Physics through Computational Thinking
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Lecture 33
Escape Velocity

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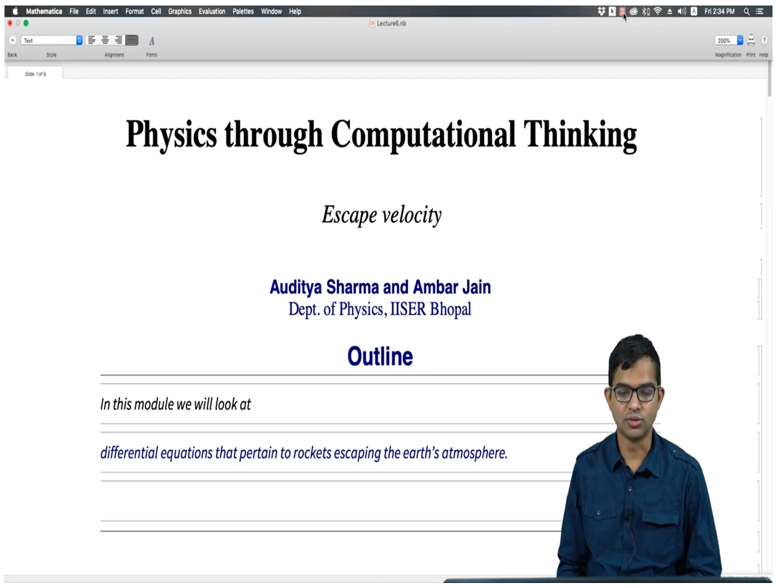

Okay hello, so in this module we are going to look at some more variants of differential equations we start simple as always and you will see that we will invoke tools which we have already learnt. So it is now an exercise in repeated application of familiar ideas and you know bringing in some small variations.

And then you will see that actually doing a number of simple tasks and apparently obvious things when you keep on putting them together certainly you will start seeing that it can open up a window to explore problems which are even sort of research level problems can open up based on the systematic approach which is the whole emphasis of this course.

Okay, so today what I want to do is talk about, you know, the notion of Escape Velocity first of all. And then build around this theme and look at a few differential equations which arise from there and then to numeric around this.

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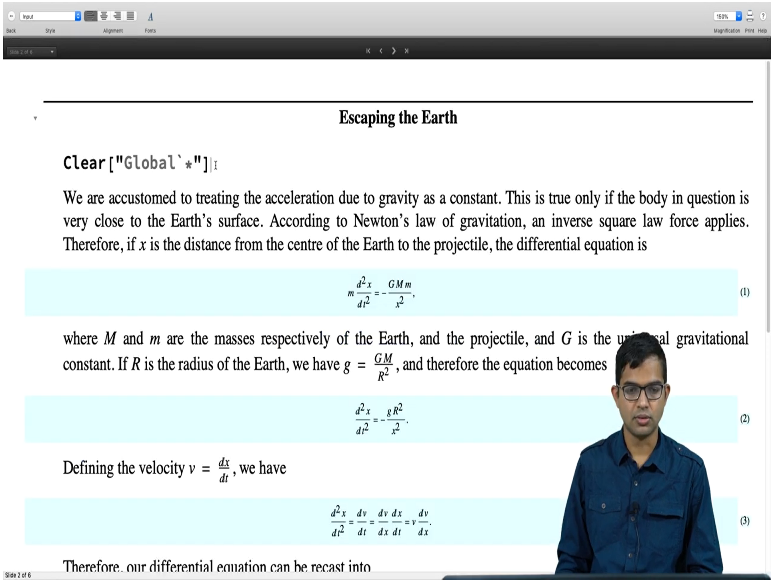

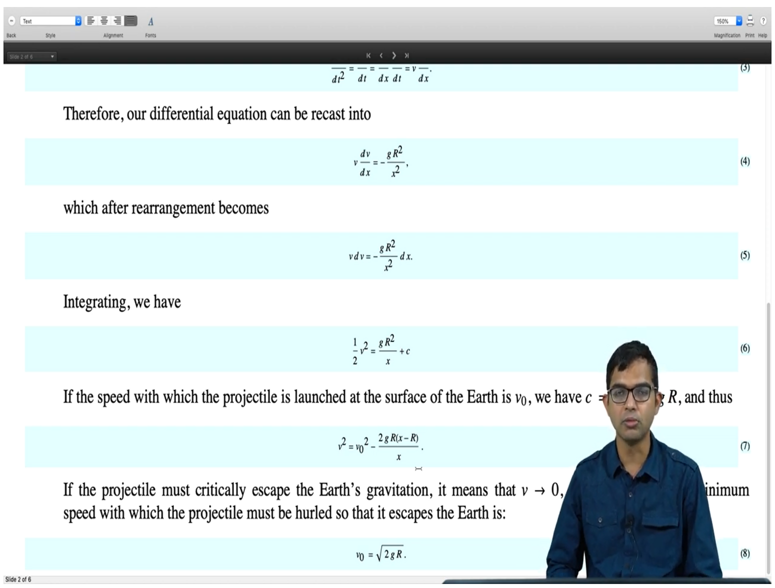

So it is useful to start a session with clear global. So it says 53 because I have already opened up before and then so many executions have already happened. If you have started a fresh Mathematica session then it will show number 1, does not matter. So I clear everything now and Mathematica remembers nothing at this point.

And so now we are going to look at Newtonian gravitation. So close to the earth we are familiar with the acceleration due to gravity to be a constant. So the differential equation we have is just simply $md^2x/dt^2 = -mg$.

So x points away from the earth, so but in fact this gravitation, the acceleration is not really meant to be a constant it is just that we are making an approximation already and treating in all distances close to the earth as if they are same with respect to the center of the earth. So the correct differential equation in fact even if you are sticking purely to the Newton's law of gravitation is actually $m d^2x/dt^2 = -GMm/x^2$ where this x is measured from the center of the earth.

And if this x is going to vary substantially with respect to capital R, which is the radius of the earth then you must use this full differential equation to solve this problem, right. So capital M and small m are the masses of the earth and the projectile and so in the very first step in fact these small m's will cancel.

So it does not matter what is the mass of the projectile itself for its motion and then we will recall that in fact small g is nothing but capital GM divided by radius square. So use this and then recast this differential equation in this form. Equation 2 here is $d^2x/dt^2 = -gR^2/x^2$, right.

So it turns out that this differential equation can actually be solved exactly and invoking you know this chain rule after defining this velocity to be just dx/dt we have $d^2x/dt^2 = dv/dt$ which. So here is the clever use of the chain rule so in fact you do not want to write this as an differential equation involving t.

So you find a way to eliminate this t because the right hand side is purely a function of x. So you write it as a differential equation in v and x we think of v as dependent on x. So the speed and now it becomes a first order differential equation, so if you plug in $v\, dv/dx$ you have $v\, dv/dx = -gR^2/x^2$.

And which after rearrangement becomes $v\, dv = -gR^2/x^2\, dx$. So this is a standard trick and quite a cleaver trick and it applies because there is no other function of t sitting in this differential equation. So now it is straight forward to go ahead and solve, so you just integrate both sides so integral of v is just v square by 2.

And on the right hand side you get gR^2/x^2. So $1/x$ is the integral of $-1/x^2$ and then there is of course a constant that you must deal with it. It is a free constant which comes from integrating out of first order differential equation.

Now if the speed with which the rocket, think of this projectile as a rocket which is being hurled from the surface of the earth and you can ask the question how fast must the rocket be hurled. So that it actually manage it to escape the earth's gravitation. So that it turns out it is already available this information comes out from just solving this differential equation at this level. You just need an expression for v in terms of x and then this argument follows.

So first of all we have this constant c can be written in terms of the initial speed v with which your rocket is hurled. And so you have $v^2 = v_0^2 - \dfrac{2gR(x-R)}{x}$. So it is not a surprise that the dependency is on $x - R$ it is only the distance with respect to the earth surface that counts in this equation.

And then there is also $1/x$ which you know is also in there, now the projectile must critically escape when do you say that it has completely manage to escape the earth's gravitation? It is when it is able to go all the way to infinity, if its velocity is going to be 0 at a finite value of x.

So then, it is still under the pull of the gravitation of earth and then it can come back. If it somehow manages to reach infinity but with a finite speed then it is not critical it is you have held it over a higher speed than the critical speed but if it has been hurled with exactly such a speed that it manages to reach infinity but only barely so.

Then you say that it is a critical velocity and what is that given by you just say v must go to 0 as x tends to infinity. And so the minimum speed with which the projectile must be hurled so that it escapes the earth is $v_0 = \sqrt{2gR}$, right. So this is probably a result which is familiar to you.

But look at the you know clever line of reasoning which goes into this and we will see that some variants of this problem which are not amenable to analytical solution can also be worked out. So before we do that let us recast this whole problem into a form that can be put on a computer.

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Nondimensionalization

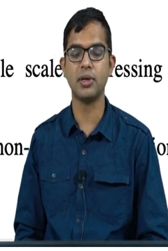
Let us rework this problem in a manner that can be tested numerically. We start by rewriting the differential equation as:

$$\frac{d^2x}{dt^2} = -\frac{gR^2}{x^2}. \tag{9}$$

Exercise

(a) Non-dimensionalize the equation by choosing suitable scale … essing the equation in dimensionless quantities.

(b) How many free parameters are left in the equation after non- … on?



Solution

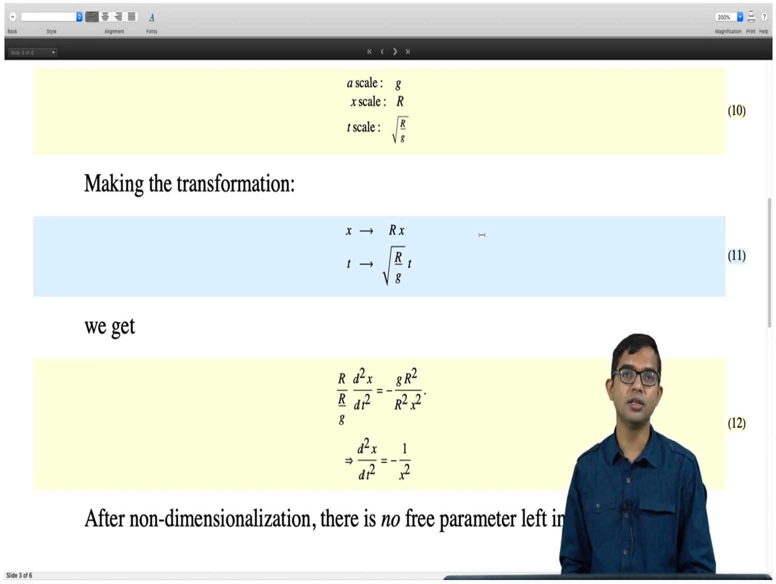
$$\begin{aligned} a\text{ scale}: &\quad g \\ x\text{ scale}: &\quad R \\ t\text{ scale}: &\quad \sqrt{\frac{R}{g}} \end{aligned} \tag{10}$$

Making the transformation:

$$\begin{aligned} x &\longrightarrow R\,x \\ t &\longrightarrow \sqrt{\frac{R}{g}}\,t \end{aligned} \tag{11}$$

we get

$$\frac{R}{\frac{R}{g}}\frac{d^2x}{dt^2} = -\frac{gR^2}{R^2x^2}. \tag{12}$$



So let us look at non-dimensionalization, so this is also a recurrent theme so what are we trying to do here? We have, so we start with this differential equation I have written it directly in the form $-gR^2/x^2$ over right hand side.

