurate number very close to 9.8.

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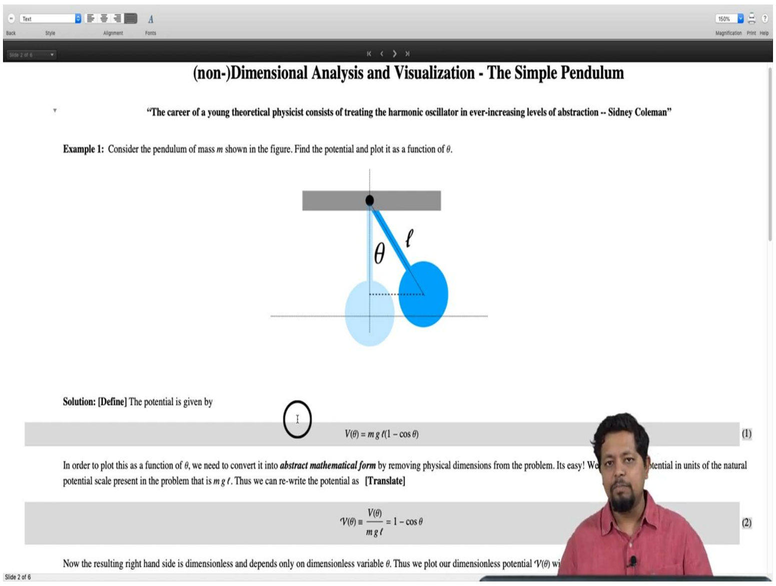

Alright, so were it, let me quickly recap what we did in this example. In this example, we understood, how to non-dimensionalize a physics problem bring into an abstract mathematical form which we can analyze on a computer or solve or plot on a computer.

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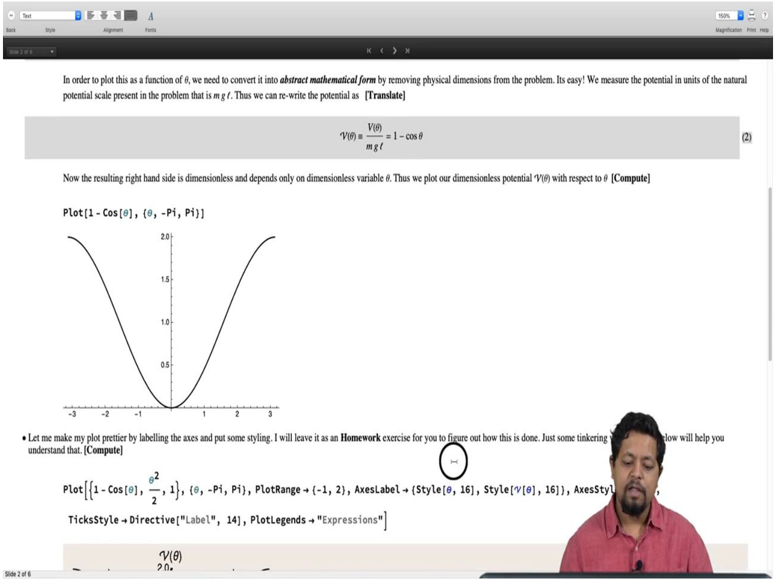

So, first we defined our problem, then we translated it into a mathematical abstract form. Then we went to the computation stage where we plotted it in our computer for visualization.

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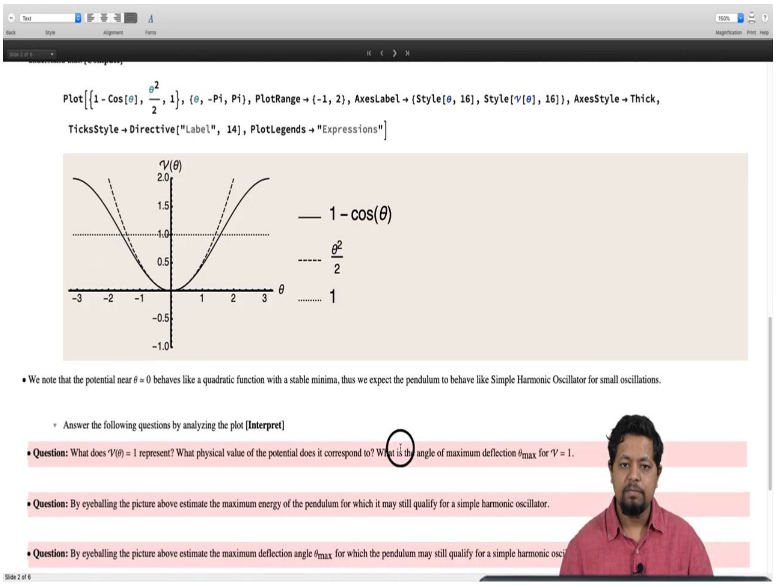

We made our plot look prettier, added a few more lines and then we use that to do an interpretation.