So if you want to pause the video now for a second for a few minutes if you want and work out the non-dimensionalization of this equation using a method very similar to what we have done in the past I urge you to do so. So it is a useful exercise non-dimensionalize find out how many free parameters are left after non-dimensionalization.

Okay, so my solution is the following I first of all extract the acceleration scale it is just g and the length scale in this problem is clearly the radius of the earth from which I can get the time scale. As a $\sqrt{R/g}$. Once I have this wherever I have x I must replace it by the length scale times x wherever I have t in my differential equation I must replace it by the time scale times t, right.

These are only 2 quantities which appear in the differential equation. So when I do this I have you know a string of cancellations happen and finally I am left with this very simple differential equation. Now these x's and t's are actually dimensionless quantities right and so this is a purely mathematical equation which can be put on a computer which can be integrated out which can be numerically solved and that is what we want to see.

And ofcourse we make the observation first of all that there is no free parameter left in the problem what does that mean? It just means that the qualitative nature of the solutions or all the physics in this actually does not really depend on. You know what precise values there were for the radius of the earth or constant gravitational acceleration close to the earth none of these things will change the nature of the solutions or nature of the physics is intact only some numbers will change.

It is just going to scale things in a different way but there is no substantial difference that will emerge does not matter whatever values you can pick for *R*'s, *R* and *g* and *m* and everything in the problem is not that crucial.

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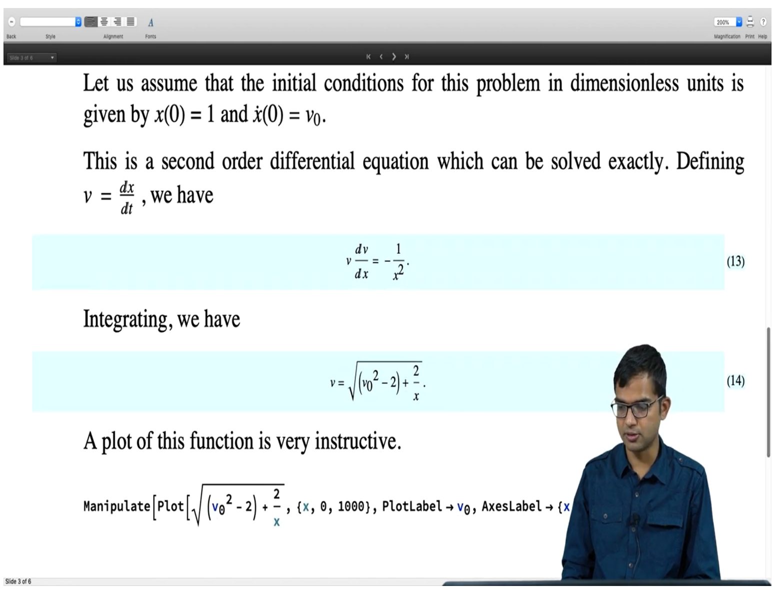

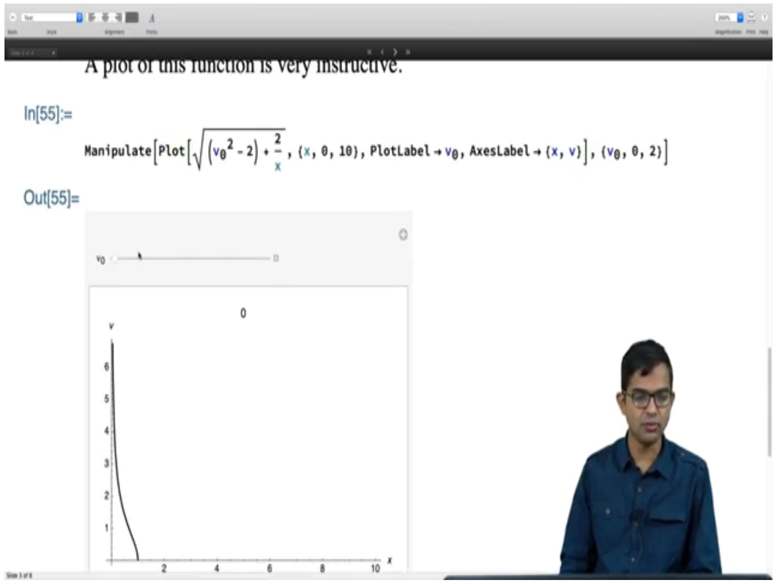

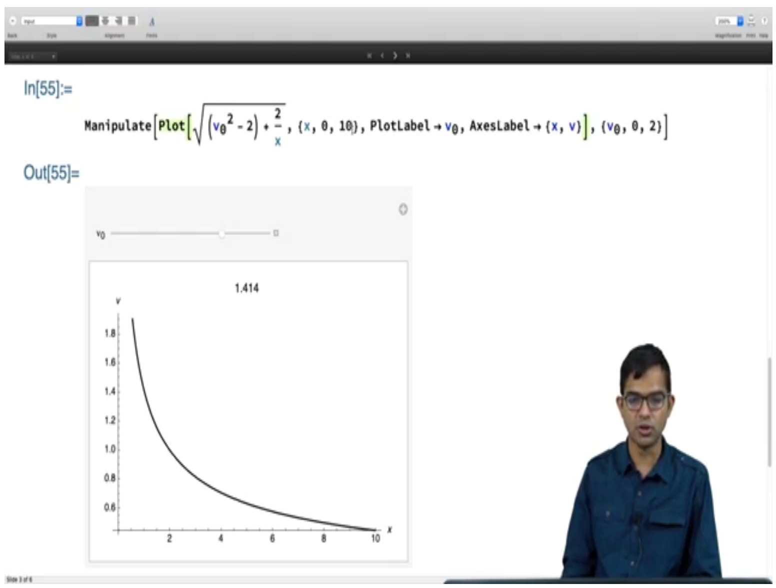

Okay so initial conditions it is convenient to assume that $x(0) = 1$ in these units. So it is a dimensionless units so $x(0) = 1$ you must think of it as meaning that you start at the surface of the earth $x = R$ because the length scale is R and then $\dot{x}(0) = v_0$. I can choose v_0 I mean it is going to be in units of in x/t.

But I can choose a number v_0 it is an open thing right now I can try it out you know go through do a numerical experiment and vary v_0 and see what the solution looks like and so on. So this is a second order differential equation which can be solved exactly I have already shown you the solution.

So let us quickly go over what we did now but in non-dimensional form, we have a non-dimensionalized differential equation. So $v\, dv/dx$ against same trick equal to $-1/x^2$ integrating $v = \sqrt{(v_0^2 - 2) + 2/x}$. It is just the same thing we did in the previous slide.

So plot of this function is very instructive so let us just plot this first as always when you have when beginning to get a feeling for a problem you must plot as many quantities out for yourself and try to extract as much information about any problem without any hard work. Before you put in a lot of hard work do lot of checks and try to develop a feeling for the problem.

And only then get into the labor you know and then finally cross check, cross check at every stage. So you see that so many be x equal to going all the way to till 1000 is too much if I make 10 let us see what happens so there you see. When v_0 is very small v is going to go up, it will keep on reducing as a function of x.

And then there is a finite value of x at which v goes to 0. But if I keep on increasing v_0 I see that the point at which it goes to 0 also keeps increasing. But it exists but then if I keep doing this keep doing this and increase v naught there comes this threshold there beyond which it does not matter.

So no matter what value of x you choose your v is going to be finite and that value we already know what is the critical value. So let me just tune it close to that and there you go.

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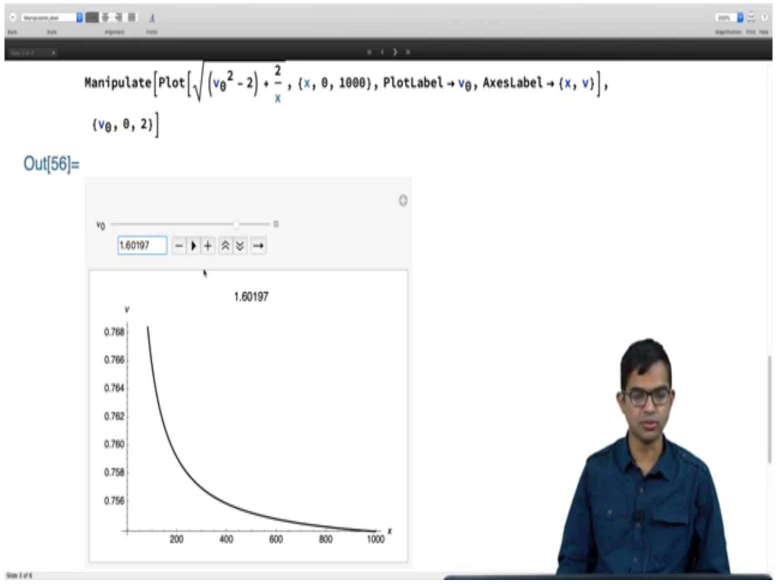

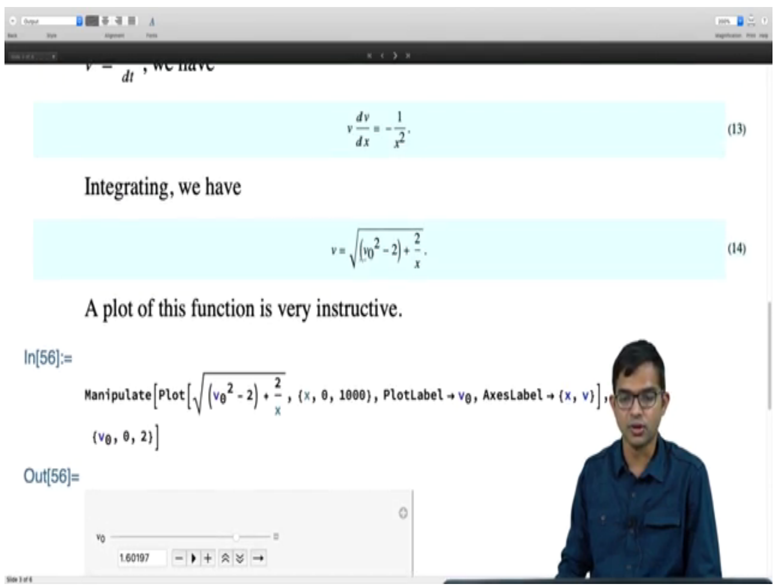

So now if I plot this all the way to 100 or 1000 it does not matter. So I have to tune this v_0 to be 1.414 if I do this then I have. So you can also tune it to keep on changing and then I will suddenly pause this and then you see there you go. So it is it does not go to 0 it will go to 0 only for $x = \infty$ that is something that we have already analytically worked out and it is a cross check against our intuition.

So this is a general approach to physics and to you know when you particularly useful when you are trying to attack problems which are already do not have solutions. You need to take solid decisive steps forward but they can be very tiny steps. And so keep on making sure that you know you understand one step ahead and then come up with an alternate method make sure that it is correct and then keep on building and then suddenly you have problem for which does not have a an understood solution and then you have made progress that is the way physics goes.

So there is by the way there is no closed form solution for $x(t)$ it is quite a messy differential equation even this one for. If you want to get x as a function of t.

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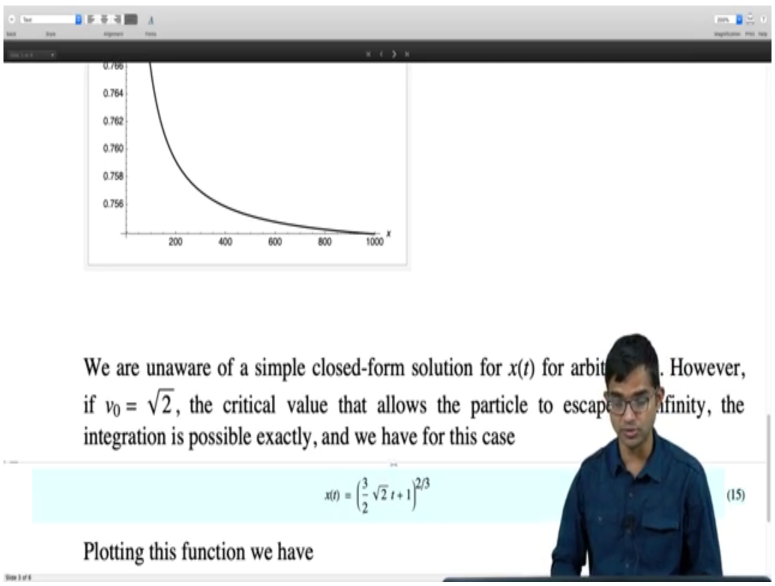

But if you choose v_0 to be exactly, so that is the case for arbitrary v_0. But if you choose v_0 to be if tune it to be exactly at the critical value. Then you can go ahead and solve this differential equation so then what happens is if you choose $v_0^2 = 2$ then v become just $\sqrt{2}/\sqrt{x}$.

And then you can go ahead and check this, right. So this is homework go back and check that the solution you will get is $x(t)$ is equal to this expression that I have here $3/2(\sqrt{2}t + 1)^{2/3}$. And then it is also useful to plot this function.

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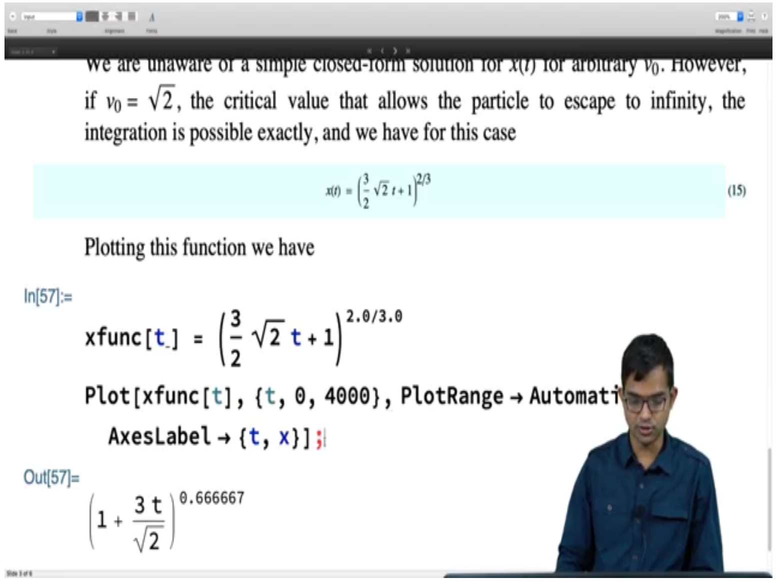

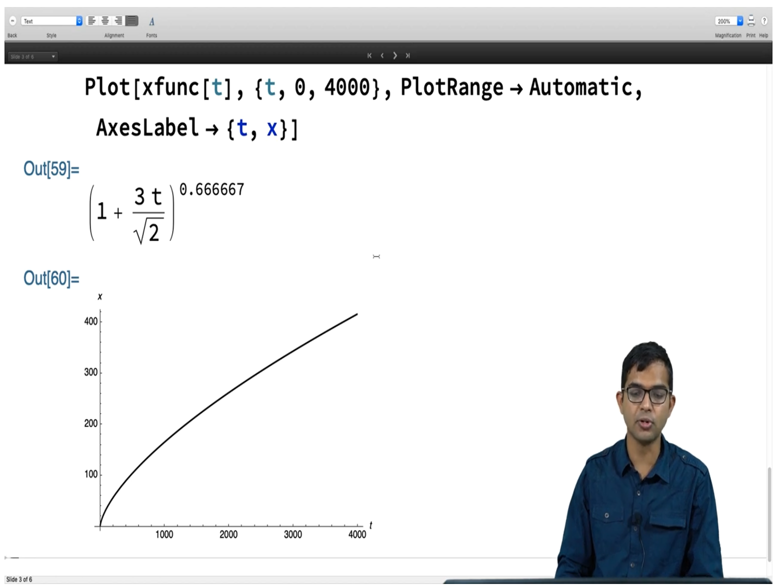

So I define this function and then I plot this, so bear in mind that this is at the critical velocity. So it is not a surprise that x keeps on increasing it will keep on it will go very slowly may be but it will go to infinity eventually. Okay so this is all you know analytical solution and visualization.

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Numerical Solution with the RK4 Method

- Lets recall how we can bring a higher order differential equation into the canonical form:

$$\dot{x} = f(t, x, y, z)$$
$$\dot{y} = g(t, x, y, z)$$
$$\dot{z} = h(t, x, y, z) \quad (16)$$

- Next we define the column vectors X and F as

$$X = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \qquad F = \begin{pmatrix} 1 \\ f \\ g \\ h \end{pmatrix} \quad (17)$$

- Then the coupled ODEs can be written as

$$\dot{X} = F \quad (18)$$

- The RK4 method is given by

$$R_1 = F(X_n)$$
$$R_2 = F\left(X_n + \frac{h}{2} R_1\right)$$

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$$\dot{X} = F \quad (18)$$

- The RK4 method is given by

$$R_1 = F(X_n)$$
$$R_2 = F\left(X_n + \frac{h}{2} R_1\right)$$
$$R_3 = F\left(X_n + \frac{h}{2} R_2\right)$$
$$R_4 = F(X_n + h R_3) \quad (19)$$

$$X_{n+1} = X_n + h \frac{R_1 + 2 R_2 + 2 R_3 + R_4}{6} \quad (20)$$

- Here we have copied its implementation.

```
rk4[F_, X0_, tf_, nMax_] :=
 Module[{h, datalist, prev, rate1, rate2, rat
   rate4, next},
```

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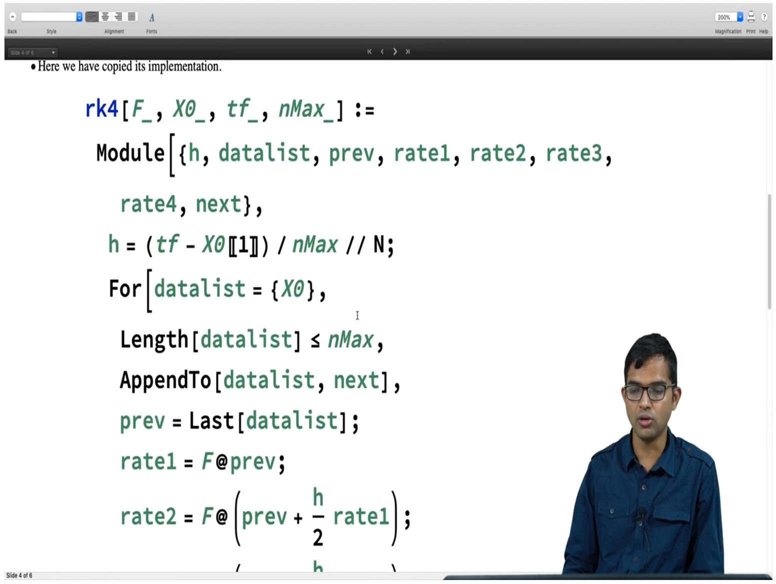

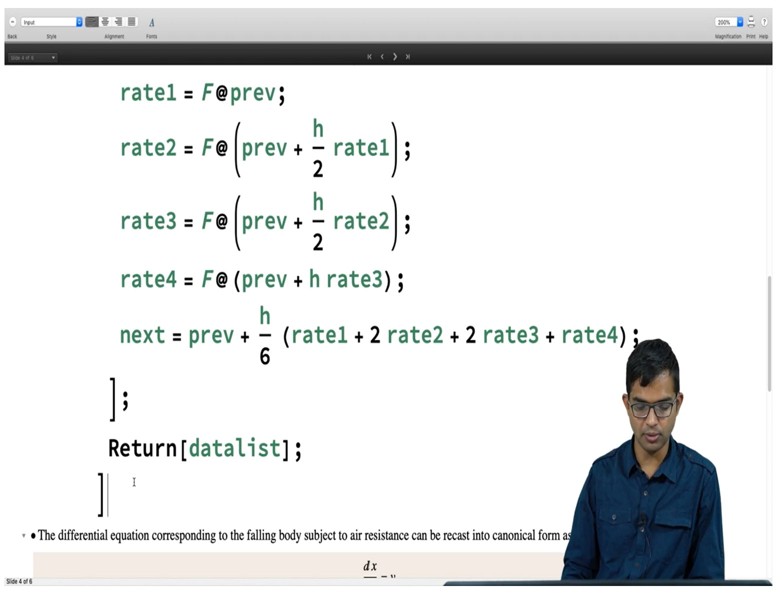

So now let us rework the whole thing using our powerful RK4 toolbox we have already created our own RK4 toolkit we have our own we have generated our own algorithm. So the algorithm is given to us and we have written our own program and then we have this Rk4 written now. All we have to do is copied on and the load it every time we have a new differential equation and then so many games can be played using that.

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- The differential equation corresponding to the falling body subject to air resistance can be recast into canonical form as:

$$\frac{dx}{dt} = v$$
$$\frac{dv}{dt} = \frac{-1}{x^2} \tag{21}$$
$$x(0) = 1$$
$$v(0) = v_0$$

- So in vector form we have:

$$X = \begin{pmatrix} t \\ x \\ v \end{pmatrix} \qquad F = \begin{pmatrix} 1 \\ v \\ \frac{-1}{x^2} \end{pmatrix} \tag{22}$$
$$\dot{X} = F$$

- So we proceed to define the functions and the initial vector:

```
rateFunc[{t_, x_, v_}] = {1, v, -1/x^2};
```

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- So we proceed to define the functions and the initial vector:

In[62]:=

```
rateFunc[{t_, x_, v_}] = {1, v, -1/x^2};
initial = {0, 1, √2};
solx[t_] = (3/2 √2 t + 1)^(2.0/3.0);
```

- Now we are ready to invoke the rk4 function:

```
data = rk4[rateFunc, initial, 4, 300];
ListPlot[data[[ ;; , 1 ;; 2]], Joined -> Tru
  PlotMarkers -> None, PlotRange -> Full];
Show[ListPlot[data[[ ;; , 1 ;; 2]], Joined
```

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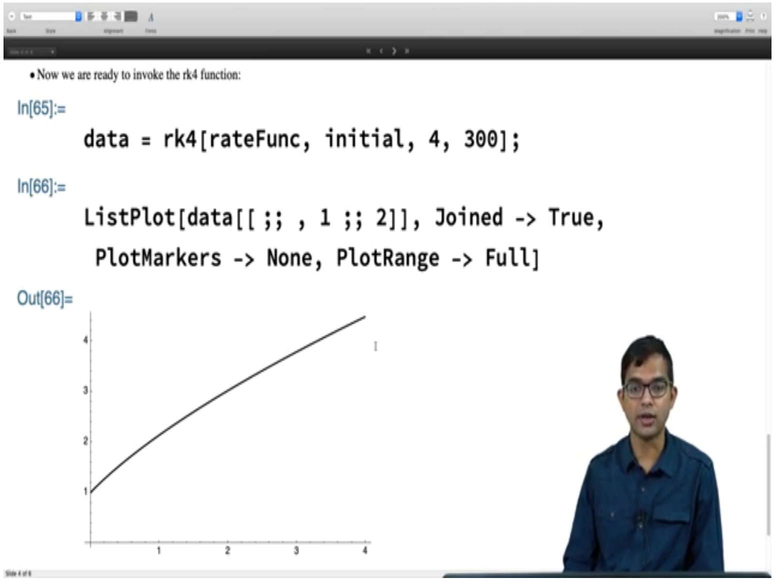

So we let us go ahead and load it and then we have to as always make these modifications then we have to bring our differential equation in to this canonical form. So $dx/dt = v$ we have already defined it and then dv/dt so it is 2 variable differential equation. It is so because you start with a second order differential equation you must convert into a first order differential equation involving more variables which you have already done.

So $dx/dt = v\ dv/dt = -1/x^2$. We no longer need to, you know, this trick where you write $d^2x/dt^2 = v\ dv/dx$ that is not the philosophy of this method. RK4 does not worry about you know the complexities of this integration. It is brute force it will know how to go ahead and numerically calculate this.

So we have data for $x(t)$ which will come out of this program. So in vector form we have $\dot{x} = F$ so I have this new F this capital X is ofcourse t, x and v. And as before we proceed we have to first write down this rate function as 1, v and $-1/x^2$ and initial I am going to take it I am going to tune it to be exactly at the critical velocity.

So homework for you would be to explore what happens when you change these velocities to different values of v_0. So then here I have so let me evaluate this run this and then so this is the syntax for generating data from the RK4 routine and do this and then let me plot this function.

So I have so this is the data which has come out from then numerical routine RK4 I have said that is the gold standard for differential equation solvers. It has just the right mix of you know optimization of the number of operations involve and also accuracy.

So if you want to get maximum accuracy using minimum number of operation RK4 is definitely one of the candidates for this, strong candidate.

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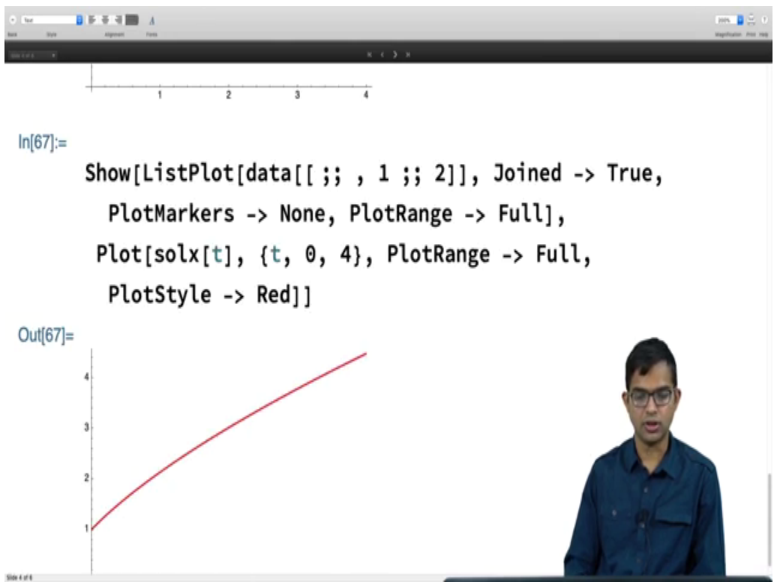

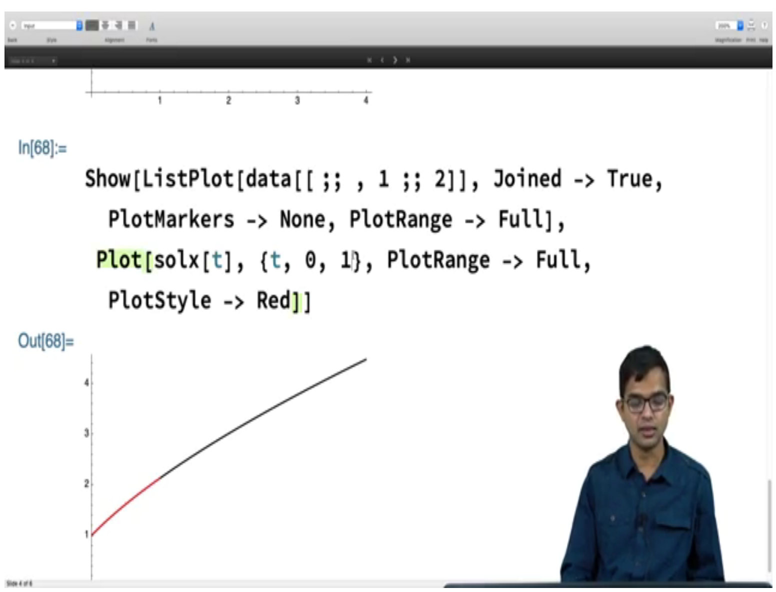

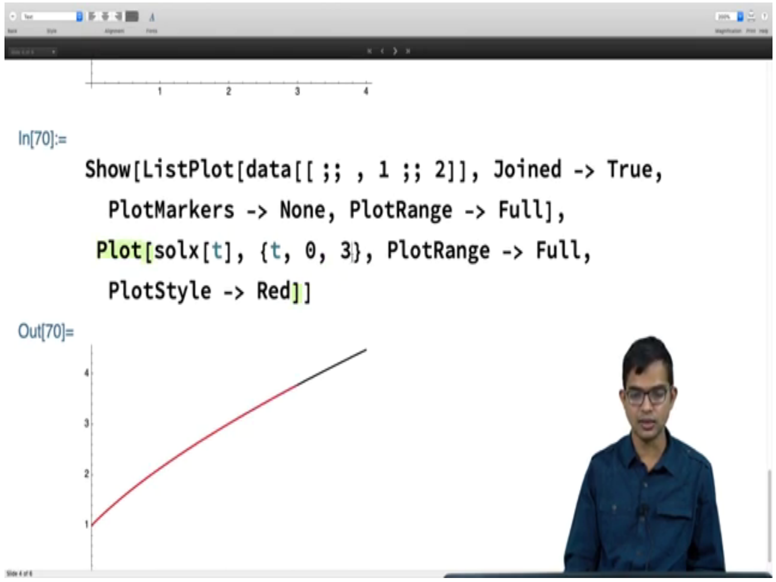

So let us go ahead and compare this so in fact the agreement is so good that the 2 curves one from the analytical and the other from this numerical approach are basically indistinguishable. So in order to check this we will reduce the time involved here, so let me only run it up to 1.

So then you see that it receive, red and black are sitting on top of each other then I will increase it to may be 2. Then you see that it is slowly inching forward but staying always on track and then 3 and then 4.

So this is a confirmation that our RK4 routine is working well it is also a confirmation that we have done the analytical calculation correctly. So this is the, at the core of the philosophy of this course. So now what we want to do we want to use this to see if we can we can stretch ourselves a little more we can be flex our muscles and attempt to solve problem which our analytical methods are unable to tackle.

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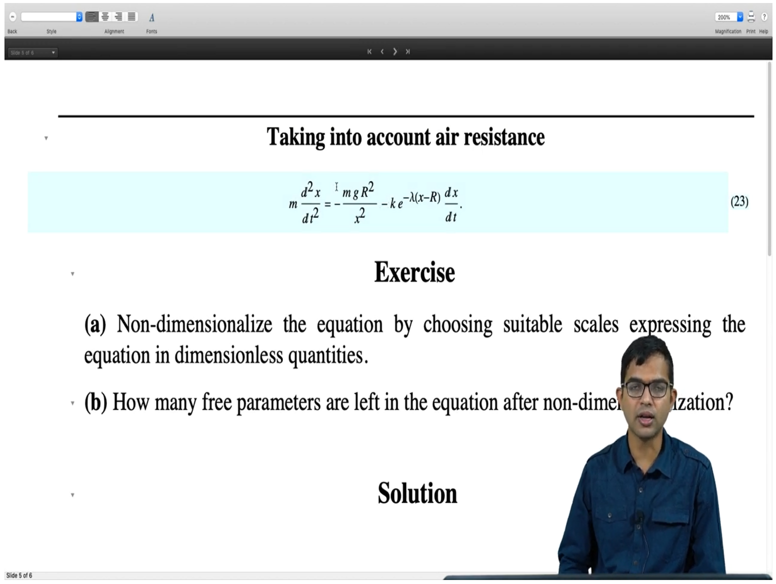

So one natural question that can come up is there could be air resistance so there is definitely air resistance. So if you are scientist in ISRO trying to actually work out the dynamics of a rocket being launched from the surface of the earth. This is a factor that you might want to consider and ofcourse that the atmosphere is finite it is not going to extend all the way till forever in the universe.

So one model I have just coped up this model I do not know whether this is the model but it is probably a reasonable model. One model is to imagine that there is exponential decay of this air resistance, so if you are very far away from the earth's surface the air resistance is minimum.

So I am going to model this as an exponential of minus lambda is some constant times $x - R$. so the closer you are to the earth, the larger is this, the magnitude of the air resistance. And I am also modeling this as being proportional to the speed and opposite in direction to the direction of motion. So I have $m\ d^2x/dt^2 = m\ g\ R^2/x^2$ like before with a minus sign ofcourse.

But also I have $-k\ e^{-\lambda(x-R)}\ dx/dt$, right. So the first step as always is to non-dimentionalize this differential equation. So once again pause your video, spend may be 5 minutes so this is a little more complicated than the previous differential equation, go ahead and non-dimentionalize. Okay, so I will show you my solution and hopefully yours and mine will agree, but they do not want to agree by the way.

It is possible for you to be right and for me to be right and for both of us to be different. So the thing is that you may work in some different units and I may work in different units, so the important thing is for you to be completely consistent. You work out your entire solution in the units that you have chosen and finally make sure that your results are reasonable, within your own units and only finally, you can compare against mine.

Right, so there is always a way to be right in multiple ways. These in particularly in when your differential equation involves lots of parameters and when it becomes more and more complicated there are multiple ways. And so there ofcourse there is some intuition which goes into choosing a more suitable set of scales.

So sometimes it is better to work in for example, when you have a problem involving resistor and LCR circuits. So you may choose one time constant which comes from L/R but you may use another time constant which comes and R and C. So depending upon which of these time constants is more relevant. It may be more appropriate to use one or the other depending upon the context.

But in the end, there is no real right or wrong in this. They are both are correct as long as or many multiple ways are correct as long as you are consistent.

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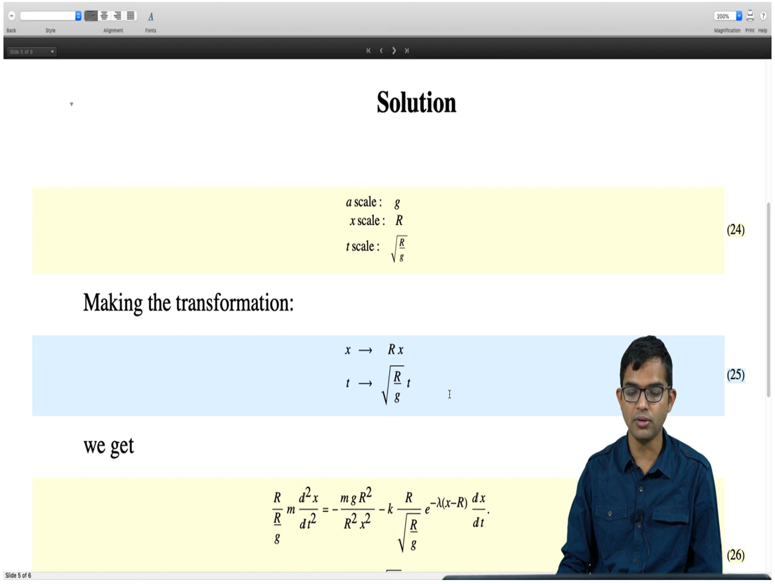

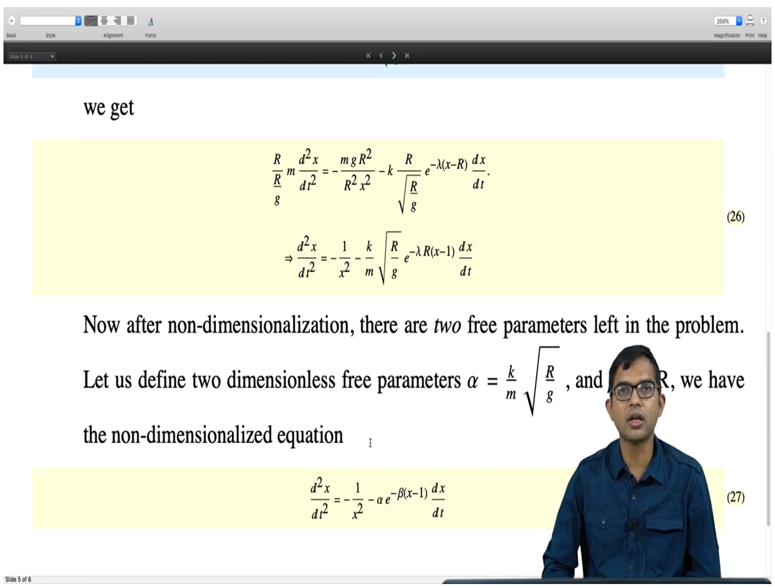

So my solution is the following. I choose the acceleration scale first because it is easy it is just g and then x scale once again is R I choose it to be R and then time scale is $\sqrt{R/g}$. If I use this, so there is also k siting here by the way. So let us it turns out that this is already enough. If I do this I replace x by $R\ x$ and t by $\sqrt{R/g}\ t$.

Now if I do this calculation carefully my differential equation becomes $d^2x/dt^2 = -1/x^2$. And this is exactly like what we had earlier, but now I have this slightly more messy term which is sitting there which is $-k/m\sqrt{R/g}\, e^{-\lambda R(x-1)}\, dx/dt$.

So now I see that it looks like I have a lot of free parameters left, but actually it is not so many parameters I can create a group of parameters combine them and call it as one parameter. So let me define two new parameters, one is I am going to call it $\alpha = k/m\sqrt{R/g}$ and then $\beta = \lambda\, R$.

So if I use these two parameters, my final differential equation is going to look like $d^2x/dt^2 = -\,1/x^2 - \alpha\, e^{-\beta\,(x-1)}\,dx/dt$. So here time and position both are non-dimensional quantities.

α is also a dimensionless quantity, β is also dimensionless. But you have some tuning possibilities here. So it is alpha comes from many of these constants put together in this manner. So now as you can see this is quite a complicated differential equation, so we started with something very simple and we used a very reasonable set of arguments to extend our problem, to the next natural question. And already you see that the complexity of a differential equation has increased manifold. And it is not going to be analytically solvable.

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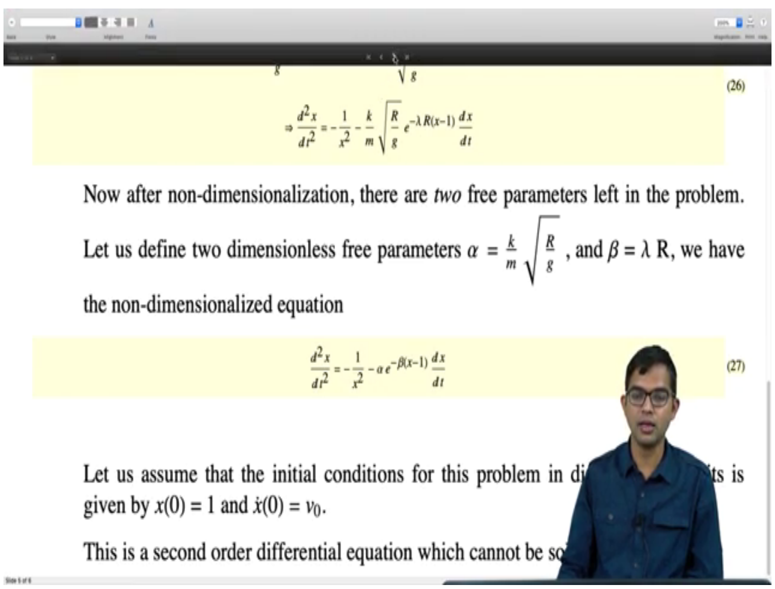

So let us just again once again assume $x(0) = 1$ and $\dot{x}(0) = v_0$. And this is a second order differential equation which cannot be solved exactly. So what we will do is, directly recast it with RK4.

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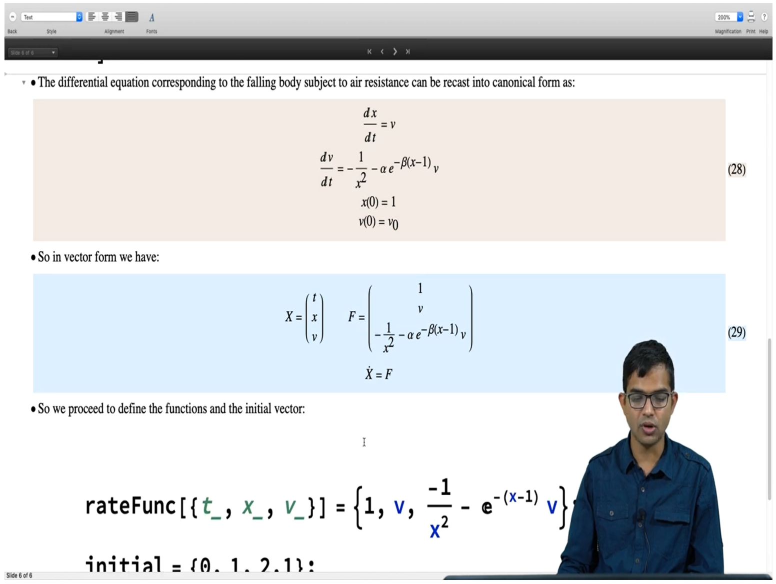
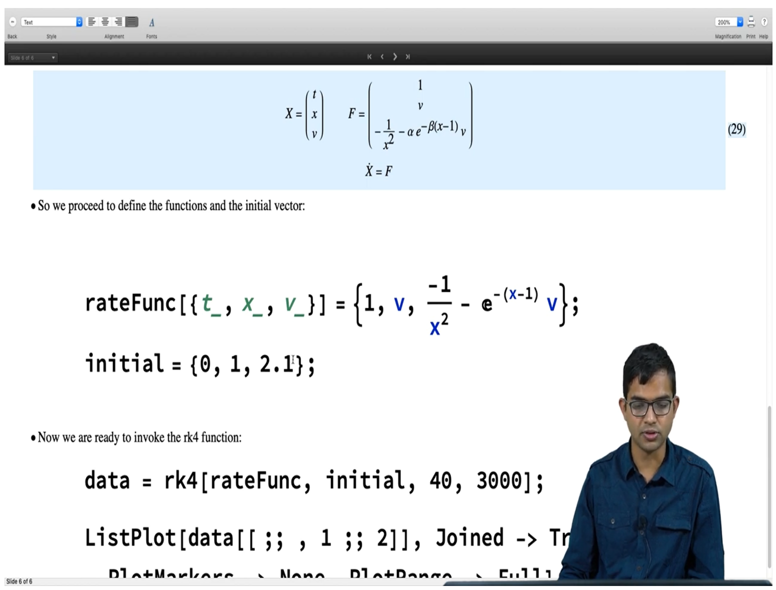
Numerical Solution with the RK4 Method

```
rk4[F_, X0_, tf_, nMax_] :=
 Module[{h, datalist, prev, rate1, rate2, rate3,
   rate4, next},
  h = (tf - X0[[1]]) / nMax // N;
  For[datalist = {X0},
   Length[datalist] ≤ nMax,
   AppendTo[datalist, next],
   prev = Last[datalist];
   rate1 = F@prev;
```


```
   prev = Last[datalist];
   rate1 = F@prev;
```


```
   rate2 = F@(prev + h/2 rate1);
   rate3 = F@(prev + h/2 rate2);
   rate4 = F@(prev + h rate3);
   next = prev + h/6 (rate1 + 2 rate2 + 2 rate3 + rate4)
  ];
  Return[datalist];
 ]
```





So I do not give you all the steps here. So once again I will load it I probably do not have to do it, because I have loaded it once. Now I have $dx/dt = v\; dv/dt = -1/x^2 - \alpha\, e^{-\beta(x-1)}\, v$, $x(0) = 1$, $v(0) = v_0$.

Compact form for this is to introduce this function capital F and then all I have to do is write down the rate function. And now I am choosing my initial to be 0 and then 1, so 0 and 1 are of course given and then I have some freedom for what value I choose for my initial velocity v_0. I can play with this, so I was I tried out a bunch of these. So let us look at what happen, so let me take this v naught to be very tiny.

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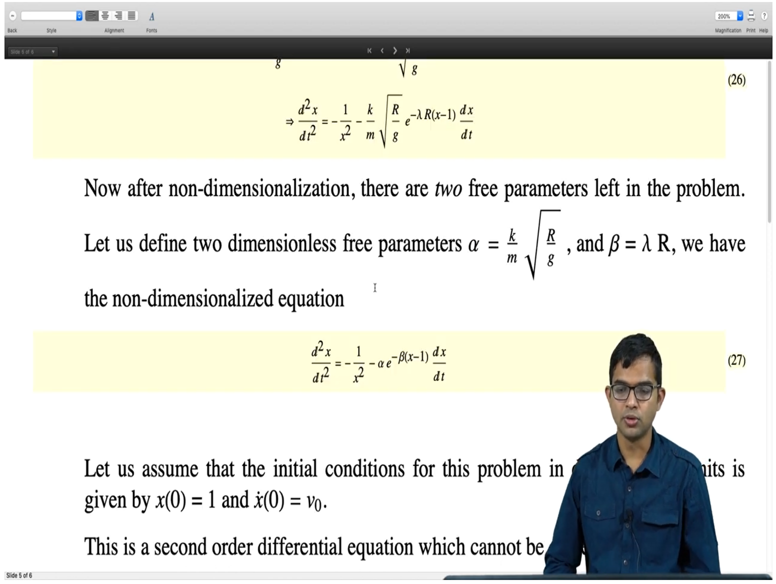

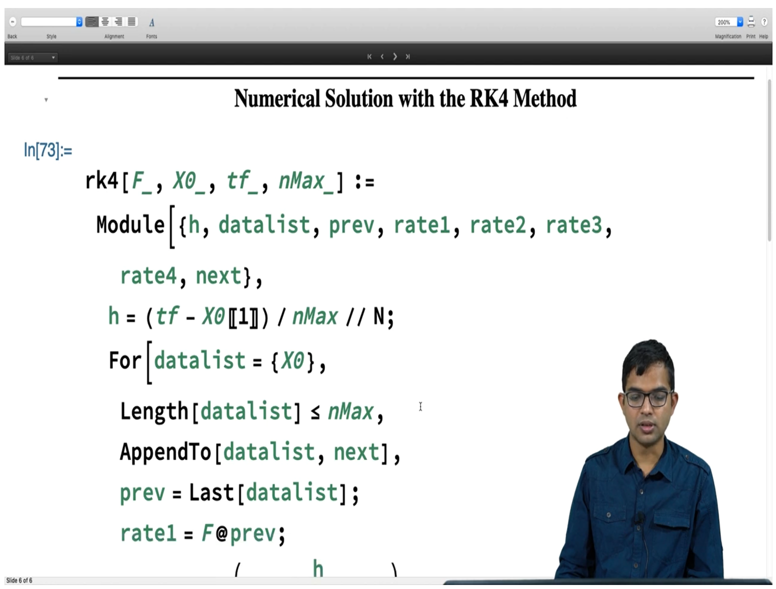

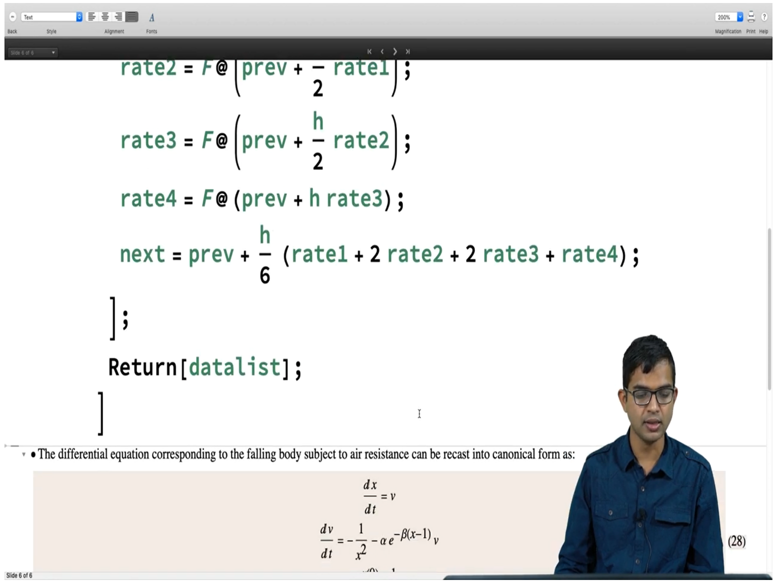
rate2 = F@(prev + h/2 rate1);
rate3 = F@(prev + h/2 rate2);
rate4 = F@(prev + h rate3);
next = prev + h/6 (rate1 + 2 rate2 + 2 rate3 + rate4);
];
Return[datalist];
]
• The differential equation corresponding to the falling body subject to air resistance can be recast into canonical form as:
dx/dt = v
dv/dt = - 1/x^2 - α e^{-β(x-1)} v
(28)
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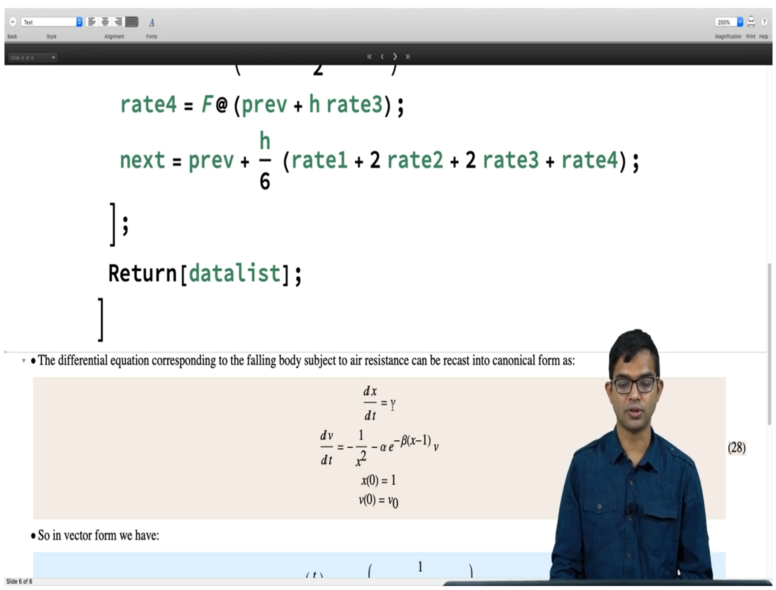
rate4 = F@(prev + h rate3);
next = prev + h/6 (rate1 + 2 rate2 + 2 rate3 + rate4);
];
Return[datalist];
]
• The differential equation corresponding to the falling body subject to air resistance can be recast into canonical form as:
dx/dt = v
dv/dt = - 1/x^2 - α e^{-β(x-1)} v
x(0) = 1
v(0) = v0
(28)
• So in vector form we have:
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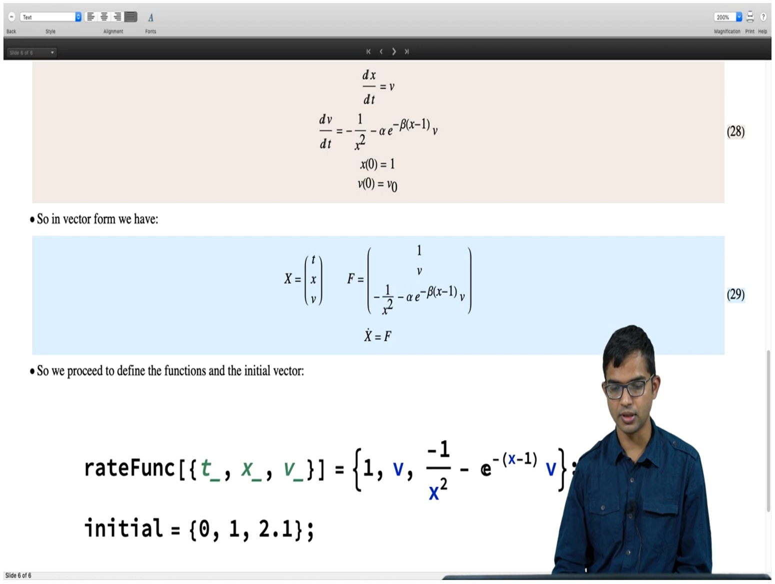

So look at what we have done, we have a differential equation now which is considerably more complicated. We started small, we wrote down initial differential equation which could be solved analytically at least as far as $v(x)$ was concern.

And then I introduced a natural next complication namely a frictional force which dies down as you go away from the surface of the earth. And suddenly we have a differential equation which is not solvable analytically.

Alright so we will directly go to RK4 with this and it is convenient to choose $x(0) = 1$, $\dot{x}(0) = v_0$. And I will just load my RK4 method again just to be safe but we have already done it before so it should be fine even otherwise. And then so now you see that the canonical form for this second order differential equation converted into first order is like here $dx/dt = v\ dv/dt$ has both these terms.

The first term is this $-1/x^2$ but also this exponential $dk * v$ is the drag force and then $x(0) = 1$, $v(0) = v_0$. So v_0 is the free parameter which I can play with in vector form you know I have this force function which I can define $\dot{x} = F$ or maybe I should not call it force, it is just $\dot{x} = F$ we have called it the function F. So then we have the rate function rather, so the function F is should be thought of as a rate function which is defined as a vector here.

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- So in vector form we have:

$$X = \begin{pmatrix} t \\ x \\ v \end{pmatrix} \qquad F = \begin{pmatrix} 1 \\ v \\ -\frac{1}{x^2} - \alpha\, e^{-\beta(x-1)}\, v \end{pmatrix} \tag{29}$$

$$\dot{X} = F$$

- So we proceed to define the functions and the initial vector:

In[75]:=

```
rateFunc[{t_, x_, v_}] = {1, v, -1/x^2 - e^(-(x-1)) v}
initial = {0, 1, 2.1};
```

Out[75]=

$$\left\{1,\ v,\ -e^{1-x}\, v - \frac{1}{x^2}\right\}$$

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$$\dot{X} = F$$

- So we proceed to define the functions and the initial vector:

In[84]:=

```
rateFunc[{t_, x_, v_}] = {1, v, -1/x^2 - e^(-(x-1)) v};
initial = {0, 1, 0.1};
```

- Now we are ready to invoke the rk4 function:

In[81]:=

```
data = rk4[rateFunc, initial , 40, 3000];
```

In[83]:=

```
ListPlot[data[[ ;; , 1 ;; 2]], Joined -> Tr
  PlotMarkers -> None, PlotRange -> Full]
```

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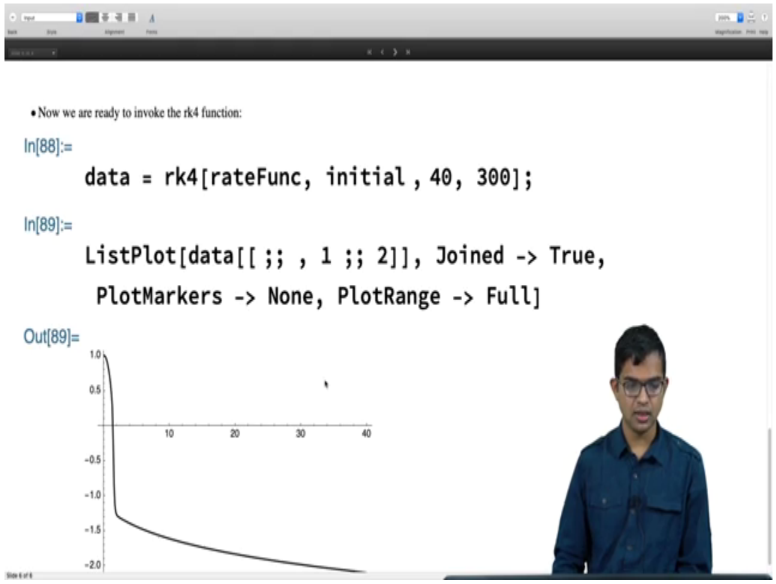

And then I will run this and then I have initial conditions, I have to define the initial conditions. So here I have some room for play. I have my initial other time $t = 0$ is where I start and $x = 1$ that is fixed because I started the surface of the earth. But I can pay with different speeds. I can ask myself what will happen if I send my rocket at various speeds and see if there is some critical value which I can extract purely numerically.

So let me start very small I will start with a very tiny speed 0.1 v_0. So I will then I can play with this, so rate, so then I have to generate this data numerical data, so let me do this I will run this, and then finally I will plot this. So it gives me data of x as a function of t, so may be 3000 is too much, so let me try to bring it down to just a 300.

So you see that beyond a certain value of time this is actually going down to 0 and any data beyond that time is not trust worthy. So you cannot it is not physical for x to go below 0, so you the main message to take away from here is that there is a finite time at which x is actually going down to 0. So it is not really so this speed is not an acceptable speed to send it at. So let us try to keep on increasing it.

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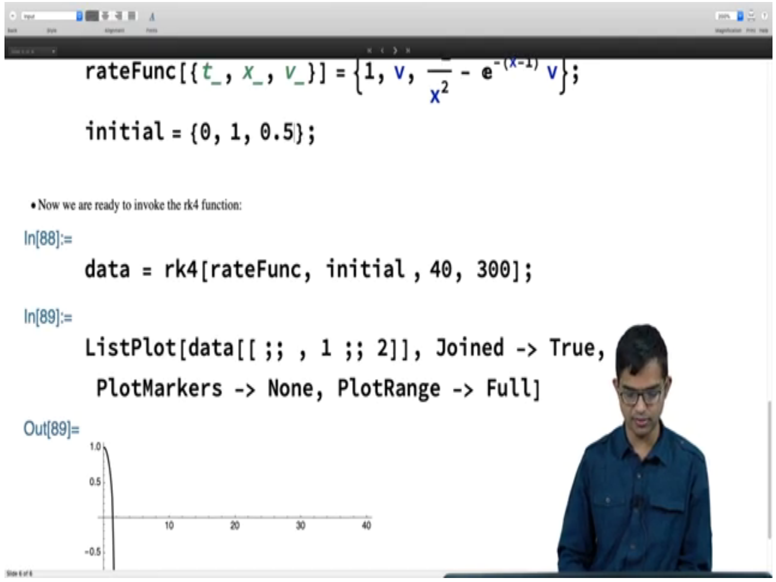

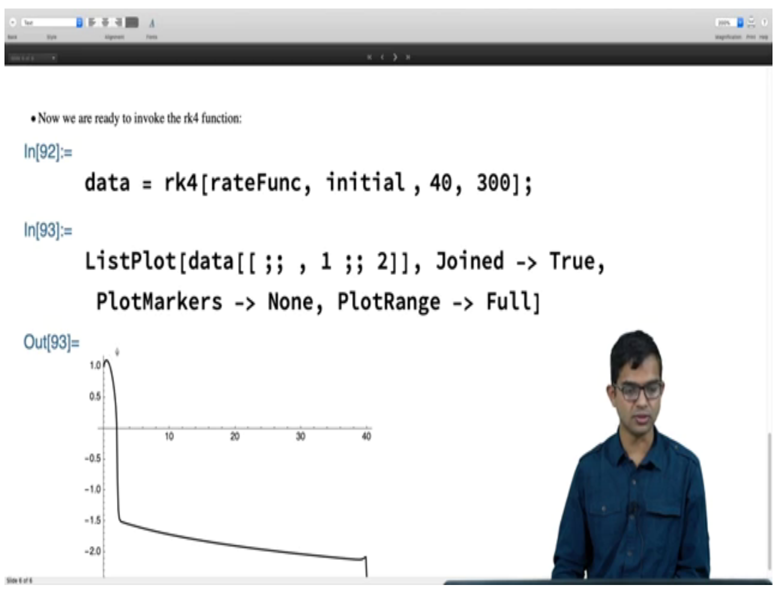

Suppose I send it at 0.5 then what happens? I run it again then I run all of this stuff again, then so it goes up and it is coming down. So in fact *x* cannot even go below 1, so it is you see that initially it goes up and then very quickly comes down. So may be 0.5 is also too small of a speed for the type of times that I am studying. So if I had studied much smaller times it would be possible, the other way it increase this is to go up.

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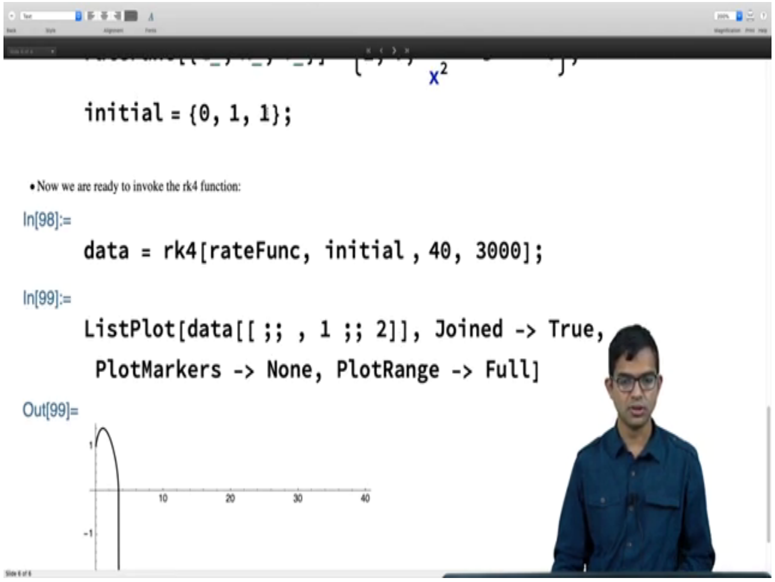

So let me go to a speed of 1, then what happens. If I have a speech of 1may be I need this also to increase. Oh, I see. So in fact I should be running this till 40 but this I should increase so this is the number 3000 there you go. So this works out alright.

And then if I plot this I see that even for a velocity of 1 it is coming down to 0 which is not acceptable so let me keep on increasing this if I make it 1.5 then what happens?

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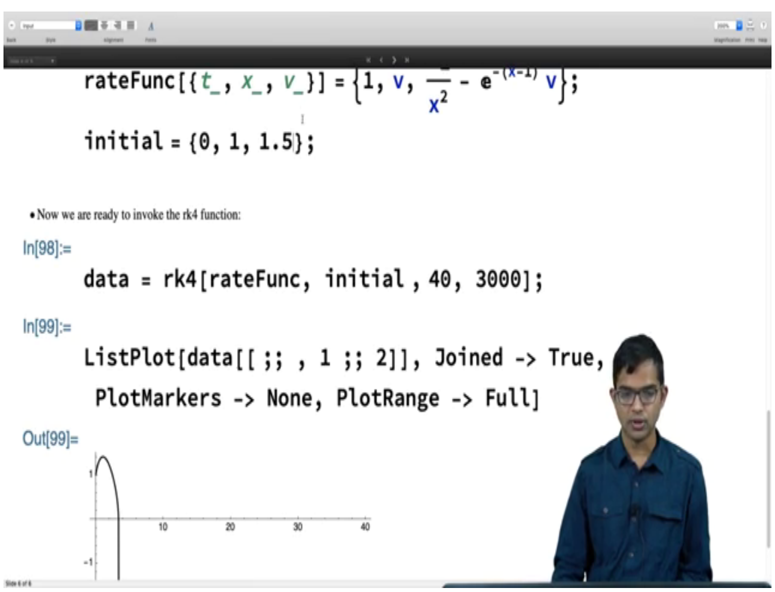

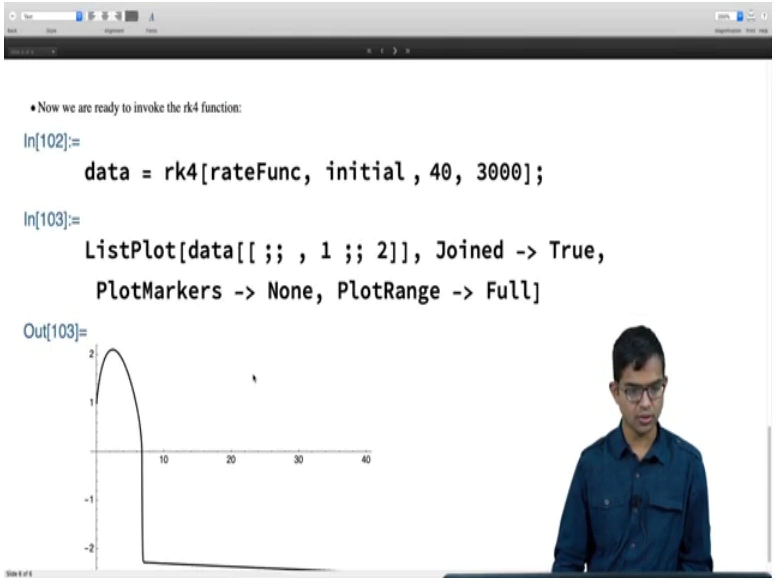

1.5 I run this then I run this then I plot this. So you see that even now there is a finite time at which it returns to the earth and may be that is not what you want. It is all experimental in nature what we are trying to do.

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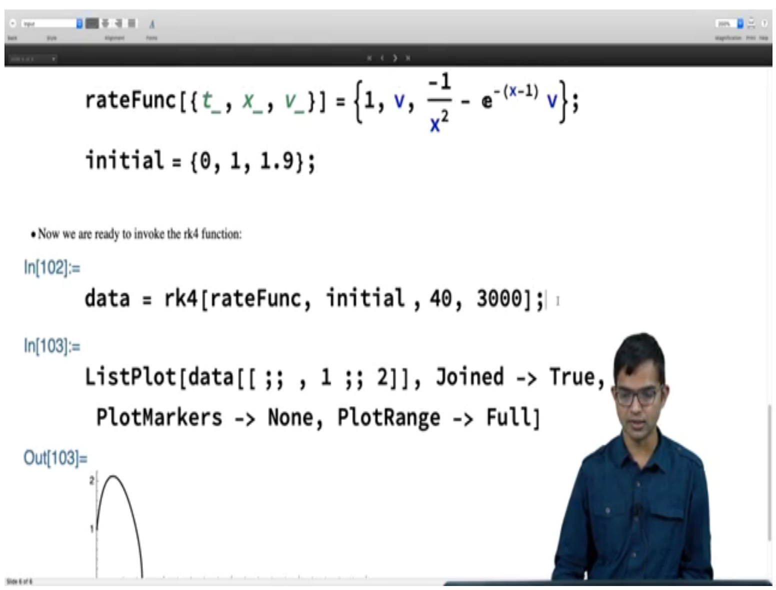

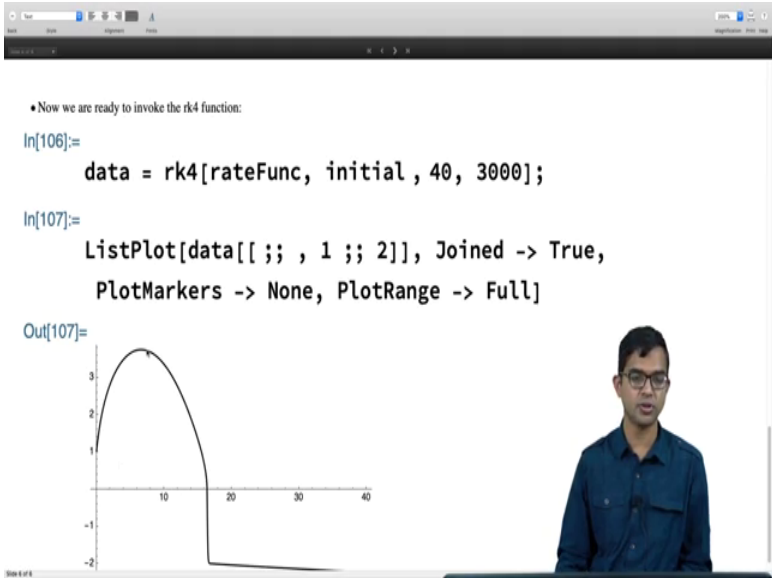

See if I take it to 1.9 then what happens? I have 1.9, something like this and there you go. So it is staying outside the surface of the earth for longer, but is it going to be always is it always going to return in a finite amount of time or is there some critical value?

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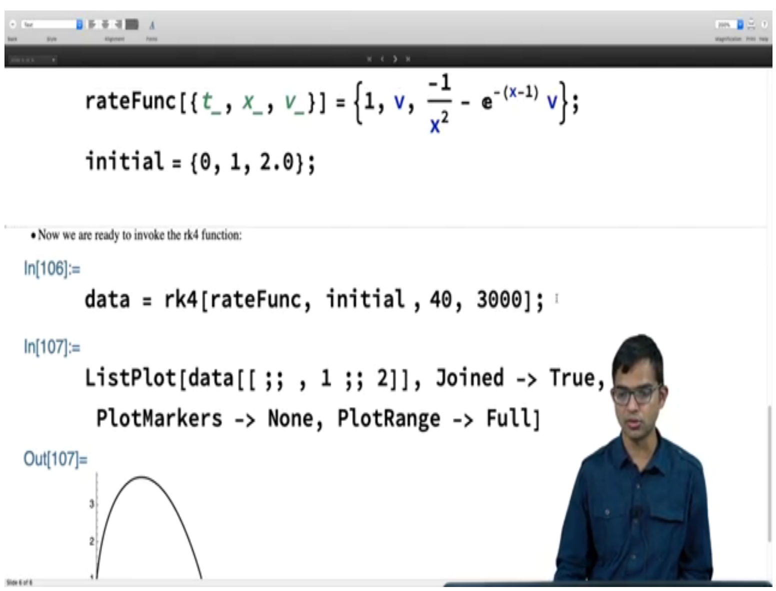

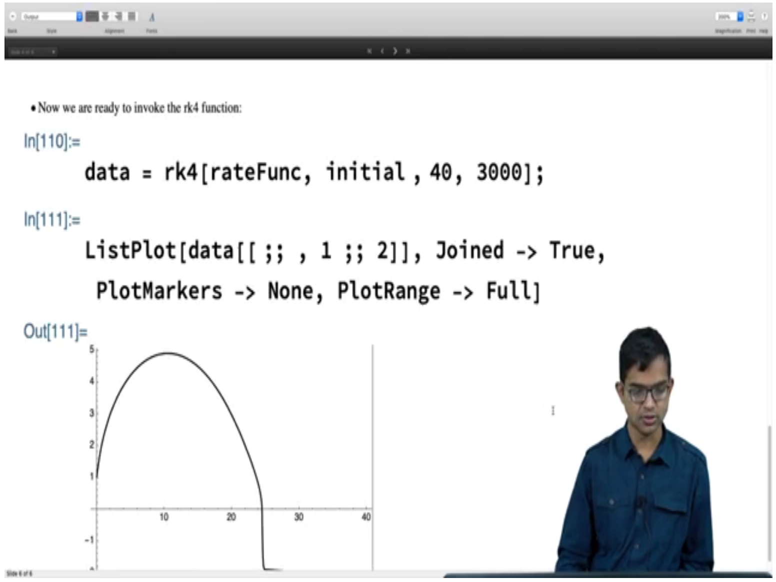

So let us keep on increasing, this is just totally experimental in nature what we are doing. It is just crude model and we are playing. And so now what happens? So I see that it is increasing, it is increasing, so, more and more.

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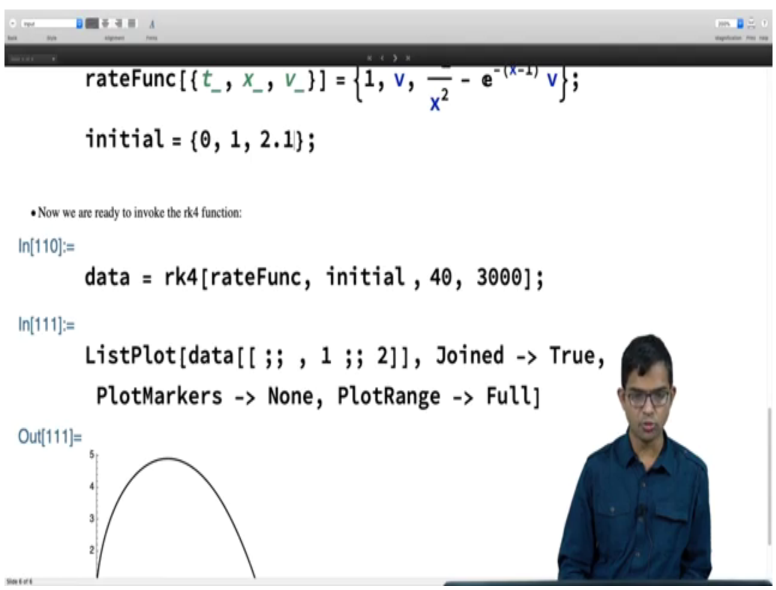

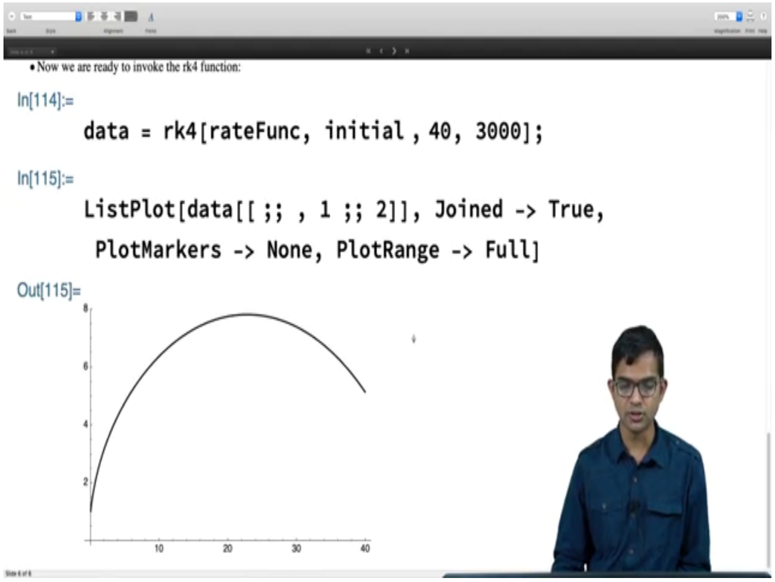

What about 2.1? 2.1 more increase I expect there you go. So you see that suddenly now 40 is not enough may be it is going to return for a longer times.

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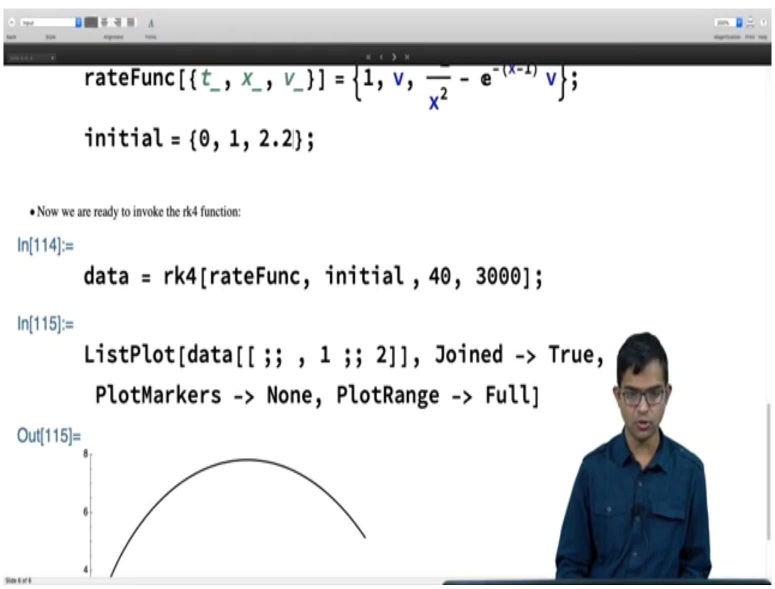

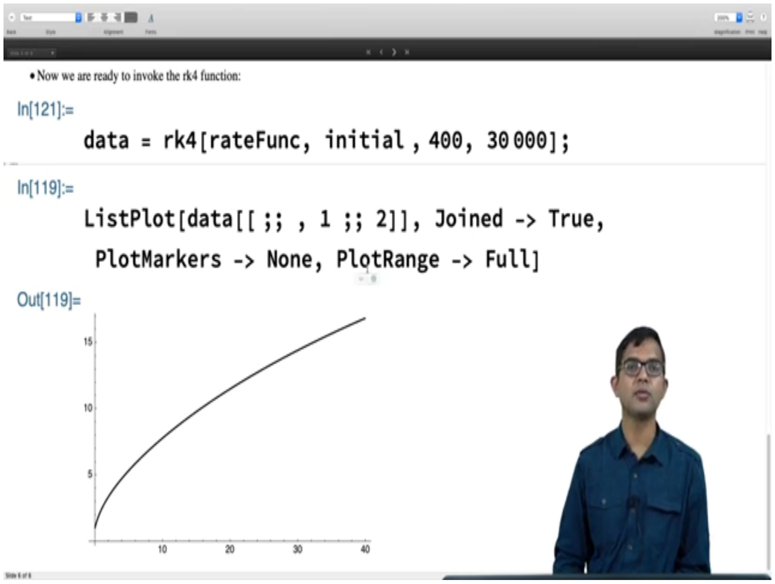

And then what about 2.2? 2.2 You see is suddenly it looks like it is not going to return at all. So I might at run it for much longer. And this is not enough so I will probably have to do, there you go. So I see that qualitatively something has changed here.

So my inference is that when v_0 is slightly more than 2.1, 2.2 there is a critical v_0 that we have crossed. Beyond which this particle is going to go away to run away to infinity. So this projectile will basically bulldoze its weight through the earth's air resistance. If this initial speed is greater than a critical value.

See now it is very different from the earlier critical value. So by the way I have to point out that here tacitly I have assumed $\alpha = 1$ and $\beta = 1$. So there is more exploration possible and that is going to be part of your homework. You must try out what happens for different values of α different values of β.

These are after all you know parameters which you can give some physical meaning to it. It has all these constants in your problem sitting together there is a k there is g there is m and all this. And you should find a way to play around with various α's and β's and then in fact also with various v_0.

And maybe a fun exercise could be to come up with a tabulated data of v_0 critical as a function of α and β. So I must point out that this the latest part of this problem is completely our own, we have we have created this problem we have built a model and we have managed to extract data and also to understand that from based on the intuition that we developed on a simple problem.

And it is potentially a research problem I do not know if study of this kind of a model has been carried out. So the main point here is that starting small and building in a very-very step by step solid way. We have managed to create a completely newer avenue for research which could be of real interests to I do not know some space scientist for example or with certainly of interest for us as academics.

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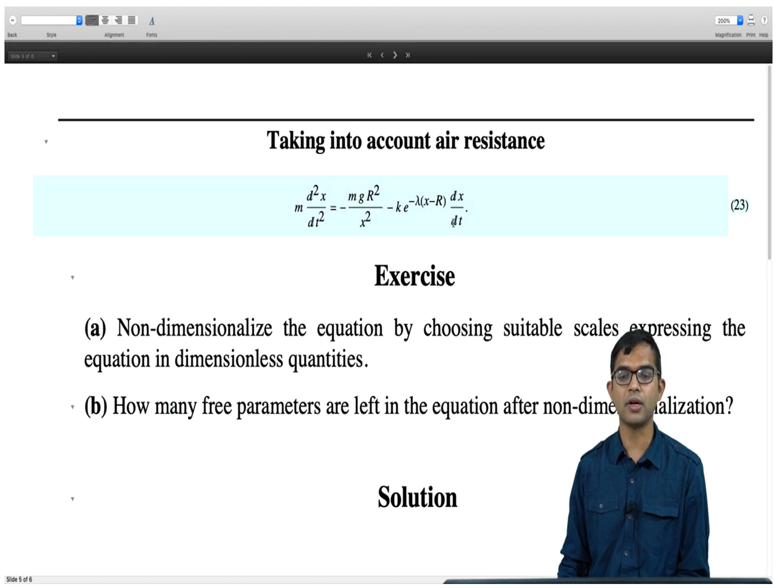

So in the spirit of the philosophy of this course, I urge you to try out more variants of this kind of a model one possibility which immediately comes to mind is to see if you can consider a different type of model where is nothing sancrosanct said about a proportionality to the speed, the air resistance does not have to be directly proportional to the speed, it could be quadratic in nature.

You could now that you have this toolbox you can simply change dx/dt to $(dx/dt)^2$. And ofcourse as you have seen last time just doing a simple change like this you will have to rework the whole thing.

All the scales of the problem will change, you have to do non-dimentionalization in a different way. And you may get very different results. Although last time we saw that for the problem of dropping a ball with air resistance there is final qualitative solution was the same in the sense that it would also hit a terminal velocity and so on.

But this is one immediate thought that comes to mind, but you can also have more complicated thing. You could replace this exponential dk by some other softer kind of dk or even harder kind of dk or you can think of a box, a model the atmosphere as being valid up to a certain distance but making it like a hard cut off. You have a minus k times a heaviside step function suitably written down so that it goes to 0 beyond a point.

And see if that is going to change quality, or if that becomes analytically tractable, maybe, right. So these are all games for you to try out and play and we will come up with more such examples and other ideas in a next module. Thank you, thank you for now